Chapter One

Simple Stresses

In any engineering structures or mechanism , the individual components will be subjected to external forces arising from the service condition or environment in which the component works . If the component or member is in equilibrium , the resultant of forces will be zero but nevertheless , they together place a load on the member which tends to deform that member and which must be reacted by internal forces which are set up within material .

Analysis of Internal Forces :

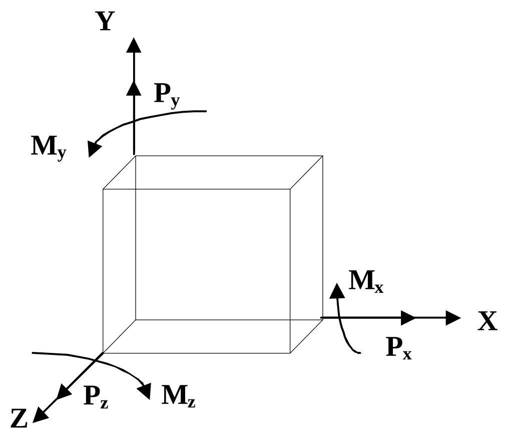
The most type of internal forces on any structure or member are :

1- P_x : Axial force represents a tensile or compressive force in (X) direction .

2- P_y, P_z : Shear force represents the resistance to sliding the portion to one side of the section past the other .

3- M_x: Torque represent to the resistance to twisting about X-axis.

4- M_y , M_z : Bending moment that measure the resistance to bending about (Y) or (Z) axis .



Internal forces on body

Types of Simple Stresses :

1) Normal stresses : (Tensile and Compressive Stresses)

One of the basic problem of the engineer is to select the proper material that is used in different applications like (structures or machines parts) to do most efficiently what it is designed to do . For this purpose , it is essential to determine the Strength , Stiffness and other properties of materials .

The unit strength of materials is usually defined as the (Stress) in the material , and the stress expressed symbolically as :

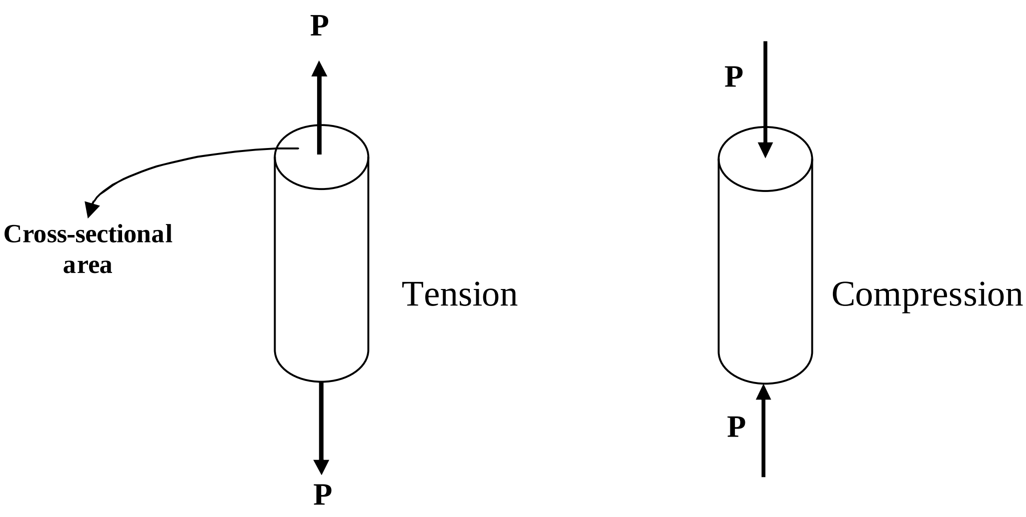
$$\sigma = \frac{P}{A_c} \qquad (1)$$

where :

σ-------Stress (force per unit area) (N/m^2) or Pascal (Pa)

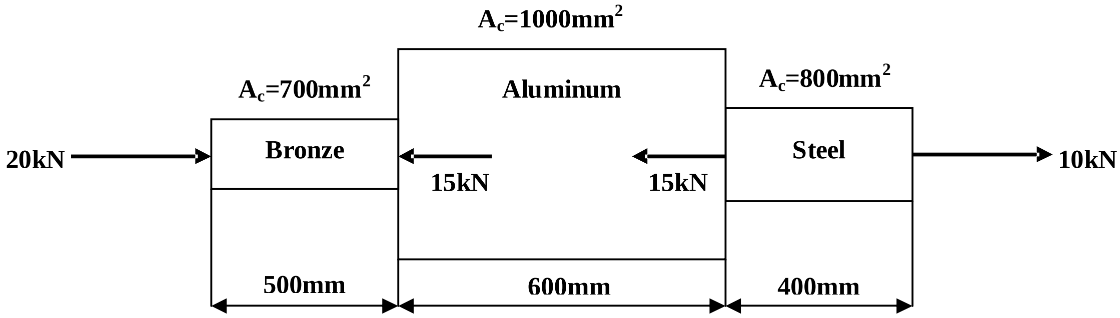
P-------Applied load (N)

