Al-Mustaqbal University / College of Engineering & Technology
Department Of Communication Engineering Class (1st)
Subject (calculus 1) / Code (TE-UOMUS-094241217-574)
Lecturer (M.Sc. Fatimatulzahraa Adnan)
2nd term – Lecture No.7 & Lecture Name (Hyperbolic function)

Hyperbolic functions : If u is any differentiable function of x , then :

21) $\frac{d}{dx}\sinh u = \cosh u.\frac{du}{dx}$

22) $\frac{d}{dx}\cosh u = \sinh u.\frac{du}{dx}$

23) $\frac{d}{dx}\tanh u = \operatorname{sech}^2 u.\frac{du}{dx}$

24) $\frac{d}{dx}\coth u = -\operatorname{csch}^2 u.\frac{du}{dx}$

25) $\frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u.\tanh u.\frac{du}{dx}$

26) $\frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u.\coth u.\frac{du}{dx}$

EX-13 - Find $\frac{dy}{dx}$ for the following functions :

a) $y = \coth(\tan x)$ b) $y = \sin^{-1}(\tanh x)$

c) $y = \ln\left|\tanh\frac{x}{2}\right|$ d) $y = x.\sinh 2x - \frac{1}{2}.\cosh 2x$

e) $y = \operatorname{sech}^3 x$ f) $y = \operatorname{csch}^2 x$

Sol. -

a) $\frac{dy}{dx} = -\operatorname{csch}^2(\tan x).\sec^2 x$

b) $\frac{dy}{dx} = \frac{\operatorname{sech}^2 x}{\sqrt{1-\tanh^2 x}} = \frac{\operatorname{sech}^2 x}{\sqrt{\operatorname{sech}^2 x}} = \operatorname{sech} x$

c) $\frac{dy}{dx} = \frac{1}{\tanh\frac{x}{2}}\operatorname{sech}^2\frac{x}{2}.\frac{1}{2} = \frac{\frac{1}{\cosh^2\frac{x}{2}}}{2.\frac{\sinh\frac{x}{2}}{\cosh\frac{x}{2}}}$

$= \frac{1}{2\sinh\frac{x}{2}.\cosh\frac{x}{2}} = \frac{1}{\sinh x} = \operatorname{csch} x$

$$d)\ \frac{dy}{dx} = x\cosh 2x.2 + \sinh 2x - \frac{1}{2}\sinh 2x.2 = 2x\cosh 2x$$

$$e)\ \frac{dy}{dx} = 3\operatorname{sech}^2 x(-\operatorname{sech} x.\tanh x) = -3\operatorname{sech}^3 x.\tanh x$$

$$f)\ \frac{dy}{dx} = 2\operatorname{csch} x(-\operatorname{csch} x.\coth x) = -2\operatorname{csch}^2 x.\coth x$$

EX-14- Show that the functions :

$$x = -\frac{2}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} \quad and \quad y = \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} + \cosh\frac{t}{\sqrt{3}}$$

Taken together , satisfy the differential equations :

$$i)\ \frac{dx}{dt} + 2\frac{dy}{dt} + x = 0 \quad and \quad ii)\ \frac{dx}{dt} - \frac{dy}{dt} + y = 0$$

Proof -

$$x = -\frac{2}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} \Rightarrow \frac{dx}{dt} = -\frac{2}{3}\cosh\frac{t}{\sqrt{3}}$$

$$y = \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} + \cosh\frac{t}{\sqrt{3}} \Rightarrow \frac{dy}{dt} = \frac{1}{3}\cosh\frac{t}{\sqrt{3}} + \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}}$$

$$i)\ \frac{dx}{dt} + 2\frac{dy}{dt} + x = -\frac{2}{3}\cosh\frac{t}{\sqrt{3}} + \frac{2}{3}\cosh\frac{t}{\sqrt{3}} + \frac{2}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} - \frac{2}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} = 0$$

$$ii)\ \frac{dx}{dt} - \frac{dy}{dt} + y = -\frac{2}{3}\cosh\frac{t}{\sqrt{3}} - \frac{1}{3}\cosh\frac{t}{\sqrt{3}} - \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} + \frac{1}{\sqrt{3}}\sinh\frac{t}{\sqrt{3}} + \cosh\frac{t}{\sqrt{3}} = 0$$

EX-15 - Prove that :

$$a)\ \frac{d}{dx}\tanh u = \operatorname{sech}^2 u.\frac{du}{dx} \quad and \quad b)\ \frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u.\tanh u.\frac{du}{dx}$$

Proof-

$$a)\ \frac{d}{dx}\tanh u = \frac{d}{dx}\left(\frac{\sinh u}{\cosh u}\right) = \frac{\cosh u.\cosh u.\frac{du}{dx} - \sinh u.\sinh u.\frac{du}{dx}}{\cosh^2 u}$$

$$= \frac{(\cosh^2 u - \sinh^2 u)\frac{du}{dx}}{\cosh^2 u} = \frac{1}{\cosh^2 u}.\frac{du}{dx} = \operatorname{sech}^2 u.\frac{du}{dx}$$

$$b)\ \frac{d}{dx}\frac{1}{\cosh u} = -\frac{1}{\cosh^2 u}.\sinh u.\frac{du}{dx} = -\operatorname{sech} u.\tanh u.\frac{du}{dx}$$