Al- Mustaqbal University

College of Science

Medical Physics Department

First Stage

Mechanics

Lecture Fife: Noninertial Reference System

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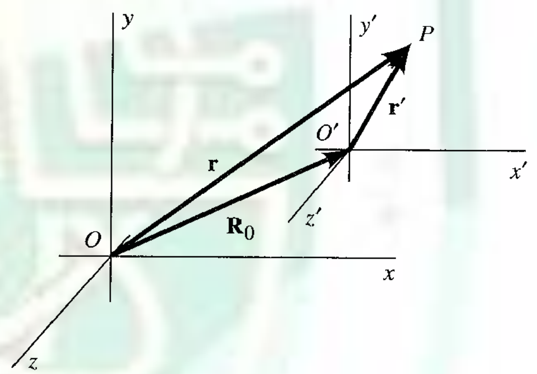
5.1. Accelerated Coordinate Systems:

Recall: Uniformly moving reference frames (e.g. those considered at 'rest' or moving with constant velocity in a straight line) are called *inertial reference frames*.

Sometimes it is necessary, to employ a coordinate system that is **not** *inertial*.

Let us first consider the case of a coordinate system that undergoes **pure translation**.

Assume $Oxyz$ are the primary fixed coordinate axes, and $O'x'y'z'$ are the moving axes. In the case of pure translation, the respective axes Ox and $O'x'$, and so on, remain parallel.



The **position vector** of a particle P is denoted by $\mathbf{r}$ in the fixed system and by $\mathbf{r}'$ in the moving system. The displacement OO' of the moving origin is denoted by $\mathbf{R}_0$. Thus, from the triangle $OO'P$, we have

$$\mathbf{r} = \mathbf{R}_0 + \mathbf{r}' \qquad (1)$$

Taking the first and second time derivatives gives

$$\mathbf{v} = \mathbf{V}_0 + \mathbf{v}' \qquad (2)$$

$$\mathbf{a} = \mathbf{A}_0 + \mathbf{a}' \qquad (3)$$

in which $\mathbf{V}_0$ and $\mathbf{A}_0$ are, respectively, the velocity and acceleration of the moving system, and $\mathbf{v}'$ and $\mathbf{a}'$ are the velocity and acceleration of the particle in the moving system.

If the moving system is **not accelerating**, i.e. it is also **inertial**, so that $\mathbf{A}_0 = 0$, then

$$\mathbf{a} = \mathbf{a}'$$

In this case we cannot specify a unique coordinate system, because Newton's laws will be the same in both systems.

For example, Newton's second law in fixed system $\mathbf{F} = m\mathbf{a}$ becomes $\mathbf{F}' = m\mathbf{a}'$ in the moving system.

On the other hand if the moving system is **accelerating**, then Newton's second law becomes

$$\mathbf{F} = m\mathbf{A}_0 + m\mathbf{a}'$$

or

$$\mathbf{F} - m\mathbf{A}_0 = \mathbf{F}' \qquad (4)$$

where $(-m\mathbf{A}_0)$ is known as the *inertial term* or *inertial force.* Such "force" is not due to interactions with other bodies; rather, it happens as a result of the acceleration of the reference system.

5.2. Rotating Coordinate Systems

In this section, we show how velocities, accelerations, and forces transform between an inertial frame of reference and a noninertial one that is **rotating**.

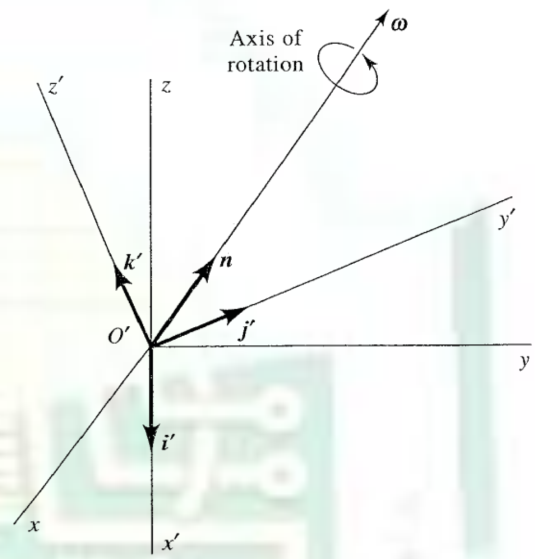
Assume that the axes of the both coordinate systems have a common origin. Let the rotation of the rotated system takes place about some specific axis of rotation, whose direction is designated by a unit vector, **n**.

The *angular velocity* of the rotating system then is;

$$\omega = \omega \mathbf{n}$$

The direction of the velocity vector is given by the right-hand rule.

