



Al-Mustaqbal University

College of Science

Artificial Intelligence Sciences Department

Lecture 3

Computer Organization and Logic Design

M.S.C. Fatima Ayad

Numbers Systems :-

1. The types of number system A-

Decimal Number System :-

This system is composed of 10 numbers or symbols, these 10 symbols are:

0 1 2 3 4 5 6 7 8 9

These symbols are called digits.

The decimal system, also called base 10 system, because it has 10 digits which is a naturally result of the fact that man has 10 fingers.

B- Binary Number System

In this system there are only two symbols or possible digit values , 0 or 1 . Even so , this base-2 system.

C- Octal Number System

This system is composed of 8 numbers or symbols:

0 1 2 3 4 5 6 7

This is a base -8 system.

D- Hexa- Decimal System

**This system is composed of 16 numbers or symbols
(digit):**

0 1 2 3 4 5 6 7 8 9 A B C D E F

It is a base – 16 systems

2. Representation of numbers

1) Decimal :-

$$(124)_{10} = 4 \times 10^0 + 2 \times 10^1 + 1 \times 10^2$$

$$(252.512)_{10} = 2 \times 10^0 + 5 \times 10^1 + 2 \times 10^2 + 5 \times 10^{-1} + 1 \times 10^{-2} + 2 \times 10^{-3}$$

2) Binary :-

$$(1011101)_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 1 \times 2^4 + 0 \times 2^5 + 1 \times 2^6$$
$$= (93)_{10}$$

$$(101.11)_2 = 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^{-1} + 1 \times 2^{-2}$$
$$= (5.75)_{10}$$

3) Octal :-

$$(537)_8 = 7 \times 8^0 + 3 \times 8^1 + 5 \times 8^2$$
$$= (351)_{10}$$

4) Hexa- Decimal :-

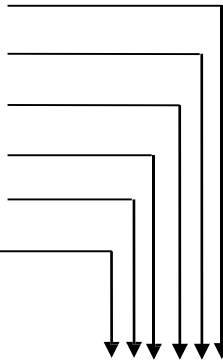
$$(A01B)_{16} = 11 \times 16^0 + 1 \times 16^1 + 0 \times 16^2 + 10 \times 16^3$$
$$= (40987)_{10}$$

1. Convert between the types of numbers systems

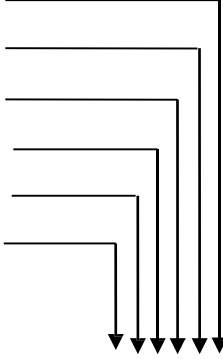
any base-to-decimal conversion :-
just use the definition given above.

Decimal-to-binary :-
divide decimal value by 2 (the base) until the value is 0

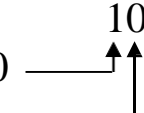
example: convert the following decimal numbers to the equivalent binary numbers (36 , 39.5).

$$\begin{array}{lcl} 36/2 = 18 & r=0 & \text{---} \\ 18/2 = 9 & r=0 & \text{---} \\ 9/2 = 4 & r=1 & \text{---} \\ 4/2 = 2 & r=0 & \text{---} \\ 2/2 = 1 & r=0 & \text{---} \\ 1/2 = 0 & r=1 & \text{---} \end{array}$$


36 (base 10) = 100100

$$\begin{array}{lcl} 39/2 = 19 & r=1 & \text{---} \\ 19/2 = 9 & r=1 & \text{---} \\ 9/2 = 4 & r=1 & \text{---} \\ 4/2 = 2 & r=0 & \text{---} \\ 2/2 = 1 & r=0 & \text{---} \\ 1/2 = 0 & r=1 & \text{---} \end{array}$$


39 (base 10) == 10011

$$\begin{array}{lcl} 0.5 \times 2 = 1.0 & \text{---} & \uparrow \\ 0 \times 2 = 0 & \text{---} & \uparrow \end{array}$$


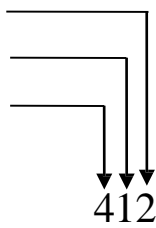
10

The binary equivalent of $(39.5)_{10}$ is $(100111.10)_2$

Decimal-to-Octal :-

A decimal integer can be converted to octal by using the same repeated-division method that we used in the decimal-to-binary conversion, but with a division of 8 instead of 2.

