جامعة المستقبل
AL MUSTAQBAL UNIVERSITY
كلية الهندسة والتقنيات الهندسية

Communication Technical Engineering Department

1st Stage

Digital Logic- UOMU028021

Lecture 5 – Karnaugh Map

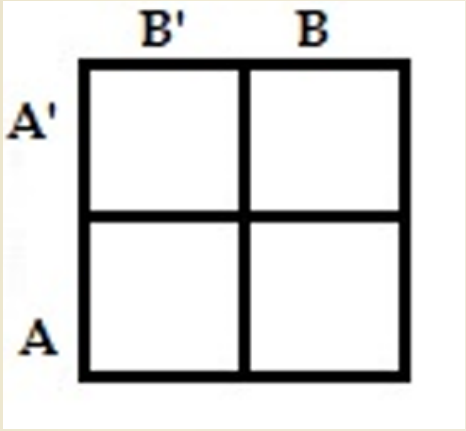
Dr. Mohammed Fadhil
PhD in Computer Networks
Email: mohammed.fadhil1@uomus.edu.iq

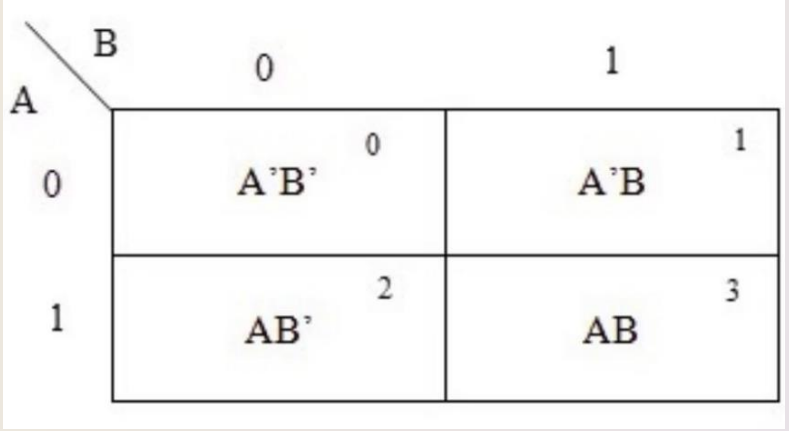
Karnaugh Map

- **Karnaugh map** is a graphical method that is used to simplify a logical equation or to convert a truth table to its corresponding logic circuit in a simple, orderly process.
- By using Karnaugh map technique, we can reduce the Boolean expression containing any number of variables, such as 2-variable Boolean expression, 3-variable Boolean expression, 4-variable Boolean expression and even 7-variable Boolean expressions, which are complex to solve by using regular Boolean theorems and laws.
- In our study we consider boxes / squares shapes called cells as follows

2-Variable K-Maps

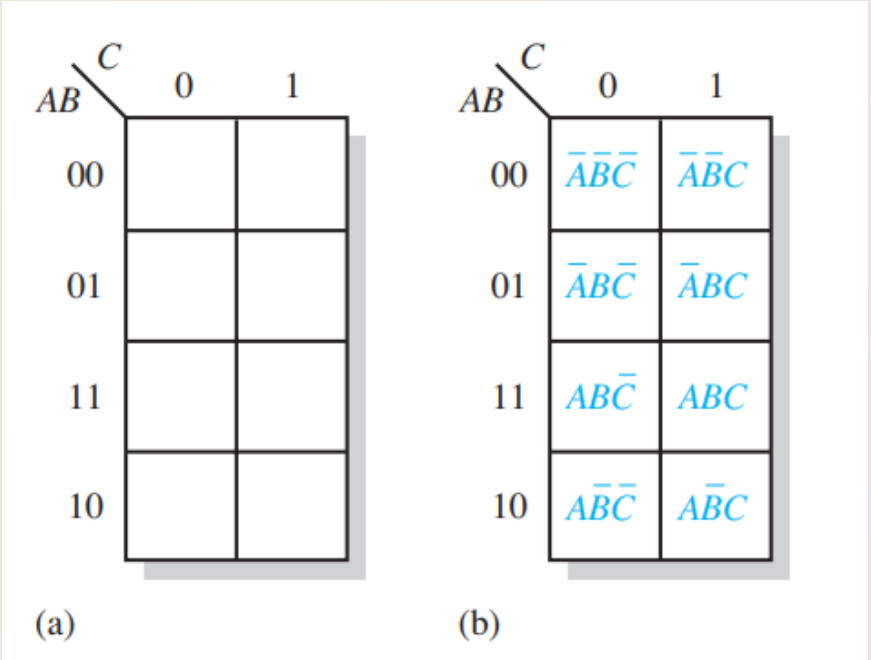
- 2-variables K-map contains 2^2 **= 4 cells**. Each cell represents a possible min-term. For variables A and B:
  - the min-terms are **A.B, A.B', A'.B, and A'.B'.**
- The following image illustrates the structure and general representation of a 2-variable K-map, showing the positions of all possible outputs for a 2-variable Boolean function.