The position of any point *P* in space can be designated by the vector **r** in the fixed system and by the vector **r'** in the rotating system.



Because the coordinate axes of the two systems have the same origin, these vectors are equal, that is,

$$\mathbf{r} = \mathbf{r}' \qquad (5)$$

or;

$$\mathbf{i}x + \mathbf{j}y + \mathbf{k}z = \mathbf{i}'x' + \mathbf{j}'y' + \mathbf{k}'z'$$

When we differentiate with respect to time to find the velocity, we must keep in mind the fact that the unit vectors **i'**, **j'**, and **k'** are **not constant**. Thus, we can write the velocity vector **v** in the fixed system as;

$$\mathbf{v} = \mathbf{v}' + x'\frac{di'}{dt} + y'\frac{dj'}{dt} + z'\frac{dk'}{dt} \qquad (6)$$

Where $\mathbf{v}'$ is the velocity in the rotating system.
From the definition of the cross product, we can write;

$$\frac{d\,\mathbf{i}'}{dt} = \omega \times i' \quad , \quad \frac{d\,\mathbf{j}'}{dt} = \omega \times j' \quad \text{and} \quad \frac{d\,\mathbf{k}'}{dt} = \omega \times k' \quad .$$

Hence;

$$x'\frac{d\mathbf{i}'}{dt} + y'\frac{d\mathbf{j}'}{dt} + z'\frac{d\mathbf{k}'}{dt} = x'(\boldsymbol{\omega} \times \mathbf{i}') + y'(\boldsymbol{\omega} \times \mathbf{j}') + z'(\boldsymbol{\omega} \times \mathbf{k}')$$
$$= \boldsymbol{\omega} \times \mathbf{r}'$$

This is the velocity of P due to rotation of the coordinate system. Accordingly, Eq. (6) can be rewritten as

$$\mathbf{v} = \mathbf{v}' + \boldsymbol{\omega} \times \mathbf{r}' \qquad (7)$$

Taking the first time derivatives gives the acceleration in the fixed system in terms of the position, velocity, and acceleration in the rotating system;

$$\mathbf{a} = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') \qquad (8)$$

If the moved system is undergoing both translation and rotation, the general equations for transforming from a fixed system to a moving and rotating system will be:

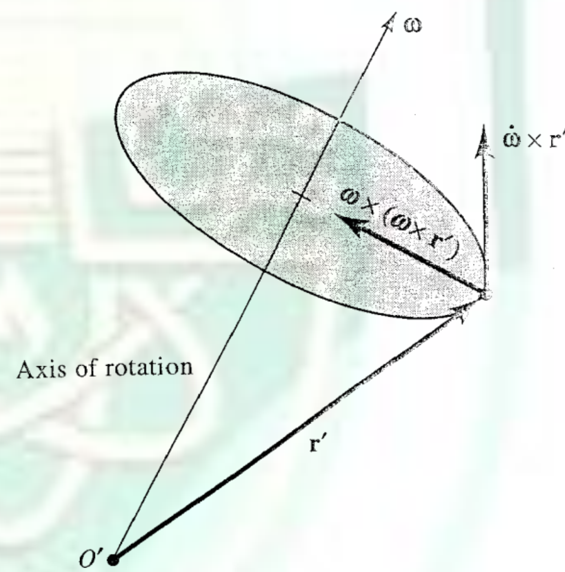
$$\mathbf{v} = \mathbf{v}' + \boldsymbol{\omega} \times \mathbf{r}' + \mathbf{V}_0 \qquad (9)$$

And;

$$\mathbf{a} = \mathbf{a}' + \dot{\boldsymbol{\omega}} \times \mathbf{r}' + 2\boldsymbol{\omega} \times \mathbf{v}' + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}') + \mathbf{A}_0 \qquad (10)$$

- The term $2\omega \times \mathbf{v}'$ is known as *the Coriolis acceleration*, which appears whenever a particle moves in a rotating coordinate system except when the velocity $\mathbf{v}'$ is parallel to the axis of rotation.

- The term $\omega \times (\omega \times \mathbf{r}')$ is called *the centripetal acceleration*. which is the result of the particle being carried around a circular path in the rotating system. It is always **directed toward the axis of rotation** and is **perpendicular to the axis** as shown in the figure.



- The term $\dot{\omega} \times \mathbf{r}'$ is called *the transverse acceleration*, because it is perpendicular to the position vector $\mathbf{r}'$. It appears whenever the rotating system has an angular acceleration, i.e. **if the angular velocity vector is changing in either magnitude or direction, or both.**

5.3. Dynamics of a Particle in a Rotating System:

It is well known that the equation of motion of a particle in an *inertial frame* of reference is ;

$$\mathbf{F} = m\mathbf{a}$$

where $\mathbf{F}$ is the sum of all real, physical forces acting on the particle.