Example: convert the following decimal number to equivalent octal number $(266)_{10}$ & $(20.75)_{10}$

$$\begin{array}{ll} 266/8 = 33 & r=2 \\ 33/8 = 4 & r=1 \\ 4/8 = 0 & r=4 \end{array}$$

$$(266)_{10} = 412_8$$

$$\begin{array}{ll} 20/8 = 2 & r=4 \\ 2/8 = 0 & r=2 \end{array} \qquad 0.75 \times 8 = 6.0$$

The equivalent octal number is $(24.6)_8$

Octal-to-binary :-

The conversion from octal to binary is performed by converting each octal digit to its 3-bit binary equivalent. The eight possible digits are converted as indicated in the following table:

Octal digit	0	1	2	3	4	5	6	7
Binary digit	000	001	010	011	100	101	110	111

Example: convert the following octal number to it's equivalent binary number $(472)_8$

$$\begin{array}{ccc} 4 & 7 & 2 \\ 100 & 111 & 010 \end{array}$$

The equivalent binary number is $(100111010)_2$

Binary-to-octal :-

1. group into 3's starting at least significant symbol (if the number of bits is not evenly divisible by 3, then add 0's at the most significant end)

2. write 1 octal digit for each group

example:

100 010 111 (binary)

4 2 7 (octal)

10 101 110 (binary)

2 5 6 (octal)

example:-

convert $(177)_{10}$ to its 8-bit binary equivalent by first converting to octal.

Solution:-

$$177/8 = 22 + \text{remainder of } 1$$

$$22/8 = 2 + \text{remainder of } 6$$

$$2/8 = 0 + \text{remainder of } 2$$

$$(177)_{10} = (261)_8$$

$$\begin{array}{ccc} 2 & 6 & 1 \\ 010 & 110 & 001 \end{array}$$

$$(177)_{10} = (261)_8 = (010110001)_2$$

Tutorials:-

- 1- Convert $(641)_8$ to decimal (Ans. 369).
- 2- Convert $(146)_{10}$ to octal then from octal to binary (Ans. 222 and 010010010).
- 3- Convert $(10011101)_2$ to octal (Ans. 235).
- 4- Write the next three numbers in this octal counting sequence: 624, 625, 626,,,
- 5- Convert $(975)_{10}$ to binary by first converting to octal (Ans. 1111001111).
- 6- Convert binary 1010111011 to decimal by first converting to octal (Ans. 699).

Decimal-to-hex :-

This conversion can be done using repeated division by 16.

Example: convert $(423)_{10}$ to hex.

$$\begin{array}{rcl} 423/16 = 26 & r=7 & \downarrow \\ 26/16 = 1 & r=10 & \downarrow \\ 1/16 = 0 & r=1 & \downarrow \\ & & 1A7 \end{array}$$

Hex-to-binary :-

Each hex digit is converted to it's 4-bit binary equivalent as show in the table below :

Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Binary	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

example:

3 9 C 8
0011 1001 1100 1000

Binary-to-hex :-

This conversion is just the reverse of the hexa-to-binary conversion process.

example:

1001 1110 0111 0000

9 E 7 0

1 1111 1010 0011

1 F A 3

Tutorials:-

- 1- Convert $(24CE)_{16}$ to decimal (Ans. 9422).
- 2- Convert $(3117)_{10}$ to hex, then from hex to binary (Ans. C2D and 110000101101)
- 3- Convert $(1001011110110101)_2$ to hex (Ans 97B5).
- 4- Write the next four numbers in this hex counting sequence: E9A, E9B, E9C, E9D,,,,
- 5- Convert $(3527)_8$ to hex (Ans. $(757)_{16}$).

Binary-coded-decimal code :-

If each digit of a decimal number is represented by its binary equivalent, this produces a code called binary-coded-decimal (BCD). Since a decimal digit can be as large as 9, 4-bits are required to code each digit. The table below shows each decimal digit and its binary equivalent.

Decimal	0	1	2	3	4	5	6	7	8	9
BCD	0000	0001	0010	0011	0100	0101	0110	0111	1000	1001

example: The BCD of decimal 874

8 7 4
1000 0111 0100

Excess-3-code :-

It is performed in the same manner as BCD except that 3 is added to each decimal digit before encoding it in binary. The following table shows this code.

Decimal	0	1	2	3	4	5	6	7	8	9
Ex-3-code	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100

Gray code :-

The Gray code belongs to a class of codes called minimum-change codes, in which only from one step to the next. The following table shows this code.

Decimal	0	1	2	3	4	5	6	7	8	9
Gray-code	0000	0001	0011	0010	0110	0111	0101	0100	1100	1101