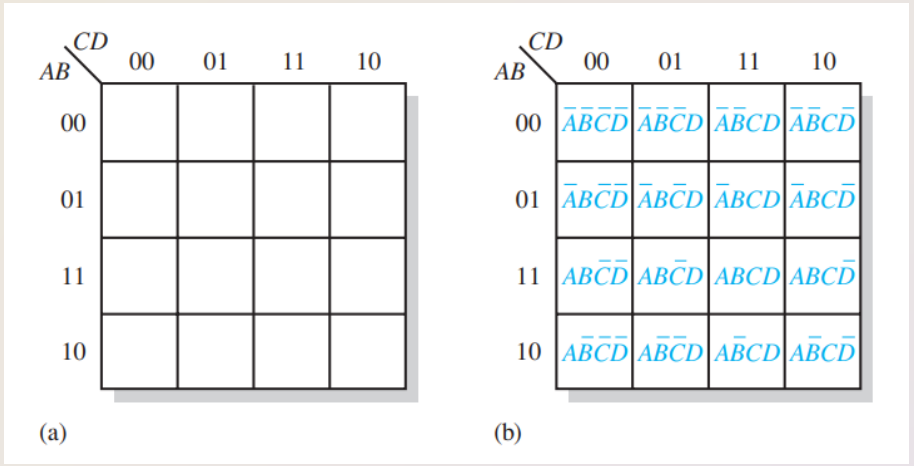
3-Variable K-Maps

- A 3-variable K-map contains 2^3 = 8 cells. For a 3-variable Boolean function, there are **8 possible output min-terms**. The general representation of these min-terms is shown below.
- A key observation in a typical 3-variable K-map plot is the **interchange of columns 10 and 11**.
- This modification ensures that only one variable changes across adjacent cells, which is crucial for minimizing logic expressions effectively.



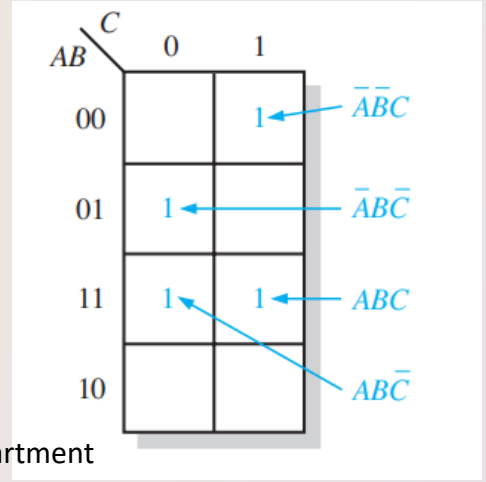
4-Variable K-Maps

- A 4-variable K-map contains 2^4 = 16 cells. There are **16 possible min-terms for a 4-variable Boolean function**. The general representation of these min-terms is shown below.
- Similar to the 3-variable K-map, a typical 4-variable K-map plot shows that both columns and rows **10 and 11 are interchanged**. This arrangement maintains the adjacency property, allowing for efficient minimization of Boolean expressions.