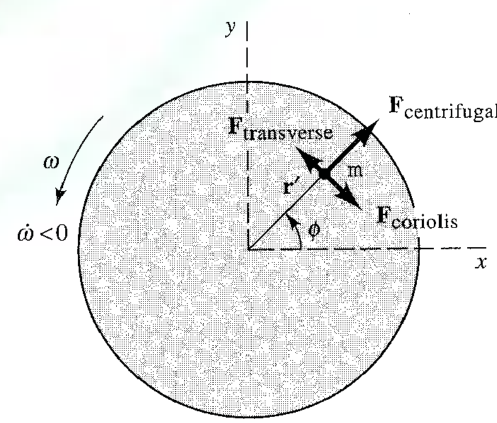
According to Eq.(10) , we can write the equation of motion of a particle in a *noninertial frame* of reference as ;

$$\mathbf{F} - m\dot{\boldsymbol{\omega}}\times\mathbf{r}' - 2m\boldsymbol{\omega}\times\mathbf{v}' - m\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}') - m\mathbf{A}_0 = m\mathbf{a}' \qquad (11)$$

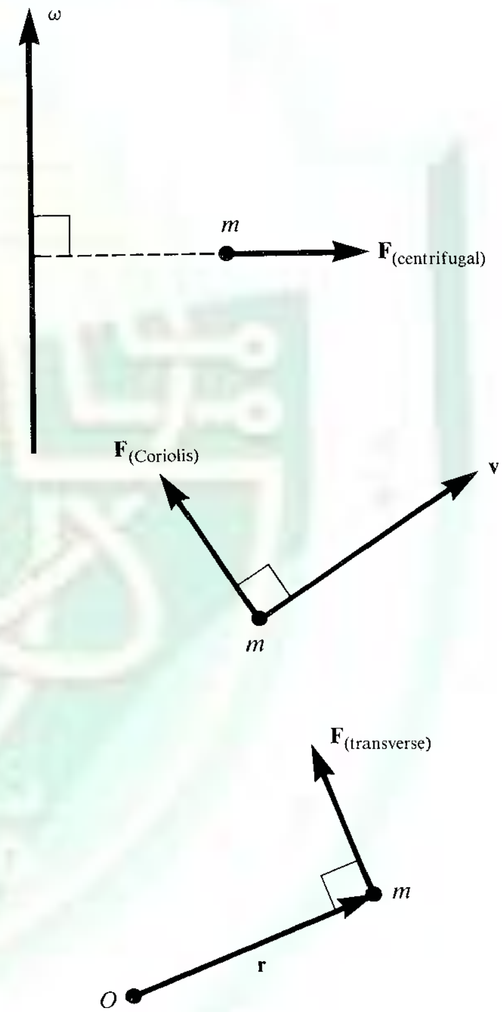
All inertial forces have names corresponding to their relevant accelerations. Thus;

- The force $-m\dot{\boldsymbol{\omega}}\times\mathbf{r}'$ is called *the transverse force*, because it is **perpendicular** to the position vector $\mathbf{r}'$. It is present **only** if there is an angular acceleration (or deceleration) of the rotating coordinate system.

- The force $-2m\boldsymbol{\omega}\times\mathbf{v}'$ is *the Coriolis force*, which appears whenever a particle moves in a rotating coordinate system. Its direction is always **perpendicular** to $\mathbf{v}'$, thus *it seems to deflect the moving particle at right angles to its direction of motion.*



❑ The force $-m\omega\times(\omega\times \mathbf{r}')$ is *the centrifugal force*, which is the result of the particle being carried around a circular path in the rotating system. It is directed **outward away** from the axis of rotation and is **perpendicular** to that axis. If $\mathbf{r}'$ is perpendicular to ω, the magnitude of the centrifugal force is $m\mathbf{r}'\omega^2$.



A no inertial observer in an accelerated frame of reference must include all, or some, of these inertial forces along with the real forces $\mathbf{F}$ to calculate the correct motion of the particle.

In other words, such an observer writes the fundamental equation of motion as;

$$\mathbf{F}' = m\mathbf{a}'$$

in which the sum of the vector forces $\mathbf{F}'$ acting on the particle is given by

$$\mathbf{F}' = \mathbf{F}_{physical} + \mathbf{F}'_{trans} + \mathbf{F}'_{Cor} + \mathbf{F}'_{cel1trif} - m\mathbf{A}_0$$

$\mathbf{F}$ (or $\mathbf{F}_{physical}$) forces are the only forces that a no inertial observer claims are actually acting upon the particle .