Digital Electronics

For

Second class

by

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Boolean Algebra

Because binary logic is used in all of today's digital computers and devices, the cost of the circuits that implement it is an important factor addressed by designers—be they computer engineers, electrical engineers, or computer scientists. Finding simpler and cheaper, but equivalent, realizations of a circuit can reap huge payoffs in reducing the overall cost of the design. Boolean algebra that will enable you to optimize simple circuits and to understand the purpose of algorithms used by software tools to optimize complex circuits involving millions of logic gates. Boolean algebra, like any other deductive mathematical system, may be defined with a set of elements, a set of operators, and a number of unproved axioms or postulates.

Boolean algebra is an algebraic structure defined on a set of elements B together with two binary operators (+) and (.) The two valued Boolean algebra is defined on a set of two elements. B ={0,1}.

❖ Boolean laws and theorems

الابدالية	1-	Commutative over +	$A + B = B + A$	$A.B = B.A$	Commutative over.
ترتيب الحدود	2-	Associative over +	$A + (B+C) = (A+B) +C$	$A. (B.C) = (A.B).C$	Associative over.
توزيع الحدود	3-	Distributive over +	$A.(B+C)= A.B+A.C$	$A+(B.C)=(A+B)(A+C)$	distributive over .
	4-	OR Operations	$A + 0 = A$	$A.1=A$	AND operations
	5-		$A + 1 = 1$	$A.0=0$	
	6-		$A + A = A$	$A.A = A$	
	7-		$A + \overline{A} = 1$	$A.\overline{A} = 0$	
	8-	Double inversion	$\overline{\overline{A}} = A$	$\overline{\overline{A}} = A$	Double inversion
	9-	Demorgan 1s't law	$\overline{A + B} = \overline{A} . \overline{B}$	$\overline{A.B} = \overline{A} + \overline{B}$	Demorgan 2nd law
التبسيط	10-		$A + A.B = A$	$A.(A+B) =A$	
	11-		$A + \overline{A}.B = A + B$	$A(\overline{A} + B) = A.B$	

❖ Duality :

This important property of Boolean algebra is called the duality principle and states that every algebraic expression deducible from the postulates of Boolean algebra remains valid if the operators and identity elements are interchanged.

1- Changing every OR by AND and vice versa.
2- Changing every 1 to 0 and every 0 to 1.

Example:- $A+0=A \ \underrightarrow{\text{duality the } 0} \ A.1=A$

$$A(B+C)=AB+AC \rightarrow A+BC=(A+B)(A+C)$$

The Proof of theorems 10 & 11

Theorem 10 $\rightarrow$ (a) $A+AB = A.1+AB = A(1+B) = A$

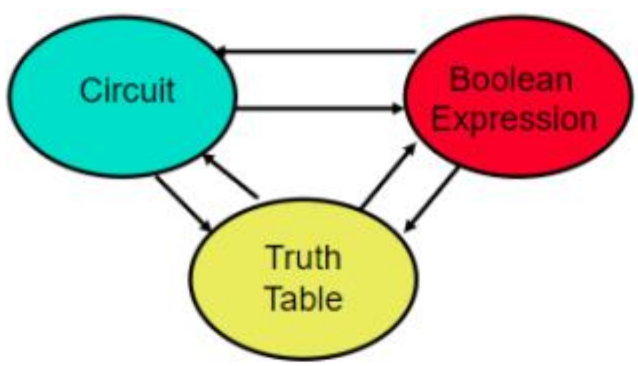
(b) $A.(A+B) = A.A + AB = A + AB = A.1 + AB$

$= A(1+B) = A$

Theorem 11 (a) $A+\overline{A}B = (A+\overline{A}).(A+B) = 1.(A+B) = A+B$

(b) $A.(\overline{A}+B) = A.\overline{A} + A.B = 0 + A.B = A.B$

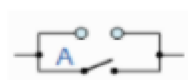
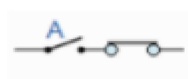
or it can proved using duality theorem

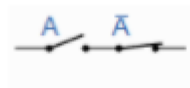
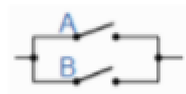
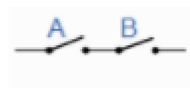


❖ truth table of each gate

INPUTS		OUTPUTS					
A	B	AND	NAND	OR	NOR	EXOR	EXNOR
0	0	0	1	0	1	0	1
0	1	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	1	1	0	1	0	0	1

Truth Tables for the Laws of Boolean

Boolean Expression	Description	Equivalent Switching Circuit	Boolean Algebra Law or Rule
A + 1 = 1	A in parallel with closed = "CLOSED"		Annulment
A + 0 = A	A in parallel with open = "A"		Identity
A . 1 = A	A in series with closed = "A"		Identity
A . 0 = 0	A in series with open = "OPEN"		Annulment
A + A = A	A in parallel with A = "A"		Idempotent
A . A = A	A in series with A = "A"		Idempotent
NOT $\overline{A}$ = A	NOT NOT A (double negative) = "A"		Double Negation
A + $\overline{A}$ = 1	A in parallel with NOT A = "CLOSED"		Complement

$A \cdot \bar{A} = 0$	A in series with NOT A = "OPEN"		Complement
$A+B = B+A$	A in parallel with B = B in parallel with A		Commutative
$A.B = B.A$	A in series with B = B in series with A		Commutative
$\overline{A+B} = \bar{A}.\bar{B}$	invert and replace OR with AND		de Morgan's Theorem
$\overline{A.B} = \bar{A}+\bar{B}$	invert and replace AND with OR		de Morgan's Theorem