جامـــــعة المـــستـقـبـل
AL MUSTAQBAL UNIVERSITY

كلية العلـــوم
قـســــــــم علوم الذكاء الاصطناعي

المحاضرة الرابعة

Discrete Structures : المادة
المرحلة : الاولى/ الكورس الثاني
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Finite Sets and Counting Principle:

A set is said to be finite if it contains exactly m distinct elements, where m denotes some nonnegative integer. Otherwise, a set is said to be infinite.

For example:
- The empty set $\varnothing$ and the set of letters of English alphabet are finite sets,
- The set of even positive integers, $\{2,4,6,\ldots..\}$, is infinite.

If a set A is finite, we let n(A) or #(A) denote the number of elements of A.

Example:
If A ={1,2,a,w} then

$$n(A) = \#(A) = |A| = 4$$

Lemma: If A and B are finite sets and disjoint Then $A \cup B$ is finite set and:

$$n(A \cup B) = n(A) + n(B)$$

Theorem (Inclusion–Exclusion Principle): Suppose A and B are finite sets. Then
$A \cup B$ and $A \cap B$ are finite and

$$|A \cup B| = |A| + |B| - |A \cap B|$$

That is, we find the number of elements in A or B (or both) by first adding n(A) and n(B) (inclusion) and then subtracting $n(A \cap B)$ (exclusion) since its elements were counted twice.
We can apply this result to obtain a similar formula for three sets:

Corollary:
If A, B, C are finite sets then

$$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B \cap C|$$

Example (1) :

A= {1,2,3}
B= {3,4}
C= {5,6}
$A \cup B \cup C = \{1,2,3,4,5,6\}$
$|A \cup B \cup C| = 6$

$|A| = 3$, $|B| = 2$, $|C| = 2$
$A \cap B = \{3\}$, $|A \cap B| = 1$
$A \cap C = \{ \}$, $|A \cap C| = 0$

$B \cap C = \{ \}$, $|B \cap C| = 0$
$A \cap B \cap C = \{ \}$, $|A \cap B \cap C| = 0$

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$
$|A \cup B \cup C| = 3 + 2 + 2 - 1 - 0 - 0 + 0 = 6$

Example (2):

Suppose a list A contains the 30 students in a mathematics class, and a list B contains the 35 students in an English class, and suppose there are 20 names on both lists. Find the number of students:

(a) only on list A
(b) only on list B
(c) on list $A \cup B$

Solution:

(a) List A has 30 names and
20 are on list B;
hence 30 − 20 = 10 names are only on list A.
(b) Similarly, 35 − 20 = 15 are only on list B.
(c) We seek $n(A \cup B)$. By inclusion–exclusion,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 30 + 35 - 20 = 45.$$

Example (3):
Suppose that 100 of 120 computer science students at a college take at least one of languages: French, German, and Russian:
65 study French (F).
45 study German (G).
42 study Russian (R).
20 study French & German $F \cap G$.
25 study French & Russian $F \cap R$.
15 study German & Russian $G \cap R$.
Find the number of students who study:

1) All three languages ($F \cap G \cap R$)
2) The number of students in each of the eight regions of the Venn diagram

Solution:
$|F \cup G \cup R| = |F| + |G| + |R| - |F \cap G| - |F \cap R| - |G \cap R| + |F \cap G \cap R|$
$100 = 65+ 45 + 42 - 20 - 25 - 15 + |F \cap G \cap R|$
$100 = 92 + |F \cap G \cap R|$
$\therefore |F \cap G \cap R| = 8$ students study the 3 languages

$20 – 8 = 12$ $(F \cap G) - R$
$25 – 8 = 17$ $(F \cap R) - G$
$15 – 8 = 7$ $(G \cap R) - F$

$65 – 12 – 8 – 17 = 28$ students study French only
$45 – 12 – 8\ 7 = 18$ students study German only
$42 – 17 – 8\ 7 = 10$ students study Russian only
$120 – 100 = 20$ students do not study any language