## *System of Units*

The principle aspects of the scientific method are accurate measurement, selective analysis, and mathematical formulation. Note that the first and most important is accurate measurements.

***Measurement*: *is the process by which one can convert physical parameters to meaningful number.***

***Instrument:*** *may be defined as a device for determining the value or magnitude of a quantity or variable.*

The standard measure of each kind of physical quantity is the unit; the number of times the unit occurs in any given amount of the same quantity is the number of measure. With out the unit, the number of measure has no physical meaning.

## *Fundamental and Derived Units*

To measure an unknown we must have acceptable unit standard for the property that is to be assessed. Since there are virtually hundreds of different quantities that man is called upon to measure, it would seem that hundreds of different standard units would be required. Fortunately, this is not the case. By choosing a small number of basic quantities as standards, we can define all the other in terms of these few.

The basic units are called ***fundamentals***, while all the others which can be expressed in terms of fundamental units are called ***derived*** units, and formed by multiplying or dividing fundamental units. The ***primary fundamental*** units which most commonly used are ***length, mass, and time,*** while measurement of certain physical quantities in ***thermal, electrical, and illumination*** disciplines are also represented by fundamental units. These units are used only when these particular classes are involved, and they may therefore be defined as ***auxiliary fundamental*** units. Every derived unit originates from some physical law defining that unit. For example, the voltage [volt]:

$$volt=\frac{workdone}{charge}=\frac{Joule}{coulomb}=\frac{J}{C}=\frac{Force\times distance}{current\times time}=\frac{Newton\times meter}{Amper\times second}\Rightarrow$$

$$volt=\frac{mass\times acceleration\times meter}{current\times time}=\frac{mass\times\frac{velocity}{time}\times meter}{current\times time}=\frac{mass\times\frac{distance}{time^2}\times meter}{current\times time}$$

$$volt=\frac{mass\times\frac{meter^2}{time^2}}{current\times time}=\frac{mass\times meter^2}{current\times time^3}=\frac{Kg.m^2}{A.\sec^3}=\left[Kg.m^2.A^{-1}.\sec^{-3}\right]$$ basic S.I units

A derived unit is recognized by its ***dimensions***, which can be defined as the complete algebraic formula for the derived unit. The dimensional symbols for the fundamental units of length, mass, and time are **L, M**, and **T**, respectively. So the dimensional symbol for the derived unit of voltage is $V=\frac{M.L^2}{I.T^3}=\left[M.L^2.I^{-1}.T^{-3}\right]$

Table (1) shows the six basic ***S.I*** quantity and units of measurement, with their unit symbol:

***Table (1):***

| *Quantity* | *Unit* | *Symbol* |
|---|---|---|
| Length | Meter | m |
| Mass | Kilogram | kg |
| Time | Second | s |
| Electrical current | Ampere | A |
| Thermodynamic temperature | Kelvin | K |
| | | |

## *Multiples and Submultiples of units*

The units in actual use are divided into submultiples for the purpose of measuring quantities smaller than the unit itself. Furthermore, multiples of units are designated and named so that measurement of quantities much larger than the unit is facilitated. Table(2) lists the decimal multiples and submultiples of units.

***Table(2):***

| *Name* | *Symbol* | *Equivalent* |
|---|---|---|
| tera | T | $10^{12}$ |
| giga | G | $10^{9}$ |
| mega | M | $10^{6}$ |
| kilo | K | $10^{3}$ |
| milli | m | $10^{-3}$ |
| micro | µ | $10^{-6}$ |
| nano | n | $10^{-9}$ |
| pico | p | $10^{-12}$ |

## *Basic Definitions:*

1. ***Speed, Velocity***: the rate of change of distance with respect to time

$$\upsilon = \frac{\partial x}{\partial t} \quad , \qquad \chi = \int_0^t \upsilon \partial t = \upsilon . t \quad , \qquad \upsilon = \frac{\chi}{t}$$

$\upsilon = \left[ LT^{-1} \right]$ basic dimensions, $\upsilon = \left[ m \sec^{-1} \right]$ basic S.I units

2. ***Acceleration:*** the rate of change of velocity during the time

$$a = \frac{\partial \upsilon}{\partial t} , \qquad \upsilon = \int_0^t a \partial t = a . t \quad , \qquad a = \frac{\upsilon}{t}$$

$a = \left[ LT^{-2} \right]$ basic dimensions, $a = \left[ m \sec^{-2} \right]$ basic S.I units

3. ***Momentum:***

$$p = mass \times velocity = m \times \upsilon$$

$p = [MLT^{-1}]$ basic dimensions , $p = [kgm\,\sec^{-1}]$ basic S.I units

4. ***Force:*** (Newton), the rate of change of momentum during the time

$F = \frac{\partial p}{\partial t} = \frac{\partial(m\upsilon)}{\partial t}$, $F = [MLT^{-2}]$ basic dimensions, $F = [kgm\,\sec^{-2}]$ basic S.I units

5. ***Energy:*** (Joule), the distance integral of force

$$E = \int_0^{\chi} F\partial\chi = F.\chi$$

$E = [ML^2T^{-2}]$ basic dimensions, $E = [kgm^2\sec^{-2}] = Joule = J$

6. ***Power:*** (Watt) , the rate of work done

$$P = \frac{\partial E}{\partial t}$$

$P = [ML^2T^{-3}]$ basic dimensions, $P = [kgm^2\sec^{-3}]$ S.I units, $P = J.\sec^{-1}$

7. ***Potential of a point (voltage):*** work done to bring a unit charge from infinity to same point.

$$V = \frac{workdone}{charge} = \frac{Joule}{coulomb}$$

$V = [ML^2I^{-1}T^{-3}]$ basic dimensions, $V = [kgm^2A^{-1}\sec^{-3}]$ basic S.I units

8. ***Electrical current:*** the rate of flow of charge

$I = \frac{\partial Q}{\partial t}$, $Q = \int_0^t I\partial t$, $Q = I.t$

$I = [Amp]$

9. ***Resistance (ohm):*** the resistance of a load to the current flow when there is voltage difference between its terminals.

$R = \frac{\partial V}{\partial I}$, $R = [ML^2I^{-2}T^{-3}]$ dimensions, $R = [kgm^2A^{-2}\sec^{-3}]$ basic S.I units

10. ***Capacitance (farad):***

$$C = \varepsilon \frac{A}{d}, \quad \text{or } C = \frac{Q}{V}, \quad C = \left[M^{-1}L^{-2}I^{2}T^{4}\right], \quad C = \left[kg^{-1}m^{-2}A^{2}\sec^{4}\right]$$

11. ***Electrical field:***

$$E = \frac{\partial V}{\partial x}, \quad E = \left[MLI^{-1}T^{-3}\right], \quad E = \left[kgmA^{-1}\sec^{-3}\right]$$

12. ***Permittivity Є***: how much electrical field lines can pass through some medium

$$\epsilon = \frac{farad}{m}, \quad \epsilon = \left[M^{-1}L^{-3}I^{2}T^{4}\right], \quad \epsilon = \left[kg^{-1}m^{-3}A^{2}\sec^{4}\right]$$

13. ***Inductance(henry):***

Induce emf = inductance x rate of change of current

$$e = -L\frac{\partial i}{\partial t}, \qquad \int_0^t e\partial t = L\int_0^i \partial i, \qquad L = \frac{et}{I}$$

$$Henry = \left[ML^{2}I^{-2}T^{-2}\right], \quad Henry = \left[kgm^{2}A^{-2}\sec^{-2}\right]$$

14. ***Reluctance (S):*** the magnetic resistance to magnetic field lines in same material

$$S = \frac{l}{\mu . A}, \quad S = \left[M^{-1}L^{-2}I^{2}T^{2}\right], \quad S = [kg^{-1}m^{-2}A^{2}\sec^{2}]$$

15. ***Magnetic flux(Φ) weber:***

$$\phi = \frac{mmf}{S} = \frac{N.I}{S}, \quad \phi = \left[ML^{2}I^{-1}T^{-2}\right], \quad \phi = \left[kgm^{2}A^{-1}\sec^{-2}\right]$$

16. ***Frequency(hertz):*** number of cycles in one second

$$f = \frac{cycles}{\sec ond} = \frac{1}{\sec}, \quad f = \left[T^{-1}\right], \quad f = \left[\sec^{-1}\right]$$

## Accuracy

**Definition**: Accuracy refers to how close a measured value is to the true or accepted value. In other words, an instrument with high accuracy yields results that are near the "correct" answer.