A_C-----Cross-sectional area (m^2)



Axial tensile Force Axial compressive force

Ex: -1- An Aluminum rod is rigidly fastened between Bronze and Steel rods as shown in figure . Axial loads are applied at the position indicated Determine the stress in each rod .

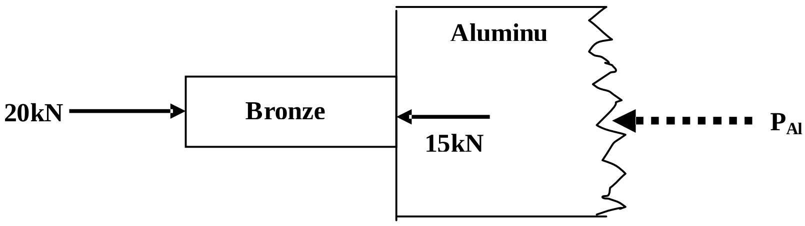


<u>Sol:</u> To calculate the stress in each rod , we first determine the total axial load in each rod .

1- For Bronze rod : By using the free-body diagram for Bronze rod

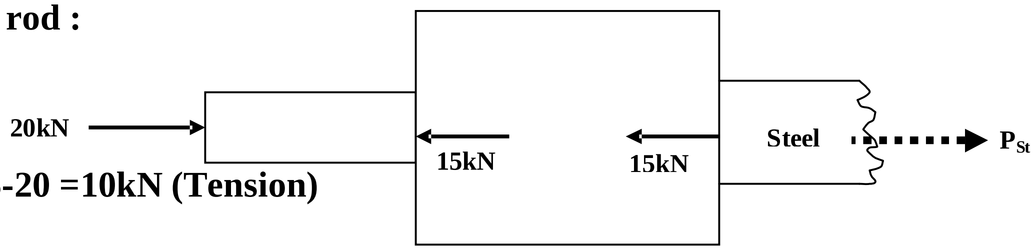
P_{Br}=20kN (Compressive)

2- For Aluminum rod :



P_{Al}=20-15 = 5kN (Compressive)

3- For Steel rod :



P_{St}=15+15-20 =10kN (Tension)

The stress in each rod now can be calculated :

$$\sigma_{Br} = \frac{P_{Br}}{A_{Br}} = \frac{20*10^3(N)}{700*10^{-6}(m^2)} = 28.6*10^6\,\frac{N}{m^2} = 28.6MPa$$ **(Compressive stress)**

$$\sigma_{Al} = \frac{P_{Al}}{A_{Al}} = \frac{5*10^3(N)}{1000*10^{-6}(m^2)} = 5MPa$$ **(Compressive stress)**

$$\sigma_{St} = \frac{P_{St}}{A_{St}} = \frac{10*10^3(N)}{800*10^{-6}(m^2)} = 12.5MPa$$ **(Tensile stress)**

Ex: -2- Determine the largest weight (W) which can be supported by the two wires as shown in figure . The stresses in wires (AB) and (AC) are not to exceed (100MPa)and (150MPa) respectively . The cross-sectional area of the two wires are (400mm^2) for wire (AB) and (200mm^2) for wire (AC).

Sol:- First we must draw the free-body diagram

$$\sum F_x = 0$$ **(Equilibrium state)**

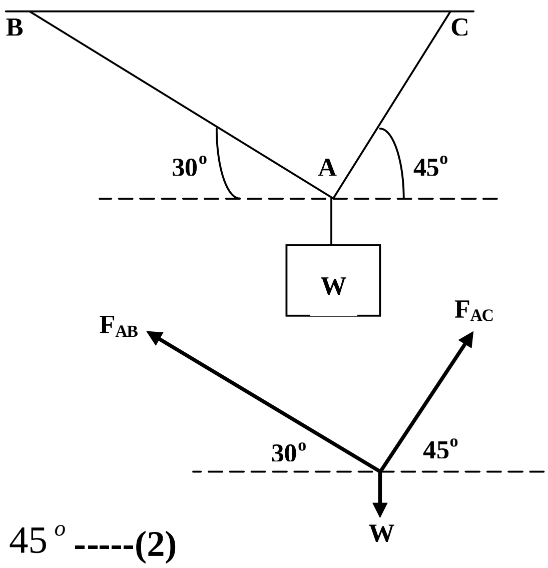
$$F_{AB} Cos\,30 = F_{AC} Cos\,45$$

$$F_{AB} = F_{AC}\,\frac{Cos\,45}{Cos\,30} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$F_{AB} = \sqrt{\frac{2}{3}}F_{AC}$$ **------(1)**

