



Integration

5-1- Indefinite integrals :

The set of all anti derivatives of a function is called indefinite integral of the function.

Assume u and v denote differentiable functions of x , and a , n , and c are constants, then the integration formulas are:-

$$1) \int du = u(x) + c$$

$$2) \int a \cdot u(x) dx = a \int u(x) dx$$

$$3) \int (u(x) \mp v(x)) dx = \int u(x) dx \mp \int v(x) dx$$

$$4) \int u^n du = \frac{u^{n+1}}{n+1} + c \quad \text{when } n \neq -1 \quad \& \quad \int u^{-1} du = \int \frac{1}{u} du = \ln u + c$$

$$5) \int a^u du = \frac{a^u}{\ln a} + c \quad \Rightarrow \quad \int e^u du = e^u + c$$

EX-1 – Evaluate the following integrals:

$$1) \int 3x^2 dx$$

$$6) \int \frac{x+3}{\sqrt{x^2+6x}} dx$$

$$2) \int \left(\frac{1}{x^2} + x \right) dx$$

$$7) \int \frac{x+2}{x^2} dx$$

$$3) \int x\sqrt{x^2+1} dx$$

$$8) \int \frac{e^x}{1+3e^x} dx$$

$$4) \int (2t + t^{-1})^2 dt$$

$$9) \int 3x^3 \cdot e^{-2x^4} dx$$

$$5) \int \sqrt{(z^2 - z^{-2})^2 + 4} dz$$

$$10) \int 2^{-4x} dx$$

Sol. –

$$1) \int 3x^2 dx = 3 \int x^2 dx = 3 \frac{x^3}{3} + c = x^3 + c$$



$$2) \int (x^{-2} + x) dx = \int x^{-2} dx + \int x dx = \frac{x^{-1}}{-1} + \frac{x^2}{2} + c = -\frac{1}{x} + \frac{x^2}{2} + c$$

$$3) \int x \sqrt{x^2 + 1} dx = \frac{1}{2} \int 2x (x^2 + 1)^{1/2} dx = \frac{1}{2} \frac{(x^2 + 1)^{3/2}}{3/2} + c = \frac{1}{3} \sqrt{(x^2 + 1)^3} + c$$

$$4) \int (2t + t^{-1})^2 dt = \int (4t^2 + 4 + t^{-2}) dt = 4 \frac{t^3}{3} + 4t + \frac{t^{-1}}{-1} + c = \frac{4}{3} t^3 + 4t - \frac{1}{t} + c$$

$$5) \int \sqrt{(z^2 - z^{-2})^2 + 4} dz = \int \sqrt{z^4 - 2 + z^{-4} + 4} dz = \int \sqrt{z^4 + 2 + z^{-4}} dz \\ = \int \sqrt{(z^2 + z^{-2})^2} dz = \int (z^2 + z^{-2}) dz = \frac{z^3}{3} + \frac{z^{-1}}{-1} + c = \frac{1}{3} z^3 - \frac{1}{z} + c$$

$$6) \int \frac{x+3}{\sqrt{x^2+6x}} dx = \frac{1}{2} \int (2x+6) \cdot (x^2+6x)^{-1/2} dx \\ = \frac{1}{2} \cdot \frac{(x^2+6x)^{1/2}}{1/2} + c = \sqrt{x^2+6x} + c$$

$$7) \int \frac{x+2}{x^2} dx = \int \left(\frac{x}{x^2} + \frac{2}{x^2} \right) dx = \int (x^{-1} + 2x^{-2}) dx = \ln x + \frac{2x^{-1}}{-1} + c = \ln x - \frac{2}{x} + c$$

$$8) \int \frac{e^x}{1+3e^x} dx = \frac{1}{3} \int 3e^x (1+3e^x)^{-1} dx = \frac{1}{3} \ln(1+3e^x) + c$$

$$9) \int 3x^3 \cdot e^{-2x^4} dx = -\frac{3}{8} \int -8x^3 \cdot e^{-2x^4} dx = -\frac{3}{8} \cdot e^{-2x^4} + c$$

$$10) \int 2^{-4x} dx = -\frac{1}{4} \int 2^{-4x} \cdot (-4 dx) = -\frac{1}{4} \cdot 2^{-4x} \cdot \frac{1}{\ln 2} + c$$