Example 3.2.12: Solve the following system of linear equations by using the Gauss - Jordan elimination method:

$$5x_1 + 6x_2 = 7$$
$$3x_1 + 4x_2 = 5$$

Solution: The system of linear equations has the following augmented matrix:

$$\left(\begin{array}{cc|c} 5 & 6 & 7 \\ 3 & 4 & 5 \end{array}\right) \xrightarrow{\frac{1}{5}R_1 \to R_1}$$

$$\left(\begin{array}{cc|c} 1 & \frac{6}{5} & \frac{7}{5} \\ 3 & 4 & 5 \end{array}\right) \xrightarrow{R_2 - 3R_1 \to R_2}$$

$$\left(\begin{array}{cc|c} 1 & \frac{6}{5} & \frac{7}{5} \\ 0 & \frac{2}{5} & \frac{4}{5} \end{array}\right) \xrightarrow{\frac{5}{2}R_2 \to R_2}$$

$$\left(\begin{array}{cc|c} 1 & \frac{6}{5} & \frac{7}{5} \\ 0 & 1 & 2 \end{array}\right) \xrightarrow{R_1 - \frac{6}{5}R_2 \to R_1} \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right)$$

The last matrix is in reduced row - echelon form . The corresponding reduced system is:

$$x_1 \quad = -1$$
$$x_2 = 2$$

Therefore the solution of the system is $x_1 = -1$ **, and** $x_2 = 2$ **.**

Example 3.2.13: **Solve the following linear system using the Gauss - Jordan elimination method:**

$$\begin{aligned} 4y + 2z &= 1 \\ 2x + 3y + 5z &= 0 \\ 3x + y + z &= 11 \end{aligned}$$

Solution: **The system of linear equations has the following augmented matrix:**

$$\left(\begin{array}{ccc|c} 0 & 4 & 2 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 1 & 1 & 11 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2}$$

$$\left(\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & 4 & 2 & 1 \\ 3 & 1 & 1 & 11 \end{array}\right) \xrightarrow{\frac{1}{2}R_1 \to R_1}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 4 & 2 & 1 \\ 3 & 1 & 1 & 11 \end{array}\right) \xrightarrow{R_3 - 3R_1 \to R_3}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 4 & 2 & 1 \\ 0 & -\frac{7}{2} & -\frac{13}{2} & 11 \end{array}\right) \xrightarrow{\frac{1}{4}R_2 \to R_2}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & -\frac{7}{2} & -\frac{13}{2} & 11 \end{array}\right) \xrightarrow{R_3 + \frac{7}{2}R_2 \to R_3}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & -\frac{19}{4} & \frac{95}{8} \end{array}\right) \xrightarrow{-\frac{4}{19}R_3 \to R_3}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & 1 & -\frac{5}{2} \end{array}\right) \xrightarrow{R_2 - \frac{1}{2}R_3 \to R_2}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & \frac{5}{2} & 0 \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{5}{2} \end{array}\right) \xrightarrow{R_1 - \frac{5}{2}R_3 \to R_1}$$

$$\left(\begin{array}{ccc|c} 1 & \frac{3}{2} & 0 & \frac{25}{4} \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{5}{2} \end{array}\right) \xrightarrow{R_1 - \frac{3}{2}R_2 \to R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & \frac{3}{2} \\ 0 & 0 & 1 & -\frac{5}{2} \end{array}\right)$$

The last matrix is in reduced row - echelon form . The corresponding reduced system is:

$$\begin{aligned} x &= 4 \\ y &= \frac{3}{2} \\ z &= -\frac{5}{2} \end{aligned}$$

Therefore the solution of the system is $x = 4$, $y = \frac{3}{2}$, **and** $z = -\frac{5}{2}$.