- **Importance in Electrical Measurements**: Accuracy is vital in fields like power measurement, signal processing, and voltage/current sensing, where small deviations can lead to significant issues or safety risks.
- **Influencing Factors**:
  - **Systematic Errors**: Errors that consistently skew measurements in one direction (e.g., calibration issues).
  - **Environmental Factors**: Temperature, humidity, and even electromagnetic interference can affect accuracy.
- **Example**: If a multimeter measures 5.1 V for a true value of 5.0 V, it's not perfectly accurate, but the error margin could be acceptable based on tolerance levels.

Accuracy is always reported with the error margin, e.g., "accuracy of ±0.2%."

## Precision

**Definition**: Precision refers to the consistency of repeated measurements. It is the ability of an instrument to give the same reading when measuring the same quantity multiple times under the same conditions.

- **Importance in Electrical Measurements**: Precision is crucial in research and quality control. High precision allows for the detection of trends, anomalies, or small changes in measured values over time.
- **Influencing Factors**:
  - **Instrument Design**: High-quality components and stable electronics improve precision.
  - **Operator Skill and Measurement Procedure**: Precise measurements require careful handling of instruments.

- **Example**: If a multimeter shows readings of 5.10 V, 5.11 V, and 5.09 V in repeated trials, it is precise even if the true value is 5.00 V.

An instrument can be precise but not accurate (e.g., consistently off from the true value).

## Resolution

**Definition**: Resolution is the smallest change in a quantity that an instrument can detect. In digital instruments, it is related to the number of digits or the least count displayed.

- **Importance in Electrical Measurements**: Resolution determines an instrument's ability to detect minute changes. In fields requiring fine measurements, such as microelectronics, resolution is critical.
- **Factors Influencing Resolution**:
  - **Display and Digital Capabilities**: For digital devices, the number of bits affects resolution. For example, an 8-bit system can detect finer changes than a 4-bit system.
  - **Noise Level**: High noise can obscure small changes, reducing effective resolution.
- **Example**: A multimeter with a resolution of 0.01 V can distinguish between 5.10 V and 5.11 V, while one with a 0.1 V resolution would display both as 5.1 V.

Resolution does not imply accuracy or precision; it only indicates the smallest unit of measurement an instrument can display.

## Comparing Accuracy, Precision, and Resolution

In electrical measurements:

- An instrument that's **accurate but not precise** could measure a voltage close to the true value but have inconsistent readings across trials.
- An instrument that's **precise but not accurate** will show consistent readings, though they may deviate from the true value.

- An instrument with **high resolution** can detect fine changes, even if those measurements aren't necessarily accurate or precise.

**Real-World Applications in Electrical Measurements**

1. **High-Precision Resistors**: Used in precise measurements, these resistors must have high accuracy and low tolerance to maintain circuit performance.
2. **Power Meters for Renewable Energy**: Power measurement requires accurate, precise, and high-resolution devices due to the varying and often small energy outputs.
3. **Oscilloscopes**: In signal analysis, accuracy and high resolution are essential for capturing small fluctuations in voltage over time.
4. **Biomedical Equipment**: Instruments like ECG monitors require high precision to detect and analyze small signals accurately.

**Challenges and Trade-Offs**

In practice, achieving high accuracy, precision, and resolution often involves trade-offs:

- **Cost**: High-resolution, accurate, and precise instruments are generally more expensive.
- **Speed vs. Resolution**: In digital instruments, higher resolution might slow down data acquisition rates, as finer measurements require more processing.

## Reliability

**Definition**: Reliability refers to the consistency of measurement results over time. In other words, a reliable instrument will produce the same results under the same conditions, regardless of when the measurement is taken.

- **Importance in Electrical Measurements**: Reliability is crucial because it ensures that measurements remain consistent across different instances. For example, in manufacturing or monitoring systems, reliable measurements enable consistent quality control and safety assurance.
- **Influencing Factors**:
  - **Instrument Durability and Quality**: High-quality components that resist wear and drift are key to maintaining reliability.
  - **Calibration**: Regular calibration keeps instruments aligned with standards, thereby enhancing reliability.
  - **Environmental Conditions**: Stability in environmental conditions (e.g., temperature, humidity) prevents measurement variability.
- **Example**: A reliable voltmeter should give the same voltage reading for a constant voltage source, regardless of how many times the measurement is repeated over a span of weeks or months.

Reliability doesn't imply accuracy—it means that repeated measurements yield the same result, even if that result is consistently off from the true value.

## Repeatability

**Definition**: Repeatability is a measure of an instrument's ability to yield the same result when the same measurement is performed multiple times in the same conditions, typically in a short period and with the same equipment, operator, and environment.

- **Importance in Electrical Measurements**: Repeatability is essential for ensuring confidence in a single measurement session. When measurements show repeatability, any observed fluctuations are likely from actual changes in the measured quantity rather than inconsistencies in the measurement process itself.
- **Factors Influencing Repeatability**:
  - **Instrument Sensitivity and Design**: A well-designed instrument minimizes random errors, contributing to better repeatability.
  - **Operator Consistency**: Ensuring that the same procedures are followed minimizes human error, improving repeatability.
  - **Stable Conditions**: External fluctuations, like electrical noise or temperature changes, should be minimized.

- **Example**: If a digital multimeter shows readings of 5.12 V, 5.12 V, and 5.13 V over three immediate trials on the same voltage source, it has high repeatability.

Repeatability focuses on short-term consistency, while reliability assesses consistency over a longer timeframe.

## Validity

**Definition**: Validity refers to the degree to which an instrument measures what it is intended to measure. In other words, validity ensures that the instrument's readings genuinely reflect the targeted quantity and are not influenced by other factors.

- **Importance in Electrical Measurements**: Validity is fundamental to ensure that the measurement data can be trusted to represent the true parameter. Validity is essential for meaningful interpretations and decisions based on the data.
- **Factors Affecting Validity**:
  - **Instrument Design and Calibration**: If an instrument is correctly designed and calibrated, it will provide valid measurements for the intended parameter.
  - **Appropriateness of Instrument**: The instrument should match the measurement's purpose. Using a DC voltmeter for AC signals, for instance, would compromise validity.
  - **Minimization of External Influences**: The setup should ensure that the instrument only measures the target quantity without interference from extraneous variables (like stray capacitance or magnetic fields).
- **Example**: A valid voltmeter measures only the actual voltage applied to it, without being affected by nearby magnetic fields or temperature fluctuations that could introduce error.

Validity means the measurement is appropriate for its intended purpose, capturing the true value of the parameter without distortion.

## Comparing Reliability, Repeatability, and Validity

To distinguish these terms, let’s consider an analogy involving a **thermometer** used to measure room temperature:

- **Reliability**: If the thermometer is reliable, it will give consistent readings each day for the same room at a stable temperature, regardless of the season or day.

