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* Area Between Two Curves

To Find The area between the Curves $f(x)$ and $g(x)$ of The iterval $[a, b]$ ~~the~~ we should follow the following steps.

$f(x) - g(x) = 0$	أول خطوة
$\int f(x) - \int g(x)$	ثاني خطوة
اضيف [ ] لحدود التكامل	ثالث خطوة

Ex: Find The total area of the region between The Two Curves $f(x) = x^2$ and $g(x) = 2x$ over The interval $[-1, 2]$

1. $f(x) - g(x) = x^2 - 2x = 0 \Rightarrow x(x-2) = 0$

$\Rightarrow x = 0$ or $x - 2 = 0 \Rightarrow x = 2$ (neg)

$$\text{area} = \left[\int_{-1}^{0} x^2 - 2x \, dx\right] + \left[\int_{0}^{2} x^2 - 2x \, dx\right]$$

$$\left[\frac{x^3}{3} - x^2\right]_{-1}^{0} + \left[\frac{x^3}{3} - x^2\right]_{0}^{2} = \frac{8}{3}$$

Ex:- Find the area of the region bounded by $F(x) = x^2$ and $g(x) = x$

1. $x^2 - x = 0 \Rightarrow x(x-1) \Rightarrow x=0, x=1$

$$\int_0^1 (x^2 - x \, dx \Rightarrow \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1 = \frac{1}{6}$$

Ex:- Find the total area of the region between Curve $y = x^3 - 4x^2 + 3x$ and the X axis of the interval $[0, 2]$

$x^3 - 4x^2 + 3x = 0$

$x(x^2 - 4x + 3) = 0$

i) $x = 0$ (neg)

ii) $x^2 - 4x + 3 = 0 \Rightarrow (x-1)(x-3)$

$x = 1, x = 3$ (neg)

$$\left[\int_0^1 x^3 - 4x^2 + 3x\right] + \left[\int_1^2 x^3 - 4x^2 + 3x\right]$$

$$\left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right]_0^1 + \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right]_1^2$$

$= 1.5$