Department of Biomedical Engineering
Electric circuit 1 / First stage
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Lecture Five

CIRCUIT THEOREMS

5 INTRODUCTION

Analyzing circuits with Kirchhoff's laws preserves the original configuration but becomes computationally tedious for complex circuits. As circuit complexity has grown, engineers have developed simplification methods like Thevenin's and Norton's theorems, which apply to linear circuits. This lecture introduces circuit linearity, superposition, source transformation, and maximum power transfer, and concludes with applications in source modeling and resistance measurement

5.1 LINEARITY PROPERTY

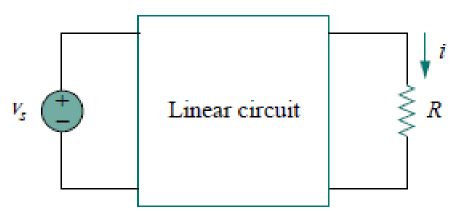
Linearity is the property of an element describing a linear relationship between cause and effect. Although the property applies to many circuit elements, we shall limit its applicability to resistors in this lecture.

For a resistor, for example, Ohm's law relates the input i to the output v,

$$v = iR$$

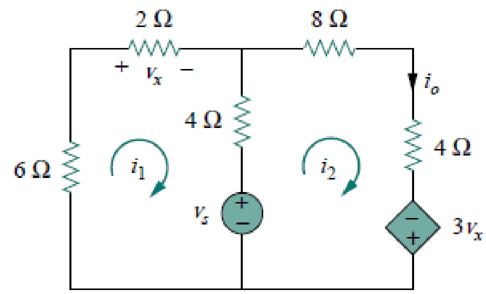
If the current is increased by a constant k, then the voltage increases correspondingly by k, that is,

$$kiR = kv$$



A linear circuit consists of only linear elements, linear dependent sources, and independent sources.

Example (1): For the circuit in the figure below, find i_o when v_s = 12 V and v_s = 24 V



Solution

Applying KVL to the two loops, we obtain

$12i_1 - 4i_2 + vs = 0$ Eq.(1)

$- 4i_1 + 16i_2 - 3v_x - v_s = 0$ Eq.(2)

But $v_x = 2i_1$, Equation (2) becomes

$-10i_1 + 16i_2 - v_s = 0$ Eq.(3)

Adding Eqs. (1) and (3) yields

$2i_1 + 12i_2 = 0 \Rightarrow i_1 = - 6i_2$

Substituting this in Eq. (1), we get

$-76i_2 + v_s = 0 \quad \Rightarrow \quad i_2 = \frac{v_s}{76}$

When v_s = 12 V,

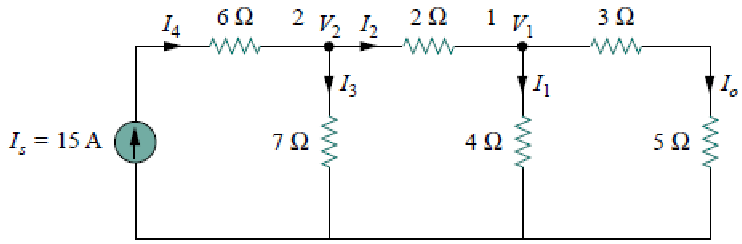
$i_o = i_2 = \frac{12}{76}$ A

When vs = 24 V,

$i_o = i_2 = \frac{24}{76}$ A

showing that when the source value is doubled, i_o doubles.

Example (2): Assume I_o = 1 A and use linearity to find the actual value of I_o in the following circuit.



Solution

If I_o = 1 A, then $V_1 = (3+5)I_o$ = 8 V and $I_1 = \frac{v_1}{4} = \frac{8}{4} = 2$ A. Applying KCL at node 1 gives

$I_2 = I_1 + I_o = 3$ A

$V_2 = V_1 + 2I_2 = 8 + 6 = 14$ V,

$I_3 = \frac{v_2}{7} = 2$ A

Applying KCL at node 2 gives

$I_4 = I_3 + I_2 = 5$ A

Therefore, $Is = 5$ A. This shows that assuming $I_o = 1$ gives $Is = 5$ A; the actual source current of 15 A will give $I_o = 3$ A as the actual value.