- **Repeatability**: If it has good repeatability, it will give nearly identical readings for the same measurement session, even if we measure several times in a short interval.
- **Validity**: If the thermometer is valid, it will accurately reflect the actual temperature of the room, without being affected by other factors (e.g., direct sunlight on the thermometer skewing the reading).

For electrical measurements:

- An **electrical instrument with high reliability** will yield consistent readings for the same parameter over time.
- An instrument with **high repeatability** will show consistent results when measurements are repeated quickly under identical conditions.
- A **valid instrument** accurately reflects the true value of the target parameter, without interference from other variables.

## Applications in Electrical Measurements

1. **Industrial Monitoring Systems**: Reliability is essential in systems that monitor electrical parameters over time (like voltage or current) to ensure consistent equipment performance.
2. **Research Laboratories**: High repeatability is critical in research settings where experiments are repeated under controlled conditions to observe subtle changes.
3. **Safety Testing**: Validity is crucial in safety-critical testing. For example, in testing insulation resistance, the instrument must be valid for that purpose, without interference from other environmental factors.
4. **Quality Control in Manufacturing**: Reliability, repeatability, and validity are all essential to ensure products meet specifications without variability due to measurement errors.

## Challenges and Considerations

Achieving high reliability, repeatability, and validity in measurements often involves specific challenges and trade-offs:

- **Balancing Cost and Quality**: High-quality instruments that provide reliable, repeatable, and valid measurements may be expensive, but they are essential in critical applications.
- **Environmental Control**: Maintaining a stable environment can be challenging but is crucial, especially for measurements sensitive to temperature, humidity, or electromagnetic interference.
- **Regular Calibration and Maintenance**: Instruments must be calibrated periodically to maintain reliability and validity. Calibration adjusts for drift and realigns the instrument with measurement standards.

# *Types of Errors*

Errors in measurement are differences between the measured value and the true value of a quantity. No measurement is completely free from errors, but understanding their types allows engineers to interpret data more accurately and refine measurement processes.

Errors can broadly be classified into:

- **Systematic Errors**: Predictable and consistent deviations.
- **Random Errors**: Unpredictable variations that scatter measurements around a mean value.

## Systematic Errors

**Definition**: Systematic errors are consistent, repeatable errors that occur due to identified causes. They either skew measurements in one direction (always too high or too low) or produce a bias in the data.

**Characteristics of Systematic Errors**:

- **Consistent**: They occur in the same way each time the measurement is taken.
- **Directional Bias**: They tend to push results consistently higher or lower than the true value.
- **Predictable and Quantifiable**: Since systematic errors are consistent, they can often be identified, quantified, and corrected.

**Types of Systematic Errors**

1. **Instrumental Errors**: Caused by imperfections or limitations in the measuring instruments.

**i)** Damaged equipment such as defective due to loading effect or worn parts.
**ii)** Calibration errors.
**iii)** Bearing fraction.
**iv)** Component nonlinearities.
**v)** Loss during transmission.
**vi)** Proper position of equipment (vertical or horizontal).
**vii)** Static charge error.

   - **Example**: A voltmeter that consistently reads 0.1 V too high due to a calibration issue. Every voltage measurement will have a 0.1 V offset.

2. **Environmental Errors**: Occur due to external factors such as :
    - **i)** Change in temperature, pressure.
    - **ii)** Humidity.
    - **iii)** Stray electric and magnetic fields.
    - **iv)** Mechanical vibration.
    - **v)** Weather variations (day, night, and four seasons).
    -
    - **Example**: High temperatures causing a thermometer to read higher voltages due to an increase in resistance in its circuitry.
3. **Observational Errors**:
    - Errors introduced by human interpretation or methodology. Parallax error, where readings are misinterpreted due to the angle of observation.
    - **Example**: Reading an analog ammeter slightly from the side instead of head-on, resulting in a consistently lower or higher current reading.
4. **Theoretical Errors**:
    - Assumptions or simplifications in measurement theories or formulas can lead to these errors. In high-frequency measurements, assuming ideal component behavior can lead to deviations from expected values.
    - **Example**: Assuming ideal resistor behavior in a high-frequency circuit can introduce error because real resistors exhibit some inductance and capacitance.

**<u>Detecting and Reducing Systematic Errors</u>**

- **Calibration**: Regular calibration of instruments with standard references minimizes systematic errors.
- **Environmental Control**: Controlling temperature, humidity, and electromagnetic interference in the measurement environment.
- **Improved Measurement Technique**: Using procedures that eliminate observational errors (e.g., using digital displays instead of analog).
- **Cross-Verification**: Measuring with different instruments or methods to check for consistency.

## Random Errors

**Definition**: Random errors are unpredictable fluctuations in measurements that vary from one measurement to another, even when conditions appear identical.

### Characteristics of Random Errors:

- **Unpredictable**: They can’t be anticipated or eliminated entirely, only minimized.
- **Scatter**: They create a spread of values around the true value, affecting precision but not necessarily the accuracy of the mean.
- **Statistically Distributable**: Random errors often follow a normal distribution (Gaussian), where most measurements cluster around the mean value.

### Sources of Random Errors

1. **Thermal Noise**:
   - Caused by the random motion of electrons in a conductor, which produces fluctuations in voltage and current.
   - Example: In sensitive circuits like low-voltage amplifiers, thermal noise can introduce random variations in readings.
2. **Electromagnetic Interference (EMI)**:
   - Random signals from external sources like nearby electronics, radio waves, or power lines.
   - Example: Power line noise can affect voltage measurements in circuits, especially at 50/60 Hz and its harmonics.
3. **Mechanical Vibrations**:
   - Physical disturbances like vibrations can introduce minor fluctuations, particularly in sensitive equipment.
   - Example: Vibrations in a laboratory can affect sensitive instruments like balances or optical sensors.
4. **Quantum Fluctuations and Low-Level Signal Variability**:
   - For very low current or voltage signals, quantum effects can introduce inherent randomness.
   - Example: In nano-electronics and quantum circuits, uncertainty in electron flow can lead to random noise in measurements.

### Quantifying and Reducing Random Errors

- **Averaging Measurements**: Taking multiple measurements and averaging them can reduce random errors since they tend to cancel each other out over many trials.
- **Signal Filtering**: Techniques like low-pass filtering help to remove high-frequency noise from the signal, reducing random fluctuations.

- **Shielding**: Using shielded cables and enclosures can prevent EMI, which is a common source of random errors.
- **Using Precision Instruments**: Higher precision instruments with low internal noise contribute to reducing random errors in measurements.

**Comparing Systematic and Random Errors**

| Feature | Systematic Errors | Random Errors |
|---|---|---|
| **Predictability** | Predictable, follow a consistent pattern | Unpredictable, scattered around true value |
| **Effect on Data** | Biases data consistently in one direction | Creates scatter around the mean |
| **Reduction** | Calibration, environmental control | Averaging, filtering, shielding |
| **Impact on Accuracy and Precision** | Lowers accuracy | Lowers precision |

**Practical Applications in Electrical Measurements**

1. **Power Systems Monitoring**: Systematic errors in current or voltage measurements can lead to incorrect readings, which may cause inefficiencies or even dangerous conditions. Regular calibration and controlled environments are essential to minimize these errors.
2. **High-Sensitivity Electronics**: Random errors like thermal noise or EMI are critical factors when dealing with small signal measurements (e.g., in biomedical devices). Shielding and filtering can reduce the impact of random noise.
3. **Signal Processing**: In applications like telecommunications, systematic and random errors can distort signals. Noise-reduction algorithms, error-correcting codes, and careful circuit design minimize these errors for

**Strategies for Error Management**

1. **Identify and Quantify Systematic Errors**: Through calibration and environmental controls, systematic errors can often be corrected or accounted for in data analysis.
2. **Reduce Random Errors through Repetition and Averaging**: By increasing the sample size of measurements, random errors can be minimized through statistical averaging.
3. **Instrument Selection and Setup**: Using high-quality instruments with known precision and ensuring proper measurement setup helps reduce both error types.

