

Al-Mustaqbal University
College of Science
Artificial Intelligence Sciences Department



The Karnaugh Map

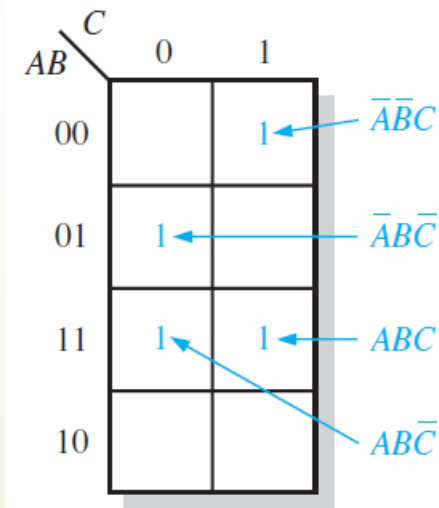
Lec.9

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The Karnaugh Map (K-Map)

A Karnaugh map provides a systematic method for simplifying Boolean expressions and, if properly used, will produce the simplest SOP or POS expression possible, known as the **minimum expression**.

➡ The number of cells in a Karnaugh map, as well as the number of rows in a truth table, is equal to the total number of possible input variable combinations. For three variables, the number of cells is $2^3 = 8$. For four variables, the number of cells is $2^4 = 16$.



| AB \ C | 0 | 1 |
|--------|-----------------------|-----------------------|
| 00 | | 1 ← $\bar{A}\bar{B}C$ |
| 01 | 1 ← $\bar{A}B\bar{C}$ | |
| 11 | 1 ← $AB\bar{C}$ | 1 ← ABC |
| 10 | | |

➤ **EXAMPLE :** Using the Karnaugh Map, find the simplified Boolean expression for the truth table given below?

| A | B | Y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

Solution: $Y = \bar{A} B + A \bar{B} + A B$, The K-Map:

| | | | |
|----------|-------------|-----------|----------|
| | | B | |
| | | \bar{B} | B |
| A | \bar{A} 0 | 0 | 1 |
| | A 1 | 1 | 1 |

The K-map shows two groupings: a vertical group of 1s in the B column (labeled **B** with an arrow) and a horizontal group of 1s in the A row (labeled **A** with an arrow).

The simplified Boolean expression: $Y=A+B$

➤ **EXAMPLE :** Map the following SOP expression on a Karnaugh map:

$$\bar{A} + A\bar{B} + AB\bar{C}.$$

➤ **Solution:** $\bar{A} + A\bar{B} + AB\bar{C}$

000 100 110

001 101

010

011

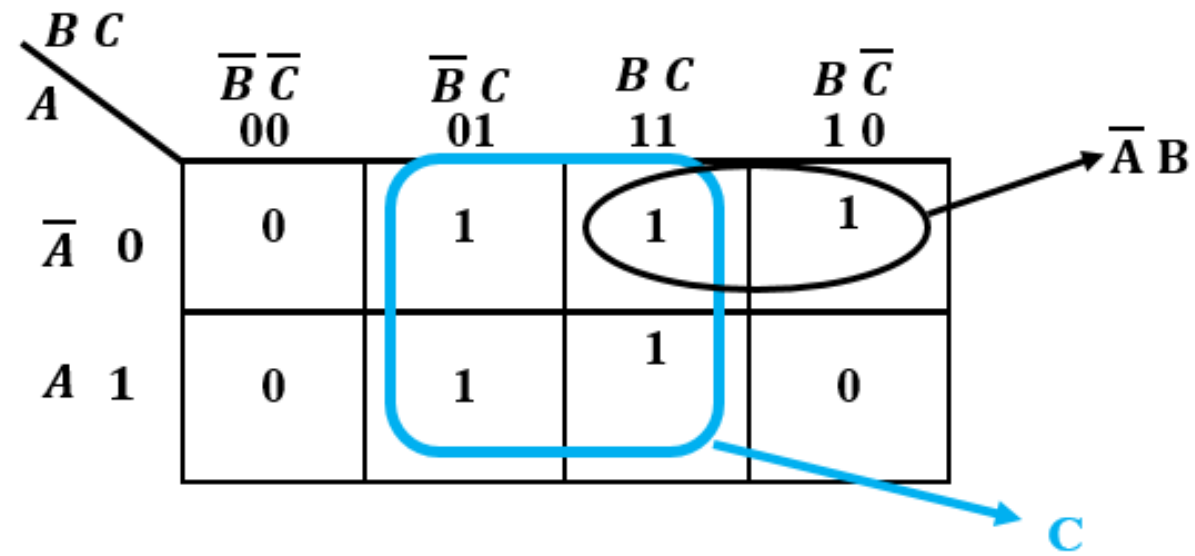
| | | C | |
|------|----|-----|---|
| | | 0 | 1 |
| AB | 00 | 1 | 1 |
| | 01 | 1 | 1 |
| | 11 | 1 | |
| | 10 | 1 | 1 |

➤ **EXAMPLE :** Using the Karnaugh Map, find the simplified Boolean expression for the logic expression: $Y = \sum(1, 2, 3, 5, 7)$

Solution: $Y = \bar{A} \bar{B} C + \bar{A} B \bar{C} + \bar{A} B C + A \bar{B} C + A B C$

001 010 011 101 111

The K-Map:



The simplified Boolean expression: $Y = C + \bar{A} B$

➤ **EXAMPLE :** Simplify the following Boolean expression using K-Map:

$$Y = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + \bar{A}BC\bar{D} + AB\bar{C}\bar{D} + ABC\bar{D} \\ + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D}$$

Solution: The K-Map:

