The inverse hyperbolic functions **: If u is any differentiable function of x, then :**

27) $\frac{d}{dx}\sinh^{-1}u=\frac{1}{\sqrt{1+u^2}}\frac{du}{dx}$

28) $\frac{d}{dx}\cosh^{-1}u=\frac{1}{\sqrt{u^2-1}}\frac{du}{dx}$

29) $\frac{d}{dx}\tanh^{-1}u=\frac{1}{1-u^2}\frac{du}{dx}$ $\quad |u|<1$

30) $\frac{d}{dx}\coth^{-1}u=\frac{1}{1-u^2}\frac{du}{dx}$ $\quad |u|>1$

31) $\frac{d}{dx}\operatorname{sech}^{-1}u=-\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}$

32) $\frac{d}{dx}\operatorname{csch}^{-1}u=-\frac{1}{|u|\sqrt{1+u^2}}\frac{du}{dx}$

EX-16 **- Find $\frac{dy}{dx}$ for the following functions :**

a) $y=\cosh^{-1}(\sec x)$ $\qquad$ b) $y=\tanh^{-1}(\cos x)$

c) $y=\coth^{-1}(\sec x)$ $\qquad$ d) $y=\operatorname{sech}^{-1}(\sin 2x)$

Sol.-

a) $\frac{dy}{dx}=\frac{\sec x.\tan x}{\sqrt{\sec^2 x-1}}=\frac{\sec x.\tan x}{\sqrt{\tan^2 x}}=\sec x \quad$ where $\tan x>0$

b) $\frac{dy}{dx}=\frac{-\sin x}{1-\cos^2 x}=\frac{-\sin x}{\sin^2 x}=-\csc x$

c) $\frac{dy}{dx}=\frac{\sec x.\tan x}{1-\sec^2 x}=\frac{\sec x.\tan x}{-\tan^2 x}=-\csc x$

d) $\frac{dy}{dx}=-\frac{2.\cos 2x}{\sin 2x.\sqrt{1-\sin^2 2x}}=-2\csc 2x \quad$ where $\cos 2x>0$

EX-17 – Verify the following formulas :

a) $\frac{d}{dx}\cosh^{-1}u=\frac{1}{\sqrt{u^2-1}}.\frac{du}{dx}$

b) $\frac{d}{dx}\tanh^{-1}u=\frac{1}{1-u^2}.\frac{du}{dx}$ $\quad |u|<1$

Proof

a) Let $y = \cosh^{-1} u \Rightarrow u = \cosh y$

$$\frac{du}{dx} = \sinh y . \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\sinh y} . \frac{du}{dx}$$

$$\cosh^2 y - \sinh^2 y = 1 \Rightarrow u^2 - \sinh^2 y = 1 \Rightarrow \sinh y = \sqrt{u^2 - 1}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{u^2-1}} . \frac{du}{dx} \Rightarrow \frac{d}{dx} \cosh^{-1} u = \frac{1}{\sqrt{u^2-1}} . \frac{du}{dx}$$

b) Let $y = \tanh^{-1} u \Rightarrow u = \tanh y$

$$\frac{du}{dx} = \sec h^2 y . \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{\sec h^2 y} . \frac{du}{dx}$$

$$\sec h^2 y + \tanh^2 y = 1 \Rightarrow \sec h^2 y + u^2 = 1 \Rightarrow \sec h^2 y = 1 - u^2$$

$$\frac{dy}{dx} = \frac{1}{1-u^2} . \frac{du}{dx} \Rightarrow \frac{d}{dx} \tanh^{-1} u = \frac{1}{1-u^2} . \frac{du}{dx}$$

The derivatives of functions like u^v : **Where u and v are differentiable functions of x , are found by logarithmic differentiation :**

Let $y = u^v \Rightarrow \ln y = v . \ln u$

$$\frac{1}{y} . \frac{dy}{dx} = \frac{v}{u} . \frac{du}{dx} + \ln u . \frac{dv}{dx}$$

$$\frac{dy}{dx} = y\left[\frac{v}{u} . \frac{du}{dx} + \ln u . \frac{dv}{dx}\right]$$

33) $$\frac{d}{dx} u^v = u^v . \left[\frac{v}{u} . \frac{du}{dx} + \ln u . \frac{dv}{dx}\right]$$

