Ex1: Solve the system of the linear equations.

$$3Z - Y - 5 = 0$$

$$5X + 2Z - 17 = 0$$

$$3X - Z + 2Y = 4$$

Solution:

Step 1: Transfer the linear equations to matrix form:

Rearrange the equations:

$$-Y + 3Z = 5$$

$$5X + 2Z = 17$$

$$3X + 2Y - Z = 4$$

$$\begin{bmatrix} 0 & -1 & 3 \\ 5 & 0 & 2 \\ 3 & 2 & -1 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 5 \\ 17 \\ 4 \end{bmatrix}$$

Step 2:

Change the location of row 2 and 1:

$$\begin{bmatrix} 5 & 0 & 2 \\ 0 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 17 \\ 5 \\ 4 \end{bmatrix}$$

Multiply row 1 by (1/5):

$$\begin{bmatrix} 1 & 0 & 0.4 \\ 0 & -1 & 3 \\ 3 & 2 & -1 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3.4 \\ 5 \\ 4 \end{bmatrix}$$

Multiply row 1 by (-3) and add it to row 3:

$$\begin{bmatrix} 1 & 0 & 0.4 \\ 0 & -1 & 3 \\ 0 & 2 & -2.2 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3.4 \\ 5 \\ -6.2 \end{bmatrix}$$

Multiply row 2 by (-1):

$$\begin{bmatrix} 1 & 0 & 0.4 \\ 0 & 1 & -3 \\ 0 & 2 & -2.2 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3.4 \\ -5 \\ -6.2 \end{bmatrix}$$

Multiply row 2 by (-2) and add it to row 3:

$$\begin{bmatrix} 1 & 0 & 0.4 \\ 0 & 1 & -3 \\ 0 & 0 & 3.8 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3.4 \\ -5 \\ 3.8 \end{bmatrix}$$

Multiply row 1 by (1/3.8):

$$\begin{bmatrix} 1 & 0 & 0.4 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3.4 \\ -5 \\ 1 \end{bmatrix}$$

Multiply row 3 by (3) and add it to row 2:

Also multiply row 3 by (-0.4) and add it to row 1:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

Step 3:

$$X = 3$$

$$Y = -2$$

Ex2: Solve the system of the linear equations.

$$3X + 2Y = 4$$

$$2X - Y = -9$$

$$X + 3Y = 13$$

Solution:

Step 1: Transfer the linear equations to matrix form:

$$\begin{bmatrix} 3 & 2 \\ 2 & -1 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 4 \\ -9 \\ 13 \end{bmatrix}$$

Step 2:

Multiply row 1 by (1/3):

$$\begin{bmatrix} 1 & 0.67 \\ 2 & -1 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1.34 \\ -9 \\ 13 \end{bmatrix}$$

Multiply row 1 by (-2) and add it to row 2:

$$\begin{bmatrix} 1 & 0.67 \\ 0 & -2.34 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1.34 \\ -11.67 \\ 13 \end{bmatrix}$$

Multiply row 1 by (-1) and add it to row 3:

$$\begin{bmatrix} 1 & 0.67 \\ 0 & -2.34 \\ 0 & 2.33 \end{bmatrix} \times \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1.34 \\ -11.67 \\ 11.67 \end{bmatrix}$$

Add row 2 to row 3:

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Computer Techniques Eng. Dept.
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Assist Lec. Anmar F. Ibadi
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$$\begin{bmatrix} 1 & 0.67 \\ 0 & -2.34 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1.34 \\ -11.67 \\ 0 \end{bmatrix}$$

Multiply row 2 by (-1/2.33):

$$\begin{bmatrix} 1 & 0.67 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1.34 \\ 5 \\ 0 \end{bmatrix}$$

All elements of row 3 have to have the value of zero since only variable are

unknown.

$$\begin{bmatrix} 1 & 0.67 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 1.34 \\ 5 \\ 0 \end{bmatrix}$$

Multiply row 2 by (-0.67) and add it to row 1:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix}$$

Step 3: Solve the matrix equation and as follows:

$X = -2$

$Y = 5$

Ex3: Solve the system of the linear equations.

$$4X - Y - Z = 1$$

$$-X + 2Y - Z = 1$$

$$-X - Y + 6.5Z = 1$$

Solution:

Step 1: Transfer the linear equations to matrix form:

$$\begin{bmatrix} 4 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 6.5 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Step 2:

Multiply row 1 by (1/4):

$$\begin{bmatrix} 1 & -0.25 & -0.25 \\ -1 & 2 & -1 \\ -1 & -1 & 6.5 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.25 \\ 1 \\ 1 \end{bmatrix}$$

Multiply row 1 by (1) and add it to row 2 and 3:

$$\begin{bmatrix} 1 & -0.25 & -0.25 \\ 0 & 1.75 & -1.25 \\ 0 & -1.25 & 6.25 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.25 \\ 1.25 \\ 1.25 \end{bmatrix}$$

Multiply row 2 by (1/1.75):

$$\begin{bmatrix} 1 & -0.25 & -0.25 \\ 0 & 1 & -0.714 \\ 0 & -1.25 & 6.25 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.714 \\ 1.25 \end{bmatrix}$$

Multiply row 2 by (1.25) and add it to row 3:

$$\begin{bmatrix} 1 & -0.25 & -0.25 \\ 0 & 1 & -0.714 \\ 0 & 0 & 5.35 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.714 \\ 2.143 \end{bmatrix}$$

Multiply row 3 by (1/5.358):

$$\begin{bmatrix} 1 & -0.25 & -0.25 \\ 0 & 1 & -0.714 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.714 \\ 0.4 \end{bmatrix}$$

Multiply row 2 by (0.25) and add it to row 1:

$$\begin{bmatrix} 1 & 0 & -0.43 \\ 0 & 1 & -0.714 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.43 \\ 0.714 \\ 0.4 \end{bmatrix}$$

Multiply row 3 by (0.714) and add it to row 2:

Also multiply row 3 by (0.43) and add it to row 1:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.6 \\ 1 \\ 0.4 \end{bmatrix}$$

Step 3: Solve the matrix equation and as follows:

$X = 0.6$

$Y = 1$

$Z = 0.4$

Homework

Solve the system of the linear equations.

$$3Y - 1 = Z - 2X$$

$$3X + 3Z = 4Y - 1$$

$$2X + Z = Y - 3$$

$$3X + Y = 4 + 2Z$$