**Tolerance:**

A certain amount of error will inevitably occur between the measured value and the true value. What is important is to specify the allowable range of errors. In terms of measurement, the difference between the maximum and minimum dimensions of permissible errors is called the "tolerance." The allowable range of errors prescribed by law

**Tolerance= accuracy x $V_{FS}$**

Measured value = true ± tolerance

$$errors = \frac{true - measured}{true} \times 100\% \qquad \text{Or } error = \frac{\pm Tolerance}{True} \times 100\%$$

***Example (1):***

*(Systematic, Human errors, the proper range of measurement)*

A 0 to 150V voltmeter has accuracy of 1% of full scale reading. The theoretical (true) expected value we want to measure it is 83V. Determine the practical (measured) value and the percentage of error.

***Sol.:***

Tolerance= accuracy x $V_{FS}$

Tolerance= 1% x 150 = 0.01 x 150 = 1.5V
Measured value= true ± tolerance
Measured value= 83 ± 1.5
Measured value= 84.5V or 81.5V

The percentage error is:

$$errors = \frac{true - measured}{true} \times 100\%$$

$$error = \frac{|83 - 84.5|}{83} \times 100\% = 1.81\% \text{, or} \qquad error = \frac{|83 - 81.5|}{83} \times 100\% = 1.81\%$$

$$\text{Or } error = \frac{|\pm Tolerance|}{True} \times 100\% = \frac{|\pm 1.5|}{83} \times 100\% = 1.81\%$$

If we want to measured anther readings on the same range and determine the error, suggest we take true 60V, and 30V.

For 60V the error is:

$$error = \frac{|\pm Tolerance|}{True} \times 100\% = \frac{|\pm 1.5|}{60} \times 100\% = 2.5\%$$

And for 30V

$$error = \frac{|\pm Tolerance|}{True} \times 100\% = \frac{|\pm 1.5|}{30} \times 100\% = 5\%$$

So we can see that the error is increased as smaller voltage is measured, thus take the proper range for every measured value, the range that give big deflection on the pointer as possible.

Resistance of voltmeter = sensitivity × $V_{FS}$

$$R_{act.} = \frac{R_{ap.} \times R_v}{R_v - R_{ap.}}$$

***Example (2):***

*(Systematic, Human errors, the difference between theoretical and practical instruments)*

To measured unknown resistor by ammeter and voltmeter method. A voltmeter of sensitivity 1000Ω/V, connect in parallel with the resistor reads 100V on its 150V scale (range), while the series ammeter read 5mA. Calculate the apparent value of the resistor, actual value, and the error.

***Sol.:***

**1-** The apparent value of the resistor is:

$$R_{ap.} = \frac{V}{I} = \frac{100}{5mA} = 20K\Omega$$

**2-** The actual value of the resistor by taking the resistance of voltmeter in consider is:

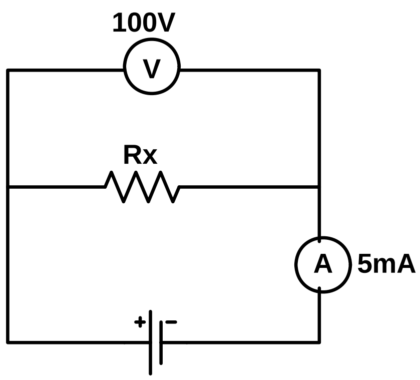
$$R_V = 1000\frac{\Omega}{V} \times 150V = 150K\Omega$$

$$R_{act.} = \frac{R_{ap.} \times R_v}{R_v - R_{ap.}} = \frac{20 \times 150}{150 - 20} = 23.05K\Omega$$

**3-** The percent error is:

$$error = \frac{actual - apparent}{actual} \times 100\%$$

$$error = \frac{23.05 - 20}{23.05} \times 100\% = 13.22\%$$




## *Estimation of Random Errors*

These errors are due to unknown causes and occur even when all systematic errors have been accounted for. In well designed experiments few random errors usually occur, but they become important in high accuracy work. The only way to offset these errors is by increasing the number of readings and using statistical means to obtain the best approximation of the true value of the quantity under measurement.

### ***Statistical Analysis of Data***

To make statistical methods useful, the systematic errors should be small compared with random errors because statistical treatment can not improve the accuracy of measurement.

## 1- *Arithmetic Mean(X\`):*

It's the value lie in the medial number of measured variable and represents the most accurate measured value for the true value. Arithmetic mean is given by:

$$\overline{X} = \frac{\sum F_i \cdot X_i}{\sum F_i}$$

, where $X_i$ is the reading values taken, and $F_i$ is the number that each reading is occur in the measurements, or the frequency number of each reading.

## 2- *Deviation From The Mean($d_i$):*

Deviation is the departure of a given reading from the mean value. It's given by:

$$d_i = X_i - \overline{X}$$

The deviation from the mean may have a positive or a negative value and the algebraic sum of all the deviation must be zero in symmetrical curve.

## 3- *Average Deviation(D):*

The average deviation is the sum of the ***absolute*** values of deviations divided by the number of readings

$$D = \frac{\sum \left| F_i \cdot d_i \right|}{\sum F_i}$$

where $\sum F_i = n$ , and n= number of all readings

## 4- *Standard Deviation(σ):*

It's the root mean square deviation, and the standard deviation represents the variation of the reading from the mean value. For a finite number of reading

$$\sigma = \sqrt{\frac{\sum F_i .(d_i)^2}{n-1}}$$

## 5- *Variance(v):*

It's defined as mean square standard deviation

$$v = \sigma^2$$

## 6- *Probable Error (r):*

It's the maximum chance (50%) that any given measurement will have a random error no greater than ±r

$$r = \pm 0.6745\sigma$$

***Example:***

The following readings were recorded for voltage measurement:

**10.1, 9.7, 10.2, 9.6, 9.7, 10.1, 9.6, 9.7, 10.1**

Calculate:

1. Arithmetic mean (X)
2. Deviation from the mean (d)
3. Average deviation (D)
4. standard deviation (σ)
5. Variance (V)
6. probable error (±r)

***Sol.:***

Rearrangement the reading in two columns with its frequency or (number of reading), thus

| *Reading values* | *No. of reading* | *di(xi-x)* |
|---|---|---|
| 10.1 | 3 | 0.3 |
| 9.7 | 3 | -0.1 |
| 9.6 | 2 | -0.2 |
| 10.2 | 1 | 0.4 |

**1-** $\overline{X} = \dfrac{\sum F_i \cdot X_i}{\sum F_i} \quad = \dfrac{3(10.1) + 3(9.7) + 2(9.6) + (10.1)}{9} = 9.8 volt$

**2-** $d_i = X_i - \overline{X}$

$d_1 = 10.1 - 9.8 = 0.3 volt$

$d_4 = 9.7 - 9.8 = -0.1 volt$

$d_7 = 9.6 - 9.8 = -0.2 volt$

$d_9 = 10.2 - 9.8 = 0.4 volt$

**3-** $D = \dfrac{\sum |F_i \cdot d_i|}{\sum F_i} \quad = \dfrac{3(0.3) + 3(0.1) + 2(0.2) + (0.4)}{9} = 0.22 volt$

**4-** $\sigma = \sqrt{\dfrac{\sum F_i .(d_i)^2}{n-1}} \quad = \sqrt{\dfrac{3(0.09) + 3(0.01) + 2(0.04) + (0.16)}{8}} = 0.26 volt$

**5-** $\nu = \sigma^2 \quad = (0.26)^2 = 0.067 volt^2$

**6-** $r = \pm 0.6745\sigma \quad = \pm 0.6745(0.26) = \pm 0.175 volt$

# Standard of Measurements

**Standards of Measurement**—a critical foundation for ensuring consistency, reliability, and accuracy in all fields of engineering. Standards of measurement allow us to define quantities, compare them consistently, and ensure interoperability across various devices, systems, and applications.