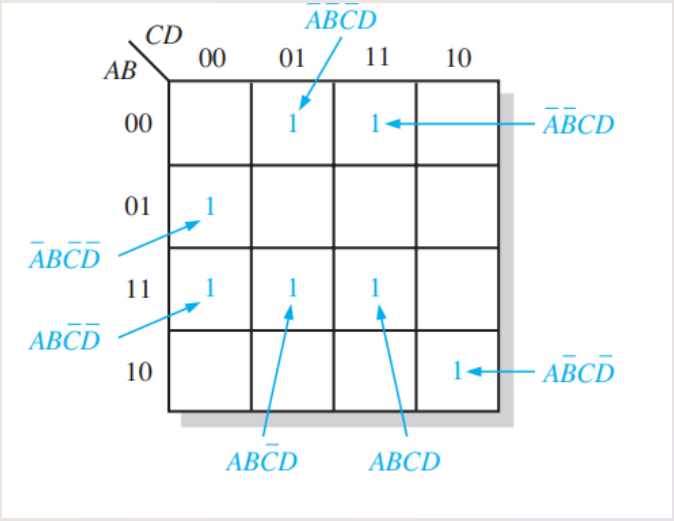
EXAMPLE

- Map the following standard SOP expression on a Karnaugh map:
  - A`B`C + A`BC` + ABC` + ABC
- Solution:
  - Evaluate the expression as shown below. Place a 1 on the 3-variable Karnaugh map in the Figure for each standard product term in the expression.
  - A`B`C + A`BC` + ABC` + ABC
  - 001 + 010 + 110 + 111



EXAMPLE

- Map the following standard SOP expression on a Karnaugh map:
  - A`B`CD + A`BC`D` + ABC`D + ABCD + ABC`D` + A`B`C`D + AB`CD`
- Solution:
  - Evaluate the expression as shown below. Place a 1 on the 4-variable Karnaugh map in Figure for each standard product term in the expression
  - A`B`CD + A`BC`D` + ABC`D + ABCD + ABC`D` + A`B`C`D + AB`CD`
  - 0011 + 0100 + 1101 + 1111 + 1100 + 0001 + 1010



Minimization with Karnaugh Maps

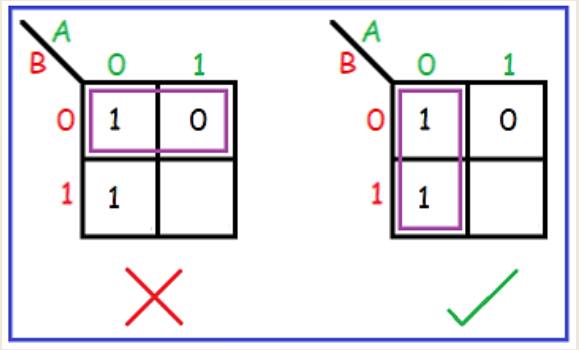
- **K-maps are used to convert the truth table of a Boolean equation into minimized SOP form.**
- Easy and simple basic rules for the simplification.
- The K-map method is faster and more efficient than other simplification techniques of Boolean algebra.
- All rows in the K-map are represented by using a square shaped cells, in which each square in that will represent a min-term.
- It is easy to convert a truth table to k-map and k-map to Sum of Products form equation.

Grouping of K-map variables

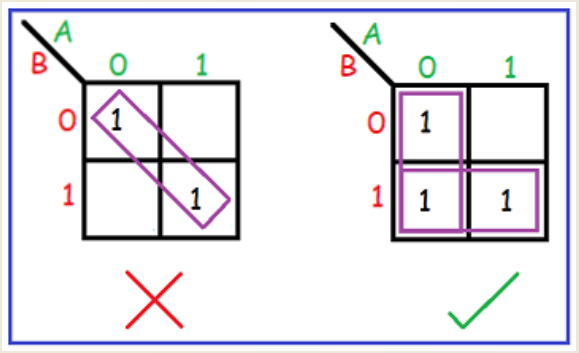
- There are some rules to follow while we are grouping the variables in K-maps. They are as follows:
  - The square that contains 1 should be taken in simplifying, at least once.
  - Group **should not** include any **zeros (0).**
  - A group should be the as large as possible.
  - Groups can be horizontal or vertical. **Grouping of variables in diagonal manner is not allowed**
  - If the square containing 1 has no possibility to be placed in a group, then it should be added to the final expression.
  - Groups **can overlap.**
  - The number of squares in a group must be equal to powers of 2, such as 1, 2, 4, 8 etc.
  - Groups **can wrap around.** As the K-map is considered as spherical or folded, the squares at the corners (which are at the end of the column or row) should be considered as they adjacent squares.
  - The grouping of K-map variables can be done in many ways, so the obtained simplified equation need not to be unique always.
  - The Boolean equation must be in must be in canonical form, in order to draw a K-map.