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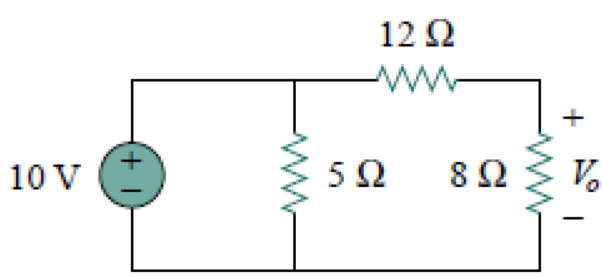
Assume that Vo = 1 V and use linearity to calculate the actual value of Vo in the following circuit.



Answer: 4 V.

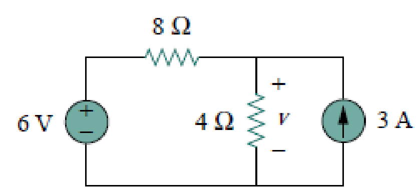
5.2 SUPERPOSITION

The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.

We apply the superposition principle in three steps:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Example (1): Use the superposition theorem to find v in the following circuit.

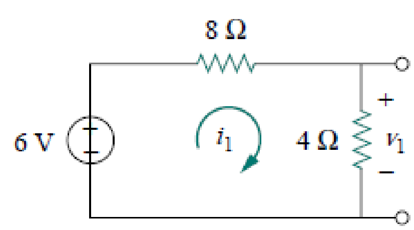


Solution:

Since there are two sources, let

$v = v_1 + v_2$

where $v1$ and $v2$ are the contributions due to the 6-V voltage source and the 3-A current source, respectively. To obtain $v1$, we set the current source to zero, as shown in the figure below. Applying KVL to the loop that gives



$12i_1 - 6 = 0 \;\Rightarrow\; i_1 = 0.5$ A

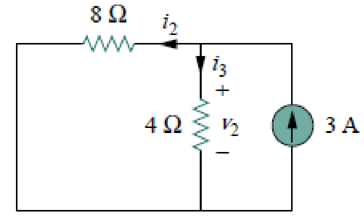
Thus,

$v_1 = 4i_1 = 2$ V

We may also use voltage division to get $v1$ by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

To get $v2$, we set the voltage source to zero, as in th figure below. Using current division



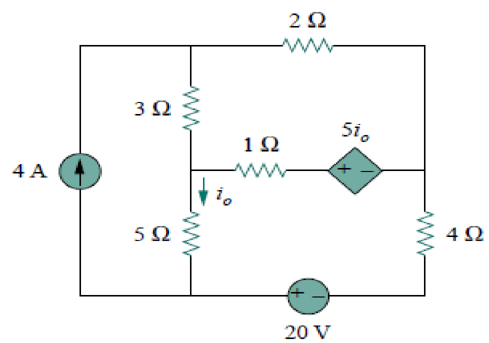
$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

Hence, $v_2 = 4i_3 = 8$ V

And we find

$v = v_1 + v_2 = 2 + 8 = 10$ V

Example (2): Find i_o in the following circuit using superposition



Solution:

The circuit in the figure involves a dependent source, which must be left intact. We let $i_0 = i_0' + i_0''$ *Eq.(1)*

where i_0' and i_0'' are due to the 4-A current source and 20-V voltage source, respectively. To obtain i_0', we turn off the 20-V source so that we have the circuit in Figure below. We apply mesh analysis to obtain i_0'.