A standard of measurement is a physical representation of a unit of measurement. A unit is realized by reference to an arbitrary material standard or to natural phenomena including physical and atomic constants. Standards are essential for engineering because:

- They provide a baseline for comparison.
- They ensure repeatability and reliability.
- They enable interoperability between devices and systems.

In electrical engineering, measurement standards ensure that quantities like voltage, current, resistance, power, and frequency are consistently defined and measured.

## Classification of Measurement Standards

Measurement standards can be classified based on their level of precision, purpose, and application. They are typically divided into four primary categories:

1. **International Standards**: Highest level of measurement standard, defined and maintained by international bodies, such as the International Bureau of Weights and Measures (BIPM) in France. These standards are universally accepted and serve as the basis for global measurement consistency.
2. **Primary Standards**: These are the highest standards in a specific country and are usually maintained by national laboratories. In the United States, for example, the National Institute of Standards and Technology (NIST) maintains primary standards. These standards are traceable to international standards and are used to calibrate secondary standards.
3. **Secondary Standards**: These standards are calibrated against primary standards and are used for calibration of working standards in laboratories, industries, and educational institutions. They are highly accurate but not as precise as primary standards.

4. **Working Standards**: Working standards are used in routine measurement processes, testing, and calibration tasks in industrial settings, workshops, and laboratories. These are less precise but are calibrated periodically against secondary or primary standards to ensure accuracy.

## Electrical Standards

Electrical standards ensure that quantities such as current, voltage, and resistance are measured accurately and consistently worldwide. They are crucial for:

- **Interoperability**: Allowing components and systems to work seamlessly together.
- **Reliability**: Ensuring measurements are accurate and reproducible.
- **Quality Control**: Supporting the calibration of devices and equipment across industries.

## 1. The Absolute Ampere

The **ampere (A)** is the SI base unit of electric current. It represents the quantity of electric charge passing through a conductor per unit time:

1 A=1 Coulomb per

Where 1 Coulomb is the amount of charge carried by approximately $6.242\times10^{18}$ electrons.

### Definition of the Ampere

Historically, the ampere was defined in terms of the force between two parallel conductors:

"The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length and negligible cross-sectional area, placed 1 meter apart in vacuum, would produce between these conductors a force equal to $2\times10^{-7}$ newton per meter of length." This definition, based on electromagnetic force, was conceptually sound but challenging to reproduce precisely in a laboratory setting.

Modern Redefinition of the Ampere, In 2019, the ampere was redefined in terms of the **elementary charge (e)**, making it independent of physical artifacts or indirect definitions. The current definition is:

"The ampere is defined by fixing the numerical value of the elementary charge e to $1.602176634\times10^{-19}$ C. With this definition, 1 ampere is the amount of current resulting from the flow of $1/e$ charges per second. This redefinition provides a stable and reproducible standard that aligns with quantum principles. The Absolute Ampere standard is realized through advanced measurement setups, such as:

1. **Single-Electron Transistors**: These devices can control the flow of individual electrons, creating a precise current.
2. **Quantum Current Pumps**: These devices transfer fixed numbers of electrons per cycle, thus generating an accurately defined current.
3. **Cryogenic Current Comparators**: Used for highly precise comparisons of current.

**2. Voltage Standard**

Voltage, or electric potential difference, is defined as the work needed to move a unit charge between two points. The SI unit for voltage is the **volt (V)**:

1 V=1 Joule per Coulomb1

Voltage standards ensure consistent measurements of electrical potential, critical in electronics, power systems, telecommunications, and other fields.

Traditionally, voltage standards were based on electrochemical cells (e.g., Weston cells). While reliable, these were sensitive to temperature and environmental changes, requiring careful maintenance.

Today, voltage standards are based on the **Josephson effect**, discovered by physicist Brian Josephson in 1962. In this effect:

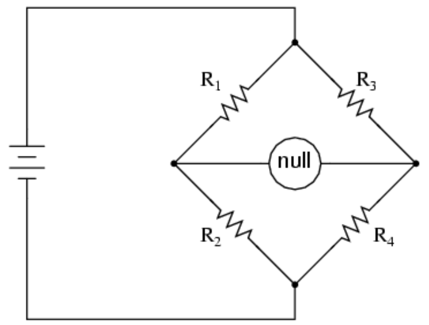
- A superconductor-insulator-superconductor (SIS) junction is exposed to microwave radiation.
- The junction generates a quantized voltage that is highly stable and precise.

# DC Bridges

### Fundamental Concept of Bridges Circuit

In DC measurement circuits, the circuit configuration known as a *bridge* can be a very useful way to measure unknown values of resistance.

To review, the bridge circuit works as a pair of two-component voltage dividers connected across the same source voltage, with a *null-detector* meter movement connected between them to indicate a condition of "balance" at zero volts:




*Basic schematic diagram of standard bridges*

Any one of the four resistors in the above bridge can be the resistor of unknown value, and its value can be determined by a ratio of the other three, which are "calibrated," or whose resistances are known to a precise degree. When the bridge is in a balanced condition (zero voltage as indicated by the null detector), the ratio works out to be this:

*In a condition of* ***balance****:*

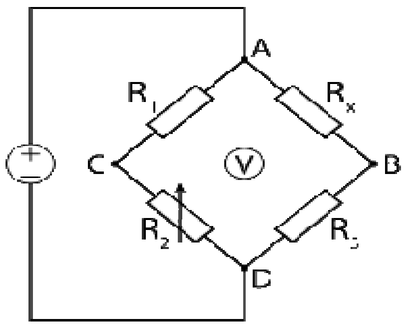
$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \qquad \text{or} \qquad R_1R_4 = R_3R_2$$

One of the advantages of using a bridge circuit to measure resistance is that the voltage of the power source is irrelevant. Practically speaking, the higher the supply voltage, the easier it is to detect a condition of imbalance between the four resistors with the null detector, and thus the more sensitive it will be. A greater supply voltage leads to the possibility of increased measurement precision. However, there will be no fundamental error introduced as a result of a lesser or greater power supply voltage unlike other types of resistance measurement schemes.

**Wheatstone Bridges**

The best-known bridge circuit, the Wheatstone bridge and is used for measuring resistance. It is constructed from four resistors, one of which has an unknown value ($R_x$), one of which is variable ($R_2$), and two of which are fixed and equal ($R_1$ and $R_3$), connected as the sides of a square. Two opposite corners of the square are connected to a source of electric current, such as a battery. A galvanometer is connected across the other two opposite corners.

The variable resistor is adjusted until the galvanometer reads zero. When the voltage between point C and the negative side of the battery is equal to the voltage between point B and the negative side of the battery, the null detector will indicate zero and the bridge is said to be "balanced." It is then known that the ratio between the variable resistor and its neigbour is equal to the ratio between the unknown resistor and its neighbour, and this enables the value of the unknown resistor to be calculated.




*Wheatstone bridge schematic diagram*

Bridges balance equation : $\frac{R_1}{R_2} = \frac{R_X}{R_3}$

From the Figure

Rx = unknown value of resistor
$R_1$ , $R_3$ = Fixed resistor
$R_2$ = Variable resistor
V= Galvanometer with high sensitivity
E = Source

The bridge is balance when no current through the galvanometer ($I_g = 0$) ;

$V_{AB} = V_{AC}$ or $V_{BD} = V_{CD}$

$$V_{AB} = V_{AC}$$

$$\frac{R_X}{R_X + R_3} \times E = \frac{R_1}{R_2 + R_1} \times E$$

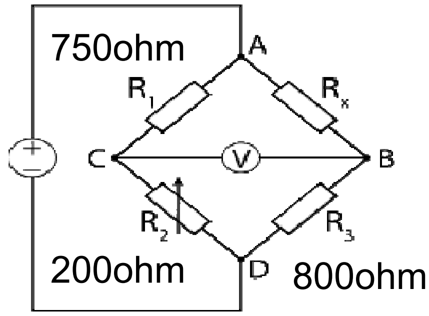
$$R_X (R_1 + R_2) = R_1 (R_X + R_3)$$

$$R_1 R_X + R_X R_2 = R_1 R_X + R_1 R_3$$

$R_X R_2 = R_1 R_3$ So $R_X = (R_1 R_3) / R_2$

EXAMPLE 1
Refer figure below, calcullaate the $R_x$ when the bridges at balance.

( The bridges is in balance condition)




*Solution:*

$R_X = (R_1 R_3) / R_2$

$R_X = (800)(750) / (200)$

$R_X = 3$ Kohm

## b) ***Kelvin Bridge:***

Kelvin Bridge, a highly specialized circuit designed for accurately measuring low resistance values. The Kelvin Bridge builds on the Wheatstone Bridge principle but introduces modifications to reduce errors that arise when measuring very small resistances. Understanding the Kelvin Bridge is essential for electronic engineering students, especially for applications requiring precise measurements of low-resistance components, such as in power systems, superconductors, and conductors.

Its operation is similar to the Wheatstone bridge except for the presence of additional resistors. These additional low value resistors and the internal configuration of the bridge are arranged to substantially reduce measurement errors introduced by voltage drops in the high current (low resistance) arm of the bridge.

It is eliminate errors due to contact and leads resistance. (**Ry**) represent the resistance of the connecting lead from $R_3$ to $R_4$. Two galvanometer connections are possible, to point (**m**) or to point (**n**).

1- ***If the galvanometer connect to point (m) then***

$R_4 = R_x + R_y$ therefore unknown resistance will be higher than its actual value by $R_y$

2- ***If the galvanometer connect to point (n) then***

$R_4 = R_3 + R_y$ therefore unknown resistance will be lower than its actual value by $R_y$

3- ***If the galvanometer connect to point (p) such that***

$$\frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2} \quad \text{.......... (1)}$$

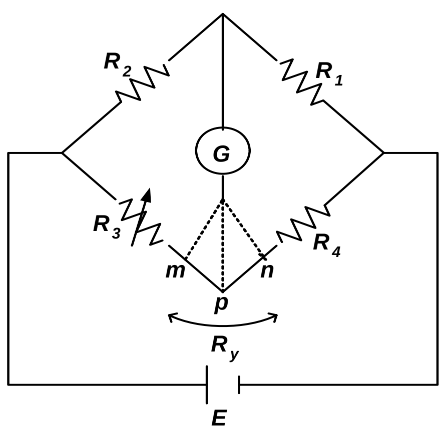
At balance condition

$$R_2\left(R_x + R_{np}\right) = R_1\left(R_3 + R_{mp}\right) \quad \text{........ (2)}$$

Substituting equ.(1) in to equ.(2) we obtain

$$R_x + \left(\frac{R_1}{R_1 + R_2}\right)R_y = \frac{R_1}{R_2}\left[R_3 + \left(\frac{R_2}{R_1 + R_2}\right)R_y\right]$$

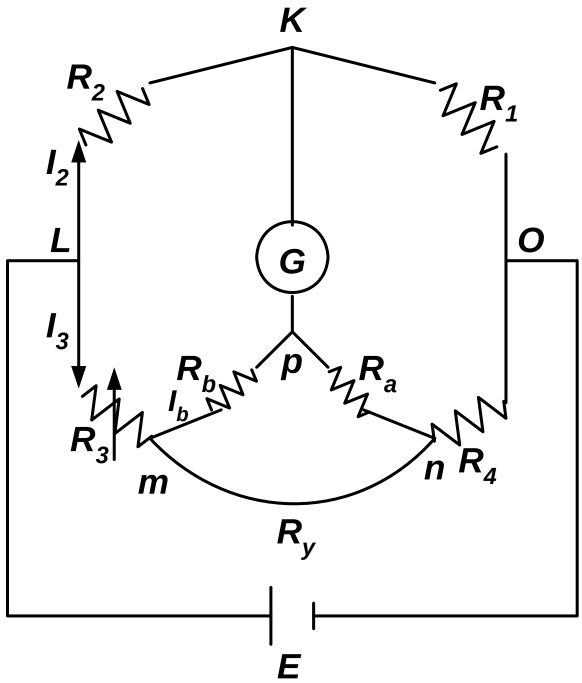
This reduces to $\boxed{R_x = \frac{R_1}{R_2} R_3}$




So the effect of the resistance of the connecting lead from point (**m**) to point (**n**) has be eliminated by connecting the galvanometer to the intermediate position (**p**).

## c) *Kelvin Double Bridge:*

Kelvin double bridge is used for measuring ***very low*** resistance values from approximately ($1\Omega$ to as low as $1x10^{-5}\Omega$). The term double bridge is used because the circuit contains a second set of ratio arms labelled Ra and Rb. If the galvanometer is connect to point (**p**) to eliminates the effect of (yoke resistance **Ry**).




$$\frac{R_a}{R_b} = \frac{R_1}{R_2}$$

At balance $V_2 = V_3 + V_b$ ………. (1)

$$V_2 = E\frac{R_2}{R_1 + R_2} \quad \text{……….. (2)}$$

$V_3 = I_3 R_3$ and $V_b = I_b R_b$ ….. (3)

$$I_b = I_3 \frac{R_y}{(Ra + Rb) + R_y} \quad \text{………… (4)}$$

$$E = I_3\left[R_3 + \frac{(Ra + Rb)R_y}{(Ra + Rb) + R_y} + R_4\right] \text{… (5)}$$

Sub.equ. (5) in to equ. (2) and equ. (4) into equ.(3) then substitute the result in equ.(1), we get

$$I_3\left[R_3 + \frac{(Ra + Rb)R_y}{(Ra + Rb) + R_y} + R_4\right]\frac{R_2}{R_1 + R_2} = I_3 R_3 + I_3\frac{R_y}{(Ra + Rb) + R_y} R_b$$

$$R_x = \frac{R_3 R_1}{R_2} + \frac{R_y Rb}{Ra + Rb + R_y}\left[\frac{R_1}{R_2} + 1 - 1 - \frac{Ra}{Rb}\right]$$

$$\boxed{R_x = \frac{R_3 R_1}{R_2} + \frac{R_y Rb}{Ra + Rb + R_y}\left[\frac{R_1}{R_2} - \frac{Ra}{Rb}\right]}$$

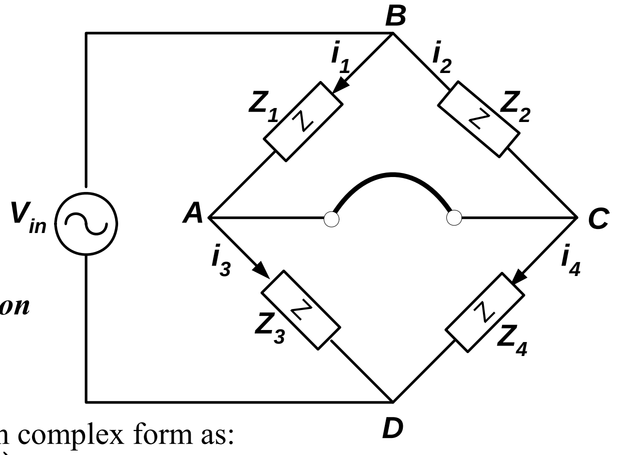
This is the balanced equation

If $\frac{R_a}{R_b} = \frac{R_1}{R_2}$ then $\boxed{R_x = \frac{R_3 R_1}{R_2}}$

## Ac Bridge :

The ac bridge is a natural outgrowth of the dc bridge and in its basic form consists of four bridge arms, a source of excitation, and a null ac detector. For measurements at low frequencies, the power line may serve as the source of excitation; but at higher frequencies an oscillator generally supplies the excitation voltage. The null ac detector in its cheapest effective form consists of a pair of headphones or may be oscilloscope.

The balance condition is reached when the detector response is zero or indicates null. Then $V_{AC} = 0$ and $V_{Z1} = V_{Z2}$




$$V_{Z1} = Vin\frac{Z_1}{Z_1 + Z_3}$$

$$V_{Z2} = Vin\frac{Z_2}{Z_2 + Z_4} \quad \text{thus}$$

$\boxed{Z_1 Z_4 = Z_2 Z_3}$ ***is the balance equation***

Or $\boxed{Y_1 Y_4 = Y_2 Y_3}$

The balance equation can be written in complex form as:

$$(Z_1\angle\theta_1)(Z_4\angle\theta_4) = (Z_2\angle\theta_2)(Z_3\angle\theta_3)$$

And $(Z_1 Z_4\angle\theta_1 + \theta_4) = (Z_2 Z_3\angle\theta_2 + \theta_3)$

So two conditions must be met simultaneously when balancing an ac bridge

1- $Z_1 Z_4 = Z_2 Z_3$

2- $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

***Example (1):***

The impedance of the basic a.c bridge are given as follows:

$Z_1 = 100\angle 80^\circ$ (inductive impedance) $Z_2 = 250\Omega$ $Z_3 = 400\angle 30^\circ$ (inductive impedance)

$Z_4 = unknown$

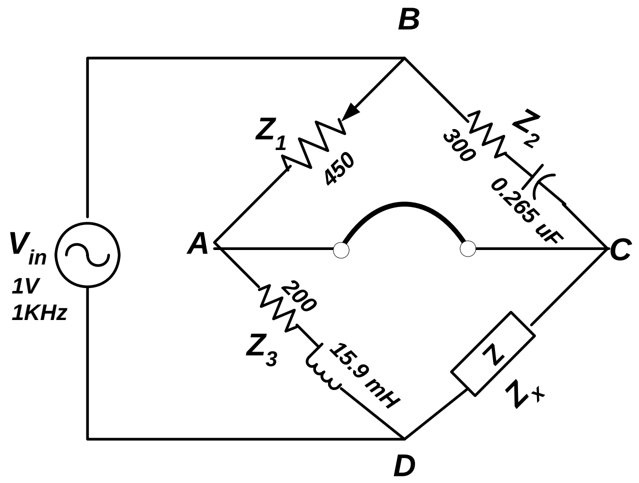
***Sol:***

$\boxed{Z_4 = \frac{Z_2 Z_3}{Z_1}}$ $Z_4 = \frac{250\times 400}{100} = 1k\Omega$ $\boxed{\theta_4 = \theta_2 + \theta_3 - \theta_1}$ $\theta_4 = 0 + 30 - 80 = -50^\circ$

$Z_4 = 1000\angle -50^\circ$ (capacitive impedance)

***Example (2):***

For the following bridge find Zx?




The balance equation $Z_1Z_4 = Z_2Z_3$

$Z_1 = R = 450\Omega$

$Z_2 = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$

$Z_2 = 300 - j600$

$Z_3 = R + j\omega L$

$Z_3 = 200 + j100$

$Z_4 = Z_x = unknown$

$Z_4 = \frac{Z_2Z_3}{Z_1}$ $\quad Z_4 = \frac{(300 - j600)(200 + j100)}{450} = 266.6 - j200$

$R = 266.6\Omega$ $\quad C = \frac{1}{2\pi F \times 200} = 0.79\mu F$

## *Comparison Bridges*

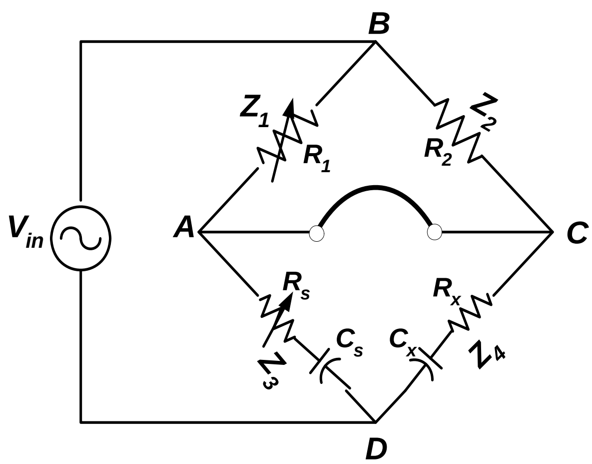
A.c comparison bridges are used to measure unknown inductance or capacitance by comparing it with a known inductance or capacitance.

**1-** ***Capacitive Comparison Bridge***:

In capacitive comparison bridge R1 & R2 are ratio arms, Rs in series with Cs are standard known arm, and Cx represent unknown capacitance with its leakage resistance Rx.

$Z_1 = R_1$ $\quad Z_2 = R_2$ $\quad Z_3 = R_s - \frac{j}{\omega C_s}$ $\quad Z_4 = R_x - \frac{j}{\omega C_x}$

At balance $\boxed{Z_1Z_4 = Z_2Z_3}$




$$R_1\left(R_x - \frac{j}{\omega C_x}\right) = R_2\left(R_s - \frac{j}{\omega C_s}\right)$$

$$R_1R_x - \frac{jR_1}{\omega C_x} = R_2R_s - \frac{jR_2}{\omega C_s}$$

By equating the real term with the real and imaginary term with imaginary we get:

$R_1R_x = R_2R_s$ $\quad \boxed{R_x = \frac{R_2R_s}{R_1}}$

$\frac{-jR_1}{\omega C_x} = \frac{-jR_2}{\omega C_s}$ $\quad \boxed{C_x = \frac{R_1C_s}{R_2}}$

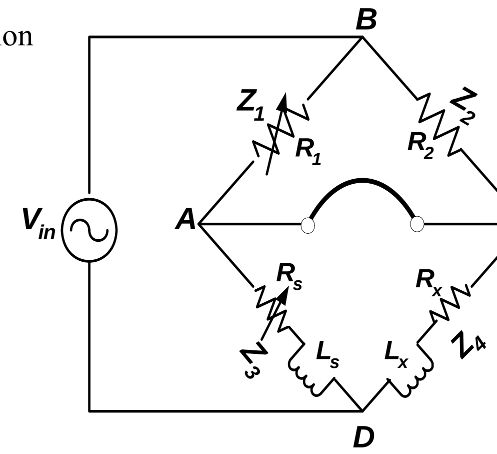
We can ***note*** that the bridge is ***independent*** on ***frequency*** of applied source.

**2-*Inductive Comparison Bridge:***

The unknown inductance is determined by comparing it with a known standard inductor. At balance we get

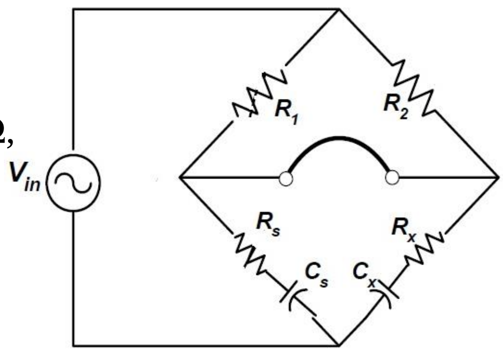
$R_x = \frac{R_2 R_s}{R_1}$ represent resistive balance equation

$L_x = \frac{R_2 L_s}{R_1}$ inductive balance equation




Example 3

Calculate the value of $R_x$ & $C_x$ in the flowing bridge if the bridge at balance and $R_1$= 5KΩ, $R_2$= 2KΩ, $R_s$ = 2.2 KΩ, $C_s$ = 0.42 **µ**f.




Solution

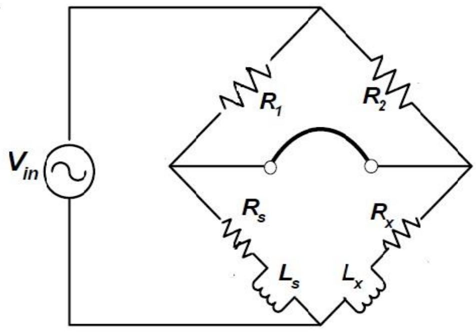
$$\mathrm{Rx} = \frac{\mathrm{R2} \times \mathrm{Rs}}{\mathrm{R1}}$$

$$\mathrm{Rx} = \frac{2\mathrm{k} \times 2.2\mathrm{k}}{5\mathrm{k}} = 880\ \Omega$$

$$\mathrm{C}_X = \frac{\mathrm{R1} \times \mathrm{C}_S}{\mathrm{R2}} \rightarrow \mathrm{C}_X = \frac{5\mathrm{K} \times 0.42\ \mu}{2\mathrm{K}} = 1.05\ \mu\mathrm{f}$$

Example 4

fined the value of $R_x$ & $L_x$ in the flowing bridge if the bridge at balance and $R_1$= 3KΩ, $R_2$= 6.5KΩ, $R_s$ = 1.6 KΩ, $L_s$ = 14 mH.




Solution

$$\mathrm{Rx} = \frac{\mathrm{R2} \times \mathrm{Rs}}{\mathrm{R1}}$$

$$\mathrm{Rx} = \frac{6.5\mathrm{k} \times 1.6\mathrm{k}}{3\mathrm{k}} = 3.46\ K\Omega$$

$$\mathrm{L}_X = \frac{\mathrm{R2} \times \mathrm{L}_S}{\mathrm{R1}} \rightarrow \mathrm{C}_X = \frac{6.5\mathrm{K} \times 14\mathrm{m}}{3\mathrm{K}} = 30\ \mathrm{mH}$$

## *Maxwell bridge*

This bridge measure ***unknown inductance*** in terms of ***a known capacitance***, at balance:

$Z_1 Z_4 = Z_2 Z_3 \qquad Z_1 = \dfrac{1}{Y_1}$ thus

$\boxed{Z_4 = Z_2 Z_3 Y_1}$ where

$Z_2 = R_2 \qquad Z_3 = R_3 \quad Y_1 = \dfrac{1}{R_1} + j\omega C_1$

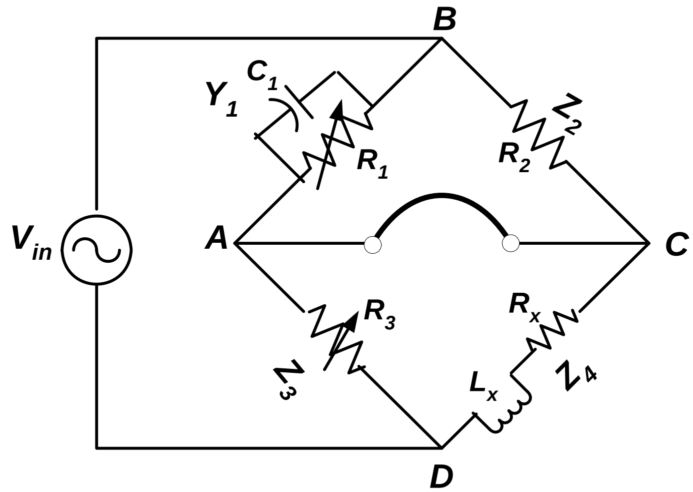
$Z_4 = R_x + j\omega L_x$

So

$$R_x + j\omega L_x = R_2 R_3 \left( \frac{1}{R_1} + j\omega C_1 \right)$$

$$R_x = \frac{R_2 R_3}{R_1}$$

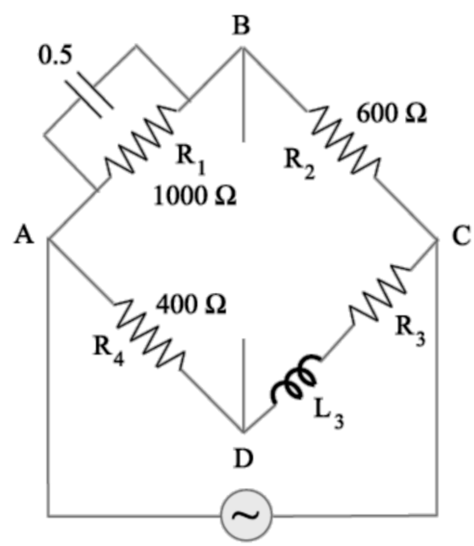
$$L_x = R_2 R_3 C_1$$




Example 5

The arms of an a.c. Maxwell bridge are arranged as follows: AB is a non-inductive resistance of 1,000 Ω in parallel with a capacitor of capacitance 0.5μF , BC is a non-inductive resistance of 600Ω, CD is an inductive impedance (unknown) and DA is a non-inductive resistance of 400 Ω. If balance is obtained under these conditions, find the value of the resistance and the inductance of the branch CD.

Solution




Since $R_1 R_3 = R_2 R_4 \quad \therefore \quad R_3 = R_2 R_4 / R_1$

$$\therefore \qquad R_3 = \frac{600 \times 400}{1000} = \mathbf{240\ \Omega}$$

Also $\quad L_3 = C R_2 R_4$

$$= 0.5 \times 10^{-6} \times 400 \times 600$$

$$= 12 \times 10^{-2} = 0.12\ \text{H}$$

## Advantages of Maxwell's Bridge

1. The balance equations are independent of each other, thus the two variables C4 and R4 can be varied independently.
2. Final balance equations are independent of frequency.
3. The unknown quantities can be denoted by simple expressions involving known quantities.
4. Balance equation is independent of losses associated with the inductor.
5. A wide range of inductance at power and audio frequencies can be measured.

## Disadvantages of Maxwell's Bridge

1. The bridge, for its operation, requires a standard variable capacitor, which can be very expensive if high accuracies are asked for. In such a case, fixed value capacitors are used and balance is achieved by varying R4 and R2.

2. This bridge is limited to measurement of low Q inductors ($1< Q < 10$).

3. Maxwell's bridge is also unsuited for coils with very low value of Q (e.g., $Q < 1$).