Grouping of K-map variables

- We need to group the cells with value 1 and may not include any cell containing zero:

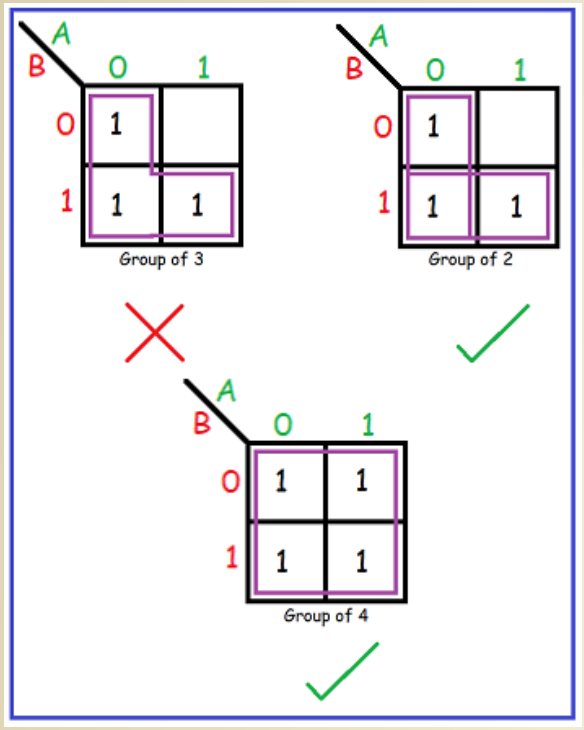


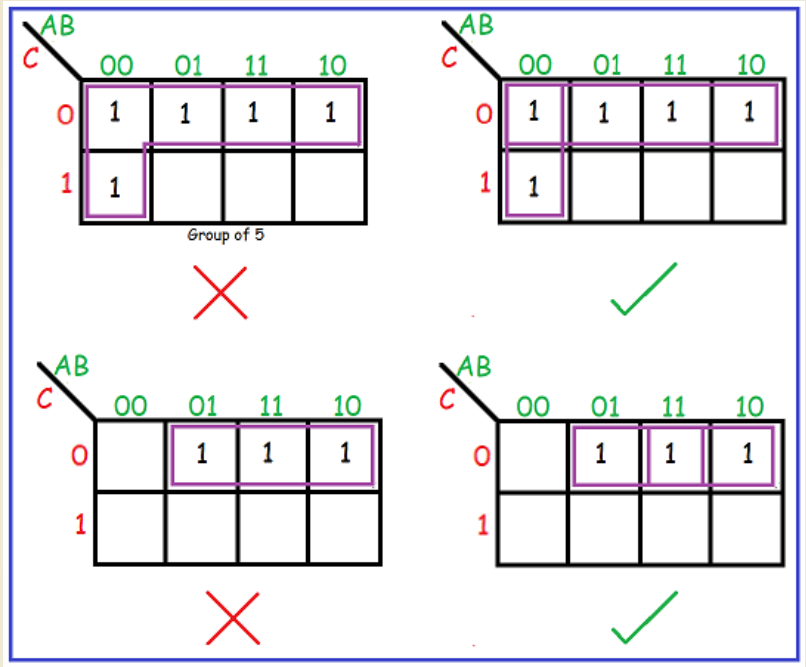
- Groups may be horizontal, vertical but not diagonal:

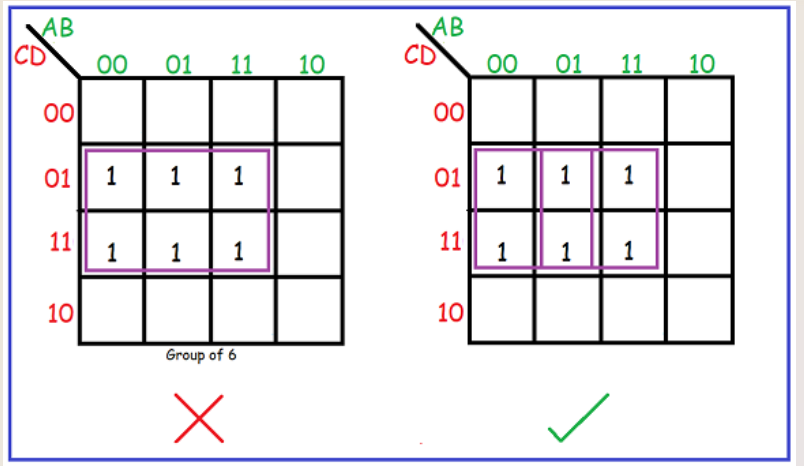


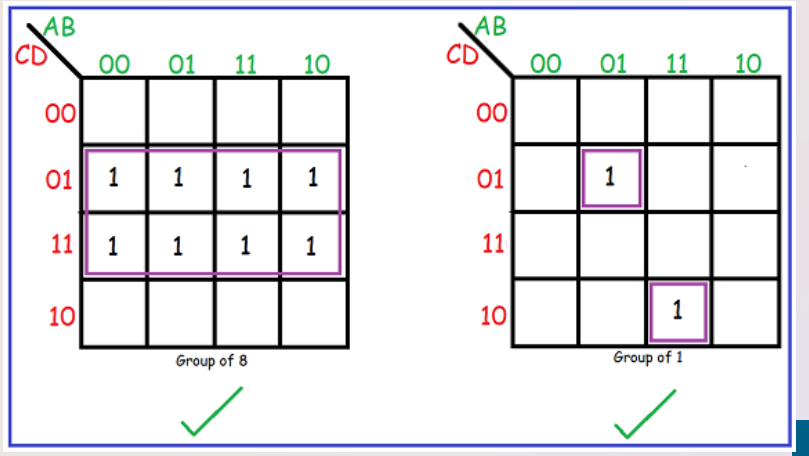
Grouping of K-map variables

- **Groups must contain 1, 2, 4, 8, …. ,2n cells:**



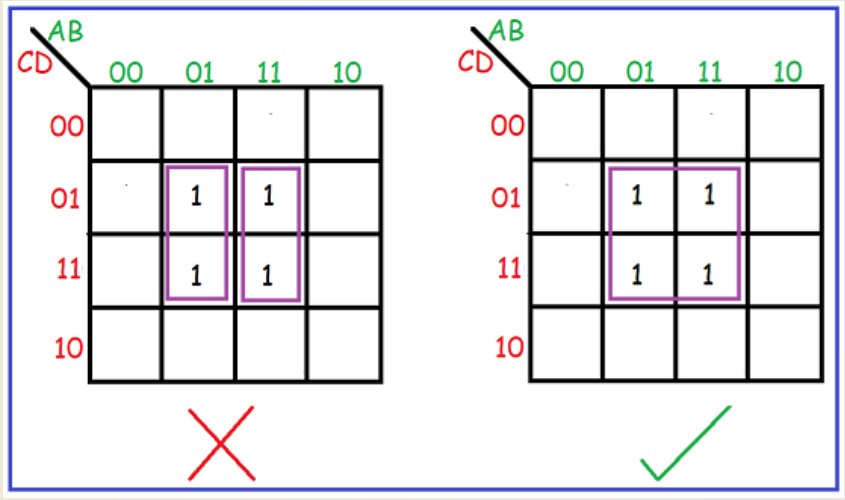






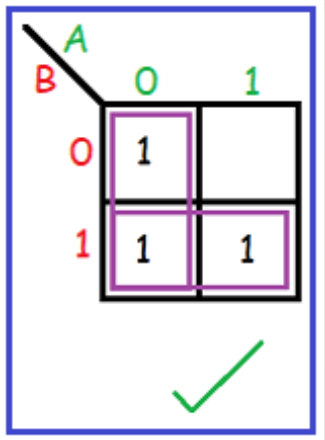
Grouping of K-map variables

- **Group should be as large as possible:**
  - **As you can see in the above diagram, we need to group the cells as large as possible. Note that the grouping in 1st image doesn't break any Boolean law but it is not sufficiently minimal.**

