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5-3- Integrals of inverse trigonometric functions:

The integration formulas for the inverse trigonometric functions are:

16) $\int \dfrac{du}{\sqrt{a^2 - u^2}} = sin^{-1}\dfrac{u}{a} + c = -cos^{-1}\dfrac{u}{a} + c$; $\forall u^2 < a^2$

17) $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a}tan^{-1}\dfrac{u}{a} + c = -\dfrac{1}{a}cot^{-1}\dfrac{u}{a} + c$

18) $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a}sec^{-1}\left|\dfrac{u}{a}\right| + c = -\dfrac{1}{a}csc^{-1}\left|\dfrac{u}{a}\right| + c$; $\forall u^2 > a^2$

EX-3 Evaluate the following integrals:

1) $\int \dfrac{x^2}{\sqrt{1 - x^6}}dx$

2) $\int \dfrac{dx}{\sqrt{9 - x^2}}$

3) $\int \dfrac{x}{1 + x^4}dx$

4) $\int \dfrac{sec^2 x}{\sqrt{1 - tan^2 x}}dx$

5) $\int \dfrac{dx}{x\sqrt{4x^2 - 1}}$

6) $\int \dfrac{2dx}{\sqrt{x}(1 + x)}$

7) $\int \dfrac{dx}{1 + 3x^2}$

8) $\int \dfrac{2cosx}{1 + sin^2 x}dx$

9) $\int \dfrac{e^{sin^{-1}x}}{\sqrt{1 - x^2}}$

10) $\int \dfrac{tan^{-1}x}{1 + x^2}dx$

Sol.-

1) $\dfrac{1}{3}\int \dfrac{1}{\sqrt{1 - (x^3)^2}}(3x^2\,dx) = \dfrac{1}{3}sin^{-1}x^3 + c$

2) $\int \dfrac{dx}{\sqrt{9 - x^2}} = sin^{-1}\dfrac{x}{3} + c$

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$3)\ \dfrac{1}{2}\displaystyle\int \dfrac{2x}{1+(x^2)^2}dx = \dfrac{1}{2}tan^{-1}x^2 + c$

$4)\ \displaystyle\int \dfrac{sec^2 x}{\sqrt{1-tan^2 x}}dx = sin^{-1}(tan\,x) + c$

$5)\ \displaystyle\int \dfrac{2\ dx}{2x\sqrt{(2x)^2 - 1}} = sec^{-1}(2x) + c$

$6)\ \displaystyle\int \dfrac{2}{\sqrt{x}(1+x)}dx = 4\int \dfrac{\frac{1}{2}\sqrt{x}\ dx}{1+(\sqrt{x})^2} = 4\,tan^{-1}\sqrt{x} + c$

$7)\ \dfrac{1}{\sqrt{3}}\displaystyle\int \dfrac{\sqrt{3}\ dx}{1+(\sqrt{3}x)^2} = \dfrac{1}{\sqrt{3}}tan^{-1}(\sqrt{3}x) + c$

$8)\ 2\displaystyle\int \dfrac{cosx\ dx}{1+(sin\,x)^2} = 2\,tan^{-1}(sin\,x) + c$

$9)\ \displaystyle\int e^{sin^{-1}x} \cdot \dfrac{dx}{\sqrt{1-x^2}} = e^{sin^{-1}x} + c$

$10)\ \displaystyle\int tan^{-1}x \cdot \dfrac{dx}{1+x^2} = \dfrac{(tan^{-1}x)^2}{2} + c$

5-4- Integrals of hyperbolic functions:

The integration formulas for the hyperbolic functions are:

$19)\ \displaystyle\int sinh\,u \cdot du = cosh\,u + c$

$20)\ \displaystyle\int cosh\,u \cdot du = sinh\,u + c$

$21)\ \displaystyle\int tanh\,u \cdot du = ln(cosh\,u) + c$

$22)\ \displaystyle\int coth\,u \cdot du = ln(sinh\,u) + c$

$23)\ \displaystyle\int sec\,h^2 u \cdot du = tanh\,u + c$

$24)\ \displaystyle\int csc\,h^2 u \cdot du = coth\,u + c$

$25)\ \displaystyle\int sec\,hu \cdot tanh\,u \cdot du = -sec\,hu + c$

$26)\ \displaystyle\int csc\,hu \cdot coth\,u \cdot du = -csc\,hu + c$