For loop 1,

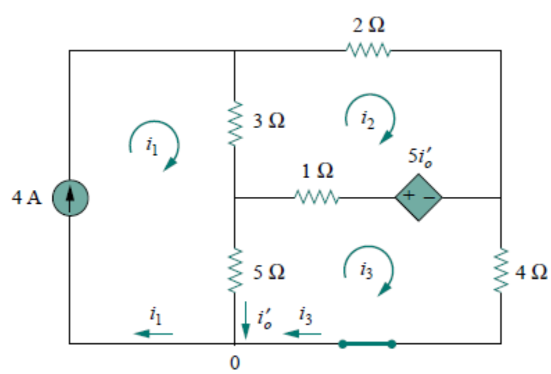
$i_1 = 4$ A *Eq.(2)*

For loop 2,

$-3i_1 + 6i_2 - 1i_3 - 5i_0' = 0$ *Eq.(3)*

For loop 3,

$-5i_1 - 1i_2 + 10i_3 + 5i_0' = 0$ *Eq.(4)*



But at node 0,

$i_3 = i_1 - i_0' = 4 - i_0'$ *Eq.(5)*

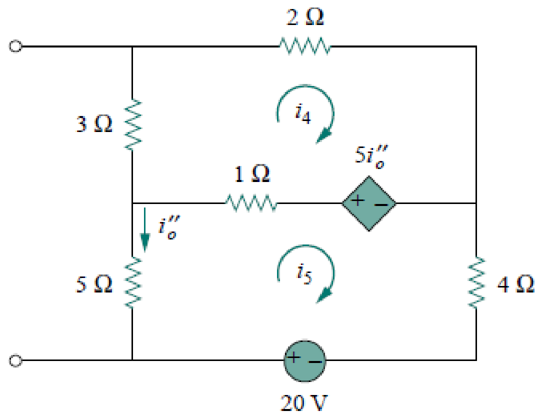
Substituting Eqs. (2) and (5) into Eqs. (3) and (4) give two simultaneous equations

$3i_2 - 2i_0' = 8$ *Eq.(6)*

$i_2 + 5i_0' = 20$ *Eq.(7)*

which can be solved to get $i_0' = \frac{52}{17}\ A$ *Eq.(8)*

To obtain i_0'' , we turn off the 4-A current source so that the circuit becomes that shown in the figure below. For loop 4, KVL gives



$6i_4 - i_5 - 5i_0'' = 0$ *Eq.(9)*

and for loop 5,

$-i_4 + 10i_5 - 20 + 5i_0'' = 0$ *Eq.(10)*

But $i_5 = -i_0''$*. Substituting this in Eqs. (9) and (10) give*

$6i_4 - 4i_0'' = 0$ *Eq.(11)*

$i_4 + 5i_0'' = -20$ *Eq.(12)*

which we solve to get

$$i_o'' = -\frac{60}{17}\ \text{A}$$

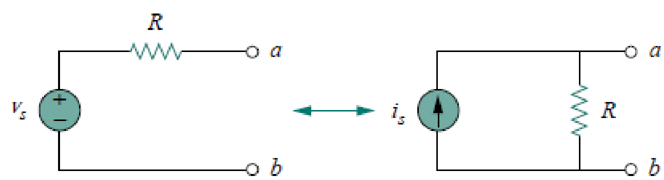
Now substituting Eqs. (8) and (13) into Eq. (1) gives

$$i_o = -\frac{8}{17} = -0.4706\ \text{A}$$

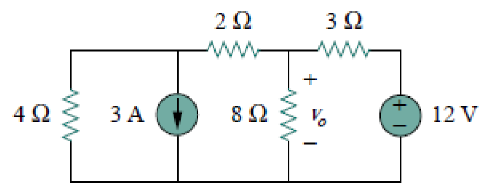
5.3 SOURCE TRANSFORMATION

We have noticed that series-parallel combination and wye-delta transformation help simplify circuits. *Source transformation* is another tool for simplifying circuits. Basic to these tools is the concept of *equivalence*. *We recall that an equivalent circuit is one whose v-i characteristics are identical to the original circuit*

It is therefore expedient in circuit analysis to be able to substitute a voltage source in series with a resistor for a current source in parallel with a resistor, or vice versa, as shown in the Figure below. Either substitution is known as a *source transformation*.



Example (1): *Use source transformation to find v_o in the following circuit.*

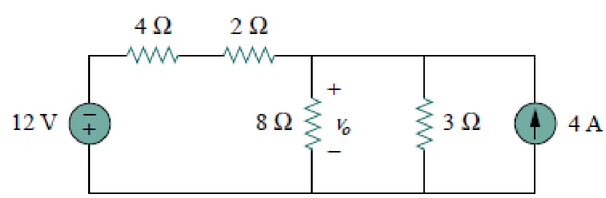


We first transform the current and voltage sources to obtain the circuit in Fig. (a). Combining the 4 Ω *and* 2Ω resistors in series and transforming the 12-V voltage source gives us Fig. (b). We now combine the 3 Ω and 6 Ω resistors in parallel to get 2 Ω. We also combine the 2-A and 4-A current sources to get a 2-A source. Thus, by repeatedly applying source transformations, we obtain the circuit in Fig. (c).

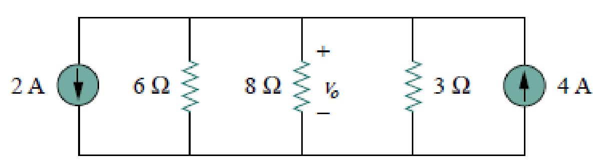
We use the current division in Fig. (c). to get

$$i = \frac{2}{2+8}(2) = 0.4$$

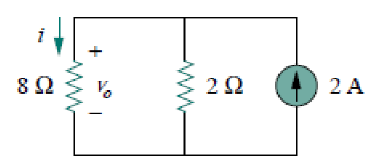
$$v_o = 8i = 8(0.4) = 3.2 \text{ V}$$



(a)



(b)



(c)

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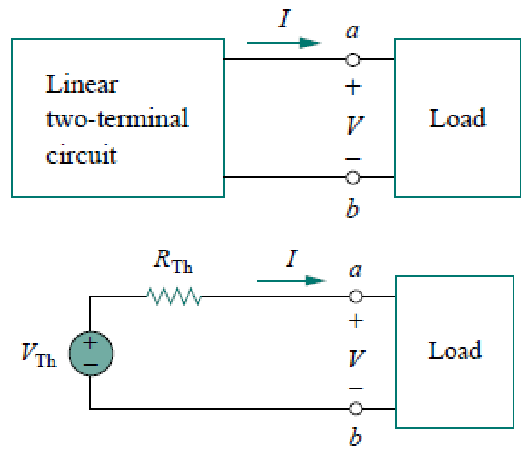
5.4 THEVENIN'S THEOREM

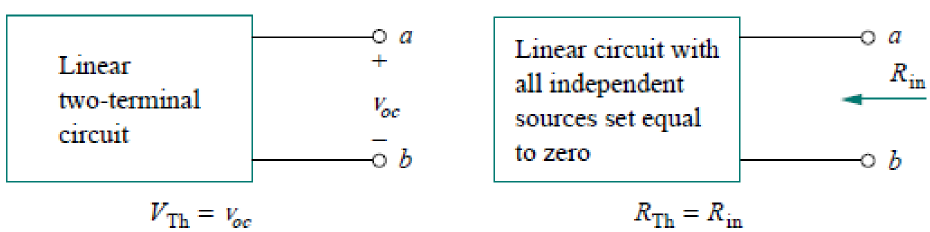
It often occurs in practice that a particular element in a circuit is variable (usually called the *load*) while other elements are fixed. As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load. Each time the variable element is changed, the entire circuit has to be analyzed all over again. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off



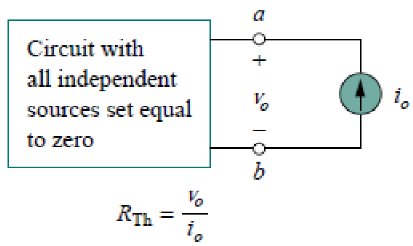
Two circuits are said to be *equivalent* if they have the same voltage-current relation at their terminals

Thus, V_{Th} is the open-circuit voltage across the terminals as shown in the figure below.



To finding the Thevenin resistance *R*Th, we need to consider two cases.

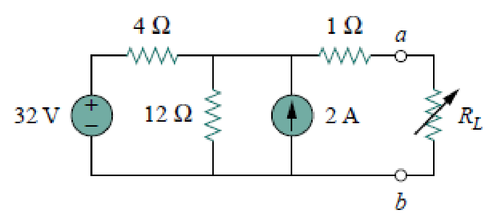
CASE 1 If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals *a* and *b*, as shown in the figure below



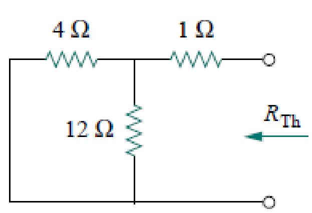
CASE 2 If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_o at terminals *a* and *b* and determine the resulting current i_o. Then $R_{Th} = v_o/i_o$, as shown in the figure below. Alternatively, we may insert a current source i_o at terminals *a-b* as shown in Fig. 4.25(b) and find the terminal voltage *vo*. Again *R*Th = *vo/io*. Either of the two approaches will give the same result. In either approach we may assume any value of *vo* and *io*. For example, we may use *vo* = 1 V or *io* = 1 A, or even use unspecified values of v_o or i_o.