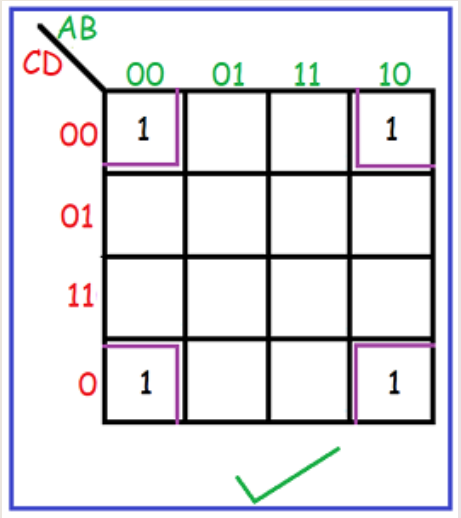


Grouping of K-map variables

- **Groups can overlap each other:**

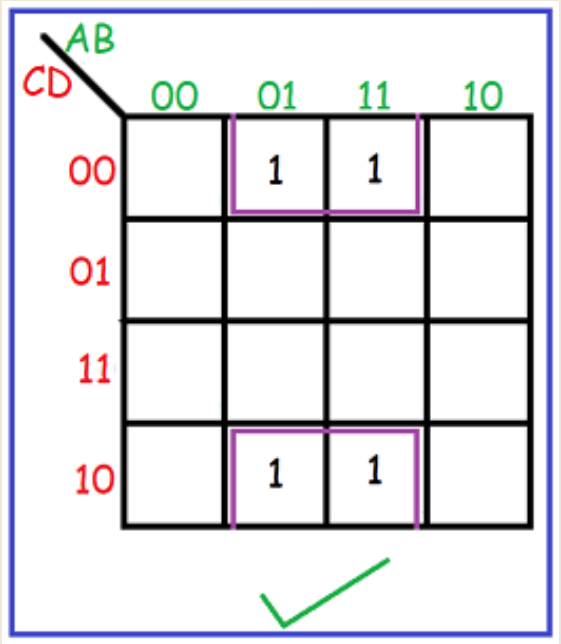


- **Opposite or corner cells can be grouped:**



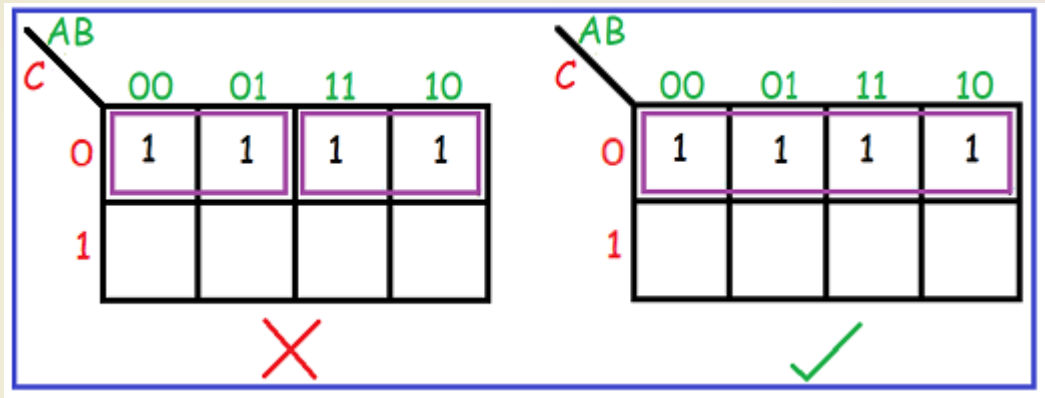
Grouping of K-map variables

- **Groups may wrap around the table. The leftmost cell in a row may be grouped with the rightmost cell and the top cell in a column may be grouped with the bottom cell:**



