5-2- Integrals of trigonometric functions :

The integration formulas for the trigonometric functions are:

$$6)\ \int \sin u \cdot du = -\cos u + c \qquad 7)\ \int \cos u \cdot du = \sin u + c$$

$$8)\ \int \tan u \cdot du = -\ln|\cos u| + c \qquad 9)\ \int \cot u \cdot du = \ln|\sin u| + c$$

$$10)\ \int \sec u \cdot du = \ln|\sec u + \tan u| + c \qquad 11)\ \int \csc u \cdot du = -\ln|\csc u + \cot u| + c$$

$$12)\ \int \sec^2 u \cdot du = \tan u + c \qquad 13)\ \int \csc^2 u \cdot du = -\cot u + c$$

$$14)\ \int \sec u \cdot \tan u \cdot du = \sec u + c \qquad 15)\ \int \csc u \cdot \cot u \cdot du = -\csc u + c$$

EX-2- Evaluate the following integrals:

$$1)\ \int \cos(3\theta - 1)\, d\theta \qquad 6)\ \int \frac{d\theta}{\cos^2 \theta}$$

$$2)\ \int x \cdot \sin(2x^2)\, dx \qquad 7)\ \int \left(1 - \sin^2 3t\right) \cdot \cos 3t\ dt$$

$$3)\ \int \cos^2(2y) \cdot \sin(2y)\, dy \qquad 8)\ \int \tan^3(5x) \cdot \sec^2(5x)\ dx$$

$$4)\ \int \sec^3 x \cdot \tan x\ dx \qquad 9)\ \int \sin^4 x \cdot \cos^3 x\ dx$$

$$5)\ \int \sqrt{2 + \sin 3t} \cdot \cos 3t\ dt \qquad 10)\ \int \frac{\cot^2 \sqrt{x}}{\sqrt{x}}\, dx$$

Sol.-

$$1)\ \frac{1}{3}\int 3\cos(3\theta-1)\,d\theta=\frac{1}{3}\sin(3\theta-1)+c$$

$$2)\ \frac{1}{4}\int 4x\cdot\sin(2x^2)\,dx=-\frac{1}{4}\cos(2x^2)+c$$

$$3)\ -\frac{1}{2}\int(\cos 2y)^2\cdot(-2\sin 2y\ dy)=-\frac{1}{2}\cdot\frac{(\cos 2y)^3}{3}+c=-\frac{1}{6}(\cos 2y)^3+c$$

$$4)\ \int\sec^2 x\cdot(\sec x\cdot\tan x\cdot dx)=\frac{\sec^3 x}{3}+c$$

$$5)\ \frac{1}{3}\int(2+\sin 3t)^{1/2}(3\cos 3t\ dt)=\frac{1}{3}\cdot\frac{(2+\sin 3t)^{3/2}}{3/2}+c=\frac{2}{9}\sqrt{(2+\sin 3t)^3}+c$$

$$6)\ \int\frac{d\theta}{\cos^2\theta}=\int\sec^2\theta\cdot d\theta=\tan\theta+c$$

$$7)\ \int(1-\sin^2 3t)\cdot\cos 3t\ dt=\frac{1}{3}\int 3\cos 3t\ dt-\frac{1}{3}\int(\sin 3t)^2\cdot 3\cos 3t\ dt$$

$$=\frac{1}{3}\sin 3t-\frac{1}{3}\cdot\frac{\sin^3 3t}{3}+c=\frac{1}{3}\cdot\sin 3t-\frac{1}{9}\sin^3 3t+c$$

$$8)\ \frac{1}{5}\int\tan^3 5x\cdot(5\sec^2 5x\ dx)=\frac{1}{5}\cdot\frac{\tan^4 5x}{4}+c=\frac{1}{20}\tan^4 5x+c$$

$$9)\ \int\sin^4 x\cdot\cos^3 x\ dx=\int\sin^4 x\cdot(1-\sin^2 x)\cdot\cos x\ dx$$

$$=\int\sin^4 x\cdot\cos x\ dx-\int\sin^6 x\cdot\cos x\ dx=\frac{\sin^5 x}{5}-\frac{\sin^7 x}{7}+c$$

$$10)\ \int\frac{\cot^2\sqrt{x}}{\sqrt{x}}dx=\int\frac{\csc^2\sqrt{x}-1}{\sqrt{x}}dx=2\int\frac{\csc^2\sqrt{x}}{2\sqrt{x}}-\int x^{-1/2}\,dx$$

$$=2\left(-\cot\sqrt{x}\right)-\frac{x^{1/2}}{1/2}+c=-2\cot\sqrt{x}-2\sqrt{x}+c$$