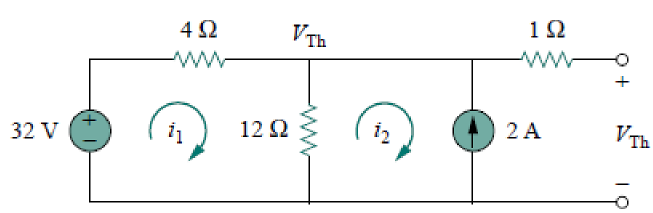
Example (1): Find the Thevenin equivalent circuit of the circuit shown below, to the left of the terminals *a-b*. Then find the current through *RL* = 6, 16, and 36 *Ω*.

Solution:



We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown below. Thus,





$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

To find V_{Th}, applying mesh analysis to the two loops, we obtain

$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\text{ V}$$

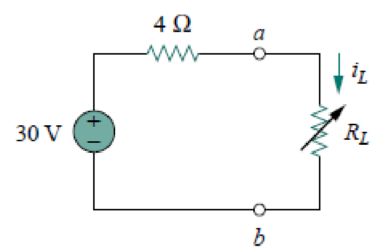
Alternatively, it is even easier to use nodal analysis. We ignore the 1-Ω resistor since no current flows through it. At the top node, KCL gives

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

$$96 - 3V_{Th} + 24 = V_{Th} \qquad \Rightarrow V_{Th} = 30\text{ V}$$

as obtained before. We could also use source transformation to find *V*Th. The Thevenin equivalent circuit is shown in Fig. b. The current through *RL* is

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + R_L}$$



When *RL* = 6, $I_L = \frac{30}{10} = 3\text{ A}$

When *RL* = 16, $I_L = \frac{30}{20} = 1.5\text{ A}$

When *RL* = 36, $I_L = \frac{30}{40} = 0.75\text{ A}$

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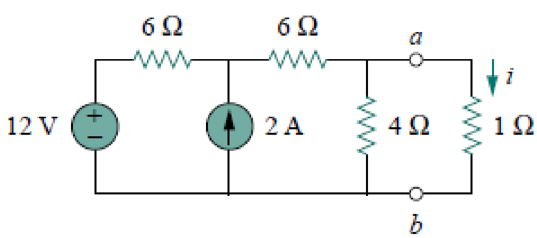
Using Thevenin's theorem, find the equivalent circuit to the left of the terminals in the circuit in the figure below. Then find *i*.



Answer: *VTh = 6 V, RTh = 3 Ω, i = 1.5*

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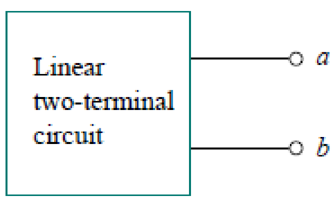
5.5 NORTON'S THEOREM

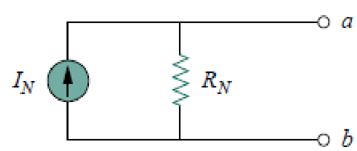
In 1926, about 43 years after Thevenin published his theorem, E. L. Norton, an American engineer at Bell Telephone Laboratories, proposed a similar theorem.

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.

Thus, the circuit in Fig. (a) can be replaced by the one in Fig. (b).



(a)



(b)

Thevenin and Norton resistances are equal

$$R_N = R_{Th}$$

To find the Norton current $\mathbf{I_N}$, we determine the short-circuit current flowing from terminal a to b in both circuits in **Fig. b**. The short-circuit current in **Fig. (b)** is $\mathbf{I_N}$. This must be the same short-circuit current from terminal a to b in **Fig. (a)**, since the two circuits are equivalent. Thus,

$$\mathbf{I_N = i_{sc}}$$

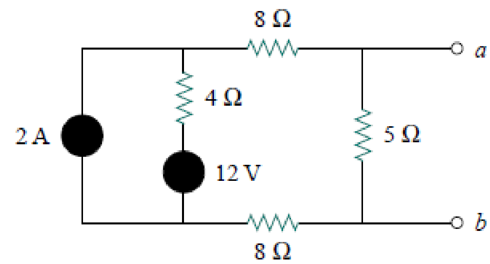
Dependent and independent sources are treated the same way as in Thevenin's theorem. Observe the close relationship between Norton's and Thevenin's theorems: $\mathbf{R_N = R_{Th}}$ and