EX-18- **Find $\frac{dy}{dx}$ for :**

a) $y = x^{\cos x}$ b) $y = (\ln x + x)^{\tan x}$

Sol. -

a) $y = x^{\cos x} \Rightarrow \ln y = \cos x . \ln x \Rightarrow \frac{1}{y} . \frac{dy}{dx} = \frac{\cos x}{x} + \ln x . (-\sin x)$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\cos x}{x} - \sin x . \ln x\right]$$

or by formula , where $u = x$ and $v = \cos x$

$$\frac{dy}{dx} = y\left[\frac{\cos x}{x} - \sin x . \ln x\right]$$

b) $y=(\ln x+x)^{\tan x} \Rightarrow \ln y=\tan x.\ln(\ln x+x)$

$$\Rightarrow \frac{1}{y}.\frac{dy}{dx}=\frac{\tan x}{\ln x+x}.(\frac{1}{x}+1)+\ln(\ln x+x).\sec^2 x$$

$$\Rightarrow \frac{dy}{dx}=y\left[\frac{(x+1).\tan x}{x(\ln x+x)}+\ln(\ln x+x).\sec^2 x\right]$$

or by formula, where $u=\ln x+x$ *and* $v=\tan x$

$$\frac{dy}{dx}=y.\left[\frac{\tan x}{\ln x+x}(\frac{1}{x}+1)+\ln(\ln x+x).\sec^2 x\right]$$

$$=y.\left[\frac{(x+1).\tan x}{x(\ln x+x)}+\ln(\ln x+x).\sec^2 x\right]$$

Al-Mustaqbal University / College of Engineering & Technology

Department Of Communication Engineering Class (1st)

Subject (calculus 1) / Code (TE-UOMUS-094241217-574)

Lecturer (M.Sc. Fatimatulzahraa Adnan)

2nd term – Lecture No.8 & Lecture Name (Inverse Hyperbolic function)

Problems -3

1. Find $\frac{dy}{dx}$ for the following functions :

1) $y=(x-3)(1-x)$ $\qquad$ $(ans.: 4-2x)$

2) $y=\frac{ax+b}{x}$ $\qquad$ $(ans.: -\frac{b}{x^2})$

3) $y=\frac{3x+4}{2x+3}$ $\qquad$ $(ans.: \frac{1}{(2x+3)^2})$

4) $y=3x^3-2\sqrt{x}+\frac{5}{x^2}$ $\qquad$ $(ans.: 9x^2-\frac{1}{\sqrt{x}}-\frac{10}{x^3})$

5) $y=\left(\sqrt{x^3}-\frac{1}{\sqrt{x^3}}\right)^2$ $\qquad$ $(ans.: \frac{3(x^6-1)}{x^4})$

6) $y=(2x-1)^2(3x+2)^3+\frac{1}{(x-2)^2}$ $\qquad$ $(ans.: (2x-1)(3x+2)^2(30x-1)-\frac{2}{(x-2)^3})$

7) $y=ln(lnx)$ $\qquad$ $(ans.: \frac{1}{x.lnx})$

8) $y=ln(Cosx)$ $\qquad$ $(ans.: -tanx)$

9) $y=Sinx^3$ $\qquad$ $(ans.: 3x^2.Cosx^3)$

10) $y=Cos^{-3}(5x^2+2)$ $\qquad$ $(ans.: \frac{30x.Sin(5x^2+4)}{Cos^4(5x^2+4)})$

11) $y=tanx.sinx$ $\qquad$ $(ans.: Sinx+tanx.Secx)$

12) $y=tan(Secx)$ $\qquad$ $(ans.: Sec^2(Secx).Secx.tanx)$

13) $y=Cot^3\left(\frac{x+1}{x-1}\right)$ $\qquad$ $(ans.: \frac{6}{(x-1)^2}.Cot^2\left(\frac{x+1}{x-1}\right).Csc^2\left(\frac{x+1}{x-1}\right))$

14) $y=\frac{Cosx}{x}$ $\qquad$ $(ans.: -\frac{x.Sinx+Cosx}{x^2})$

15) $y=\sqrt{tan\sqrt{2x+7}}$ $\qquad$ $(ans.: \frac{Sec^2\sqrt{2x+7}}{2\sqrt{2x+7}\sqrt{tan\sqrt{2x+7}}})$

16) $y=x^2.Sinx$ $\qquad$ $(ans.: x^2.Cosx+2x.Sinx)$

17) $y=Csc^{-\frac{2}{3}}\sqrt{5x}$ $\qquad$ $(ans.: \frac{5}{3\sqrt{5x}}.\frac{Cot\sqrt{5x}}{Csc^{\frac{2}{3}}\sqrt{5x}})$

18) $y=x[Sin(lnx)+Cos(lnx)]$ $\qquad$ $(ans.: 2.Cos(lnx))$