Grouping of K-map variables

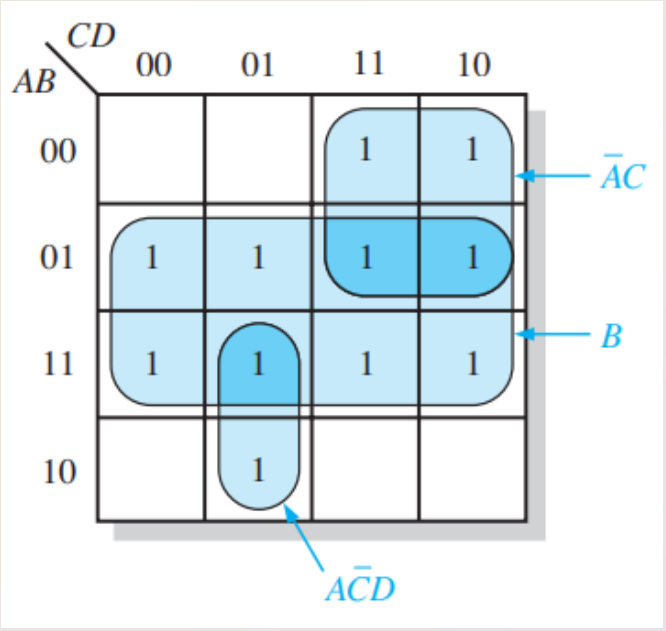
- **There should be as few groups as possible, as long as this does not contradict any of the previous rules:**



EXAMPLE

- Determine the product terms for the Karnaugh map in the below Figure and write the resulting minimum SOP expression.
- **Solution:**
  - **Step 1: Group the 1s**
    - **8-Cell Group** → Covers all cells where *B*=1.
    - **4-Cell Group** → Top-right corner ($\overline{A}$ C).
    - **2-Cell Group** → Two cells in $\overline{CD}$ column ($A\overline{C}\ D$).
  - **Step 2: Eliminate Changing Variables**

Group	Variables Eliminated	Remaining Variables	Term
8 cells	A,C,D (all vary)	Only B remains	B
4 cells	B,D (vary)	$\overline{A}$, C	$\overline{A}$ C
2 cells	B (varies)	A , $\overline{C}$, D	$A\overline{C}$ D

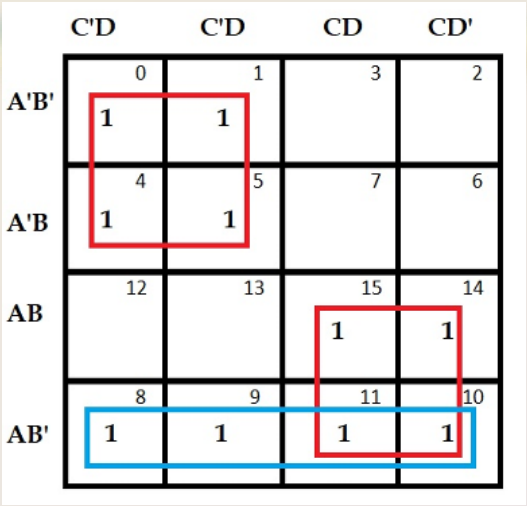


Final Simplified Expression

$$B + \overline{A}\,C + A\overline{C}\,D$$

Example

- Simplify the following function using Karnaugh map:
  - $Y = \sum_1 (0,1,4,5,8,9,10,11,14,15)$
- **Solution**
  - **Group 1s (largest possible power-of-2 groups)**
    - Group 1: Bottom row (AB = 10) → minterms 8, 9, 10, 11
      - A = 1, B = 0 → A B′
    - Group 2: Top two rows, left two columns → minterms 0, 1, 4, 5
      - A = 0 → A′
      - C = 0 → C′
      - Group result: A′ C′
    - Group 3: Bottom right corner (minterms 14, 15) and parts of Group 1
      - A = 1, C = 1 → A C
  - **Final simplified expression**
    - $Y = \overline{A}\,\overline{C} + AC + A\overline{B}$



AB \ CD	00	01	11	10
00	1	1	0	0
01	1	1	0	0
11	0	0	1	1
10	1	1	1	1

Try Yourself!

- **Use a Karnaugh map to find the minimum SOP form for each expression:**

1. **’A’BC+AB’C+ABC’**
2. **AC(B’+C)**
3. **A’(BC+B’C)+A(BC’+B’C)**
4. **A’BC+A’BC’+AB’C+ABC**

THANK YOU ☺