جامـــــعـة المـــسـتـقـبـل
AL MUSTAQBAL UNIVERSITY

كلية العلـــوم
قـســــــــم علوم الذكاء الاصطناعي

المحاضرة السادسة

المادة : Discrete Structures

المرحلة : الاولى/ الكورس الثاني

اسم الاستاذ: م.د. رياض حامد سلمان

Relations

The important aspect of the any set is the relationship between its elements. The association of relationship established by sharing of some common feature proceeds comparing of related objects. For example, assume a set of students, where students are related with each other if their sir names are same.

Conversely, if set is formed a class of students then we say that students are related if they belong to same class etc.
Relation is a predefined alliance of objects. The examples of relations are viz. brother and sister, and mathematical relation such as less than, greater than, and equal etc.

The relations can be classifying on the basis of its association among the objects. For example, relations said above are all association among two objects so these relations are called binary relation. Similarly, relations of parent to their children, boss and subordinates, brothers and sisters etc. are the examples of relations among three/more objects known as tertiary relation, quadratic relations and so on. In general an *n*-ary relation is the relation framed among *n* objects.

Product sets:
Consider two arbitrary sets A and B. The set of all ordered pairs (a,b) where $a \in A$ and $b \in B$ is called the product, or Cartesian product, of A and B.

$$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$$

Example
R denotes the set of real numbers and so :
$\mathbf{R}^2 = \mathbf{R} \times \mathbf{R}$ is the set of ordered pairs of real numbers.
The geometrical representation of $\mathbf{R}^2$ as points in the plane as in Fig.-1. Here each point P represents an ordered pair (a, b) of real numbers and vice versa; the vertical line through P meets the x-axis at a, and the horizontal line through P meets the y-axis at b. $\mathbf{R}^2$ is called the *Cartesian plane*.

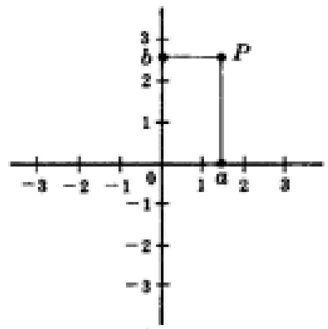


Fig. -1

Example:

a)Let A = {1,2} and B = {a ,b ,c} then

$A \times B = \{(1,a), (1,b),(1,c),(2,a),(2,b),(2,c)\}$, Also,

$A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

The order in which the sets are considered is important, so

$A\times B \neq B \times A$.

$n(A\times B) = n(A)\times n(B) = 2 \times 3 = 6$

Binary relation:

A relation between two objects is a binary relation and it is given by a set of ordered couples.

Let *A* and *B* be sets. A *binary relation* from *A* to *B* is a subset of $A \times B$.

Suppose *R* is a relation from *A* to *B*. Then *R* is a set of ordered pairs where each first element comes from *A* and each second element comes from *B*. That is, for each pair $a \in A$ and $b \in B$, exactly one of the following is true:

(i) $(a, b) \in R$; we then say "*a* is *R*-related to *b*", written *aRb*.

(ii) $(a, b) \notin R$; we then say "*a* is not *R*-related to *b*", written $a \not R\, b$.

Example

(a) A = (1, 2, 3) and B = {x, y, z}, and let

R = {(1, y), (1, z), (3, y)}. Then R is a relation from A to B since R is a subset of A × B.

With respect to this relation

1Ry, 1Rz, 3Ry but $(1,x) \notin R$ & $(2,x) \notin R$

(b) Set inclusion ⊆ is a relation on any collection of sets. For, given any pair of set *A* and *B*, either $A \subseteq B$ *or* or $A \nsubseteq B$.

(c) Consider the set *L* of lines in the plane. Perpendicularity, written "⊥," is a relation on *L*. That is, given any pair of lines *a* and *b*, either $a \perp b$ or *a* not⊥ *b*. Similarly, "is parallel to," written "‖" is a relation on *L* since either $a \parallel b$ or $a \nparallel b$.

(d) Let *A* be any set. Then $A \times A$ and ∅ are subsets of $A \times A$ and hence are relations on *A* called the *universal relation* and *empty relation*, respectively.

