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1st Stage Mathematics I
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1st term

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Exercise on Integration

1.1 Substitution

Use a suitable substitution to evaluate the following integral.

1. $\int \dfrac{dx}{\sqrt{2-5x}}$

2. $\int \dfrac{e^{3x}+1}{e^x+1}\,dx$

3. $\int \dfrac{x}{\sqrt{1-x^2}}\,dx$

4. $\int x^2\sqrt[3]{1+x^3}\,dx$

5. $\int \dfrac{x\,dx}{(1+x^2)^2}$

6. $\int \dfrac{dx}{\sqrt{x}(1+x)}$

7. $\int \dfrac{1}{x^2}\sin\dfrac{1}{x}\,dx$

8. $\int xe^{-x^2}\,dx$

9. $\int \dfrac{(\ln x)^2}{x}\,dx$

10. $\int \dfrac{e^x\,dx}{2+e^x}$

11. $\int \dfrac{dx}{e^x+e^{-x}}$

12. $\int \dfrac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

13. $\int \tan x\,dx$

14. $\int \dfrac{dx}{1+e^x}$

15. $\int x(x^2+2)^{99}\,dx$

16. $\int \dfrac{x}{\sqrt{25-x^2}}\,dx$

17. $\int \dfrac{x}{\sqrt{3x^2+1}}\,dx$

18. $\int \dfrac{x^2}{\sqrt{9-x^3}}\,dx$

19. $\int x(x+2)^{99}\,dx$

20. $\int \dfrac{x\,dx}{\sqrt{4x+5}}$

21. $\int x\sqrt{x-1}\,dx$

22. $\int (x+2)\sqrt{x-1}\,dx$

23. $\int \dfrac{x\,dx}{\sqrt{x+9}}$

24. $\int x^3(1+3x^2)^{\frac{1}{2}}\,dx$

1.2 Integration By Parts

1. $\int \ln x\,dx$

2. $\int x^2\ln x\,dx$

3. $\int \left(\dfrac{\ln x}{x}\right)^2\,dx$

4. $\int xe^{-x}\,dx$

5. $\int x^2 e^{-2x} dx$

6. $\int x \cos x dx$

7. $\int x^2 \sin 2x dx$

8. $\int (\ln x)^2 dx$

9. $\int \sin^{-1} x dx$

10. $\int x \tan^{-1} x dx$

11. $\int \ln(x + \sqrt{1 + x^2}) dx$

12. $\int x \sin^2 x dx$

13. $\int \sin(\ln x) dx$

14. $\int x \sin 4x dx$

15. $\int x \cos^{-1} x dx$

16. $\int \tan^{-1} x dx$

17. $\int x^{99} \ln x dx$

18. $\int \frac{\ln x}{x^{101}} dx$

19. $\int x \sec^2 x dx$

20. $\int e^{2x} \cos 3x dx$

1.3 Reduction Formula

Prove the following reduction formulas.

1. $I_n = \int x^n e^{ax} dx; \; I_n = \frac{x^n e^{ax}}{a} - \frac{n}{a} I_{n-1}, \; n \geq 1$

2. $I_n = \int \cos^n x dx; \; I_n = \frac{\sin x \cos^{n-1} x}{n} + \frac{n-1}{n} I_{n-2}, \; n \geq 2$

3. $I_n = \int \frac{1}{\sin^n x} dx; \; I_n = -\frac{\cos x}{(n-1) \sin^{n-1} x} + \frac{n-2}{n-1} I_{n-2}, \; n \geq 2$

4. $I_n = \int x^n \cos x dx; \; I_n = x^n \sin x + n x^{n-1} \cos x - n(n-1) I_{n-2}, \; n \geq 2$

5. $I_n = \int \frac{dx}{(x^2 - a^2)^n}; \; I_n = -\frac{x}{2a^2(n-1)(x^2 - a^2)^{n-1}} + \frac{2n-3}{2a^2(n-1)} I_{n-1},$

6. $I_n = \int \frac{x^n dx}{\sqrt{x+a}}; \; I_n = \frac{2x^n \sqrt{x+a}}{2n+1} - \frac{2an}{2n+1} I_{n-1}, \; n \geq 1$

7. $I_n = \int (\ln x)^n dx; \; I_n = x(\ln x)^n - n I_{n-1}, \; n \geq 1.$

8. $I_n = \int_0^1 x^n \sqrt{1-x} dx; \; I_n = \frac{2n}{2n-3} I_{n-1}, \; n \geq 2.$

1.4 Trigonometric Integrals

Evaluate

1. $\int \dfrac{dx}{1 - \cos x}$

2. $\int \sin^5 x \cos x\, dx$

3. $\int \sin 3x \sin 5x\, dx$

4. $\int \cos \dfrac{x}{2} \cos \dfrac{x}{3}\, dx$

5. $\int \cos^3 x\, dx$

6. $\int \sin^4 x\, dx$

7. $\int \sec^2 x \tan^2 x\, dx$

8. $\int \sec x \tan^3 x\, dx$

9. $\int \cot^2 x\, dx$

10. $\int \dfrac{dx}{\cos x \sin^2 x}$

11. $\int \dfrac{\sin x \cos^3 x}{1 + \cos^2 x}\, dx$

12. $\int \tan^5 x\, dx$

13. $\int \dfrac{dx}{\sin^4 x \cos^4 x}\, dx$

14. $\int \sin 5x \cos x\, dx$

15. $\int \cos x \cos 2x \cos 3x\, dx$

16. $\int \cos^5 x \sin^3 x\, dx$

17. $\int \cos^5 x \sin^4 x\, dx$

18. $\int \sin^2 x \cos^4 x\, dx$

1.5 Trigonometric Substitution

Evaluate the following integrals by trigonometric substitution.

1. $\int \dfrac{x^2}{1 + x^2}\, dx$

2. $\int \dfrac{dx}{(1 - x^2)^{\frac{3}{2}}}$

3. $\int \sqrt{\dfrac{1 + x}{1 - x}}\, dx$

4. $\int \dfrac{dx}{(1 + x^2)^{\frac{3}{2}}}$

5. $\int \dfrac{x^2\, dx}{\sqrt{9 - x^2}}$

6. $\int \dfrac{dx}{\sqrt{4 + x^2}}$

7. $\int x^2 \sqrt{16 - x^2}\, dx$

8. $\int \dfrac{dx}{x^2 \sqrt{x^2 + 4}}$

9. $\int \dfrac{dx}{(4x^2 + 1)^{3/2}}$

10. $\int \dfrac{1}{(2x - x^2)^{3/2}}$