

Department Of Communication Engineering Class (1st)

Subject (calculus 1) / Code (TE-UOMUS-094241217-574)

Lecturer (M.Sc. Fatimatulzahraa Adnan)

2nd term - Lecture No.5 & Lecture Name (integration by parts)

# أولا : التكامل بالتجزنة : Integration by Parts

x النين لـ v = v(x) و u = u(x) دالتين لـ u = u(x)

 $u\,dv=u\,v-\int v\,du$ 

هناك طريقتان : الاولى تُسمى طريقة الاسهم وتصلح لتكاملات الدوال

 $x^n \cos ax$ ,  $x^n e^{ax}$ ,  $e^{ax} \cos bx$  and  $e^{ax} \sin bx$  $x^n \sin ax$ 

 $n \in N^+$  ,  $a,b \in \mathcal{R}$   $\leq x$ 

والمثال التالي يوضح هذه الطريقة

مثال (١): جد

$$1. \int x^2 \cos 2x \, dx \qquad \qquad 2. \int x^3 e^{2x} \, dx$$

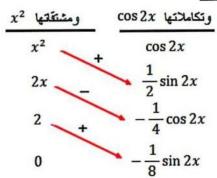
$$2. \int x^3 e^{2x} \, dx$$

$$3. \int e^{3x} \sin 2x \ dx$$

<u>الحل :</u>

فان

1.



$$\therefore \int x^2 \cos 2x \, dx = x^2 \times \frac{1}{2} \sin 2x + 2x \times \frac{1}{4} \cos 2x - 2 \times \frac{1}{8} \sin 2x + c$$
$$= \frac{1}{4} (2x^2 - 1) \sin 2x + \frac{x}{2} \cos 2x + c$$





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2.  $\frac{x^{3} + e^{2x}}{x^{3} + e^{2x}}$   $3x^{2} - \frac{1}{2}e^{2x}$   $6x + \frac{1}{4}e^{2x}$   $\frac{1}{2}e^{2x}$ 

0

$$\int x^3 e^{2x} dx = \frac{x^3}{2} e^{2x} - \frac{3x^2}{4} e^{2x} + \frac{6x}{8} e^{2x} - \frac{6}{16} e^{2x} + c$$
$$= \frac{1}{8} e^{2x} (4x^3 - 6x^2 + 6x - 3) + c$$

 $\frac{e^{3x}}{e^{3x}} + \frac{\sin 2x}{\sin 2x}$   $3e^{3x} + \frac{1}{2}\cos 2x$   $9e^{3x} + \frac{1}{4}\sin 2x$ 



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والطريقة الثانية هي التجزئة الاعتيادية وتصلح لتكاملات الدوال

$$\ln w$$
 ,  $\sin^{-1} w$  ,  $\cos^{-1} w$  ,  $\tan^{-1} w$  ,  $\cot^{-1} w$  ,  $\sec^{-1} w$  and  $\csc^{-1} w$  حيث  $w=w(x)$  من الدوال اعلاه وما تبقى يكون  $w=w(x)$  والمثال التالى يوضح هذه الطريقة

مثال (٢): جد

$$1.\int \ln x \ dx$$

2. 
$$\int \sin^{-1} 2x \, dx$$
 3.  $\int x \sec^{-1} x \, dx$ 

$$3. \int x \sec^{-1} x \, dx$$

الحل:

1. 
$$u = \ln x$$
 and  $dv = dx$ 

$$dv = dx$$

$$du = \frac{dx}{x}$$
 and  $v = x$ 

$$v = x$$

$$\int u\ dv = u\ v - \int v\ du$$

$$\therefore \int \ln x \, dx = x \ln x - \int x \cdot \frac{dx}{x}$$
$$= x \ln x - \int dx$$
$$= x \ln x - x + c$$

$$2. \ u = \sin^{-1} 2x \qquad and \qquad dv = dx$$

$$du = \frac{2dx}{\sqrt{1 - 4x^2}} \quad and \quad v = x$$

$$\int u \, dv = u \, v - \int v \, du$$

$$\therefore \int \sin^{-1} 2x \, dx = x \sin^{-1} 2x - \int \frac{2x dx}{\sqrt{1 - 4x^2}}$$
$$= x \sin^{-1} 2x + \frac{1}{2} \sqrt{1 - 4x^2} + c$$



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3. 
$$u = \sec^{-1} x$$
 and  $dv = xdx$ 

$$du = \frac{dx}{x\sqrt{x^2 - 1}} \qquad v = \frac{1}{2}x^2$$

$$\int u \, dv = u. \, v - \int v \, du$$

$$\int x \sec^{-1} x \, dx = \frac{1}{2}x^2 \sec^{-1} x - \int \frac{1}{2}x^2 \, \frac{dx}{x\sqrt{x^2 - 1}}$$

$$= \frac{1}{2}x^2 \sec^{-1} x - \frac{1}{2} \int \frac{xdx}{\sqrt{x^2 - 1}}$$

$$= \frac{1}{2}x^2 \sec^{-1} x - \frac{1}{2}\sqrt{x^2 - 1} + c$$

## تمارين

جد الحل للتكاملات التالية:

$$1. \int x \ln x \, dx$$

$$2. \int x^3 e^{3x} \, dx$$

$$3. \int \tan^{-1} x \, dx$$

$$4. \int e^{2x} \cos 3x \ dx$$

$$5. \int x^3 \sin 3x \ dx$$

$$6. \int \cos^{-1} \sqrt{x} \ dx$$

$$7. \int \sec^{-1} \sqrt{x} \ dx$$