$$\mathbf{I_N} = \frac{V_{Th}}{R_{Th}}$$

Also, we have

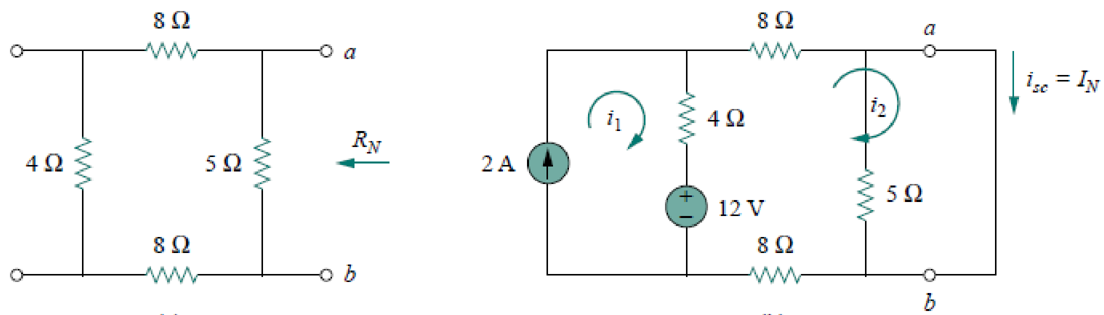
$$V_{th} = V_{oc}, \qquad I_N = i_{sc}, \qquad R_{Th} = \frac{voc}{isc} = R_N$$

Example (1): *Find the Norton equivalent circuit of the circuit in the figure below*



Solution:

We find R_N in the same way we find R_{Th} in the Thevenin equivalent circuit. Set the independent sources equal to zero. This leads to the circuit in Fig. (a), from which we find R_N. Thus,



$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N, we short-circuit terminals *a* and *b*, as shown in Fig. (b). We ignore the 5Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

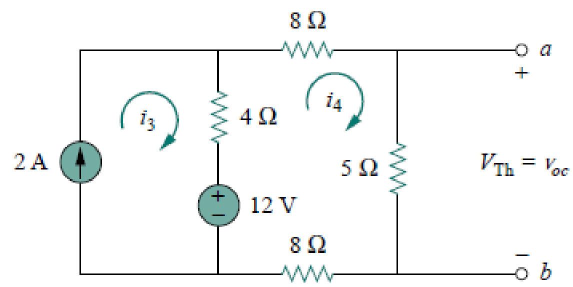
From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

Alternatively, we may determine I_N from V_{Th}/R_{Th}. We obtain V_{Th} as the open-circuit voltage across terminals *a* and *b* in Fig. (c). Using mesh analysis, we obtain

$$i_3 = 2\text{ A}$$

$$25i_4 - 4i_3 - 12 = 0 \quad \Rightarrow i_4 = 0.8\text{ A}$$



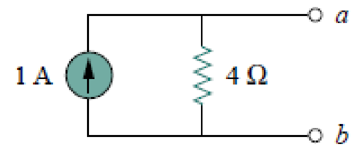
(c)

And $v_{oc} = V_{\text{Th}} = 5i_4 = 4$ V

Hence,

$$I_N = \frac{V_{\text{Th}}}{R_{\text{Th}}} = \frac{4}{4} = 1 \text{ A}$$

as obtained previously. This also serves to confirm that $R_{\text{Th}} = v_{oc}/i_{sc} = 4/1 = 4\Omega$. Thus, the Norton equivalent circuit is as shown below



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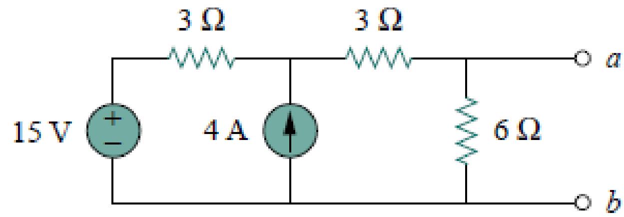
Find the Norton equivalent circuit of the following circuit.



Answer: $RN = 3\ \Omega$, $I_N = 4.5$ A.