Example :
Let A = {1, 2, 3}. Define a relation R on A by writing
(x, y) ∈ R , such that a≥b, list the element of R
aRb ↔ a≥b , a,b∈A
∴ R = {(1,1),(2,1), (2,2), (3,1), (3,2), (3,3)}.

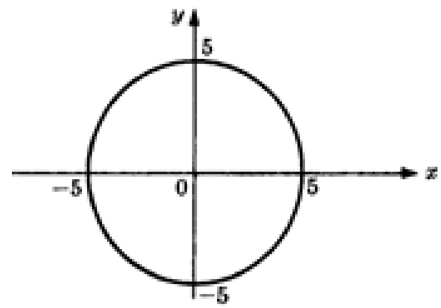
Pictorial representation of relations

There are various ways of picturing relations:

I - By coordinate plane

Let *S* be a relation on the set R of real numbers; that is, *S* is a subset of $R^2 = \mathbf{R} \times \mathbf{R}$. Frequently, *S* consists of all ordered pairs of real numbers which satisfy some given equation
$E(x, y) = 0$ (such as $x2 + y2 = 25$).
Since R^2 can be represented by the set of points in the plane, we can picture *S* by emphasizing those points in the plane which belong to *S*. The pictorial representation of the relation is called the *graph* of the relation.

For example,
the graph of the relation $\boldsymbol{x^2+y^2 = 25}$ is a circle having its center at the origin and radius 5.



$x^2 + y^2 = 25$

II -Directed Graphs of Relations on Sets

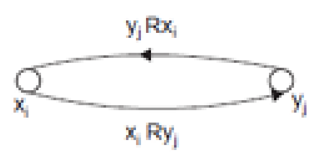
Relation can be represented pictorially by drawing its *graph* (directed graph). Consider a relation R be defined between two sets:

$X = \{x_1, x_2, \ldots\ldots, x_l\}$ and

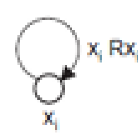
$Y = \{y_1, y_2, \ldots\ldots, y_m\}$

i.e., $x_i \, R \, y_j$, that is ordered couple $(x_i, y_j) \in R$ where $1 \le i \le l$ and $1 \le j \le m$. The elements of sets X and Y are represented by small circle called nodes. The existence of the ordered couple such as (x_i, y_j) is represented by means of an edge marked with an arrow in the direction from x_i to y_j.

While all nodes related to the ordered couples in R are connected by proper arrows, we get a directed graph of the relation R. For the ordered couples $x_i \, R y_j$ and $y_j \, R x_i$ we draw two arcs between nodes x_i and y_j,



If ordered couple is like $x_i R x_i$ or $(x_i, x_i) \in R$ then we get self loop over the node x_i.



Example,

Relation R on the set A = {1, 2, 3, 4}:

R = {(1, 2), (2, 2), (2, 4), (3, 2), (3, 4), (4, 1), (4, 3)}

Fig. 3 shows the directed graph of R

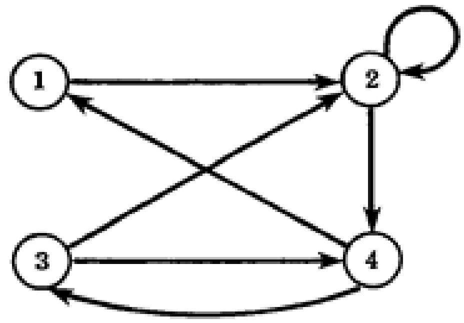


Fig. -3

III - matrix

Form a rectangular array (matrix) whose rows are labeled by the elements of A and whose columns are labeled by the elements of B. Put a 1 or 0 in each position of the array according as $a \in A$ is or is not related to $b \in B$. This array is called the *matrix of the relation*.

Example,

let A = {1, 2, 3} and B = {x, y, z}.

R = {(1,y),(1,z),(3,y)}

Fig. 4 shows the matrix of R.

	x	y	z
1	0	1	1
2	0	0	0
3	0	1	0

Fig. 4

IV - arrow from

Write down the elements of A and the elements of B in two disjoint disks, and then draw an arrow from a ∈ A to b ∈ B whenever a is related to b. This picture will be called the arrow diagram of the relation.

Fig. 5 pictures the relation R in the previous example by the arrow form.