$$F_{AB} = 0.8165F_{AC}$$

$$\sum F_y = 0 \longrightarrow W = F_{AB}\,Sin\,30^o + F_{Ac}\,Sin\,45^o$$ **-----(2)**



Free-body diagram

Sub (1) in (2) :

$$W = 0.8165F_{AC}Sin30^o + F_{AC}Sin45^o$$

$$\sigma = \frac{F}{A} \longrightarrow F_{AC} = \sigma_{AC} * A_{AC} = 150*10^6 * 200*10^{-6} = 30kN$$

$$W = 0.8165 * 30 * 10^3 Sin 30^o + 30 * 10^3 Sin 45^o$$

$$(W = 33.5kN)$$

Another solution :

For $F_{AC} = \frac{F_{AB}}{0.8165}$ **sub in equ.(2)**

$$W = F_{AB} Sin 30^o + F_{AC} Sin 45^o$$

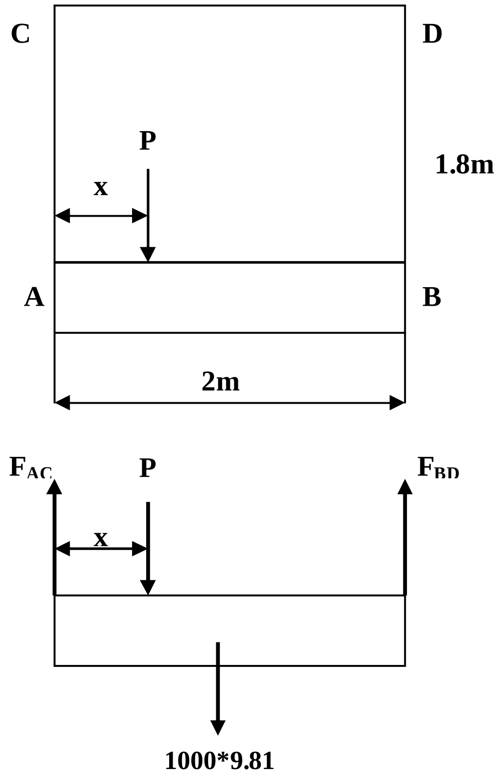
$$F_{AB} = \sigma_{AB} * A_{AB} = 100*10^6 * 400*10^{-6} = 40kN$$

$$W = 40*10^3 Sin 30^o + \frac{40*10^3}{0.8165} Sin 45^o = 54.7kN$$

we choose **(*W=33.5kN*)**

Ex:-3- A (1000kg) homogenous bar (AB) is suspended from two cables (AC) and (BD) , each with cross-sectional area ($400mm^2$) , as shown in figure . Determine the magnitude of load (P) and location (x) that is additional force which can be applied to the bar . The stresses in the cable (AC) and (BD) are limited to (100MPa) and (50MPa) respectively

<u>Sol:</u>



Free-body diagram

$$\sigma = \frac{F}{A} \Rightarrow F = \sigma * A$$

$$F_{AC} = 100 * 10^{6} * 400 * 10^{-6} = 40\,kN$$

$$F_{BD} = 50 * 10^{6} * 400 * 10^{-6} = 20\,kN$$

From F.B.D and for equilibrium state :

$$\sum F_y = 0$$

$$P + 9810 = 40 * 10^{3} + 20 * 10^{3}$$

$$P = 50.2\,kN$$

$$\sum M_A = 0 \Rightarrow 9810 * 1 + P * x = F_{BD} * 2$$

$$x = \mathbf{0.602\ m}$$

2) Direct Shear Stress :

Shearing stress differs from both tensile and compressive stress in that is caused by forces acting along or parallel to the area resisting the force where's tensile and compressive stress are caused by forces perpendicular to the area on which they act . For this reason , tensile and compressive stresses are frequently called (*Normal Stresses*) where's shearing stress may be called (*Tangential Stresses*).

$$\tau = \frac{V}{A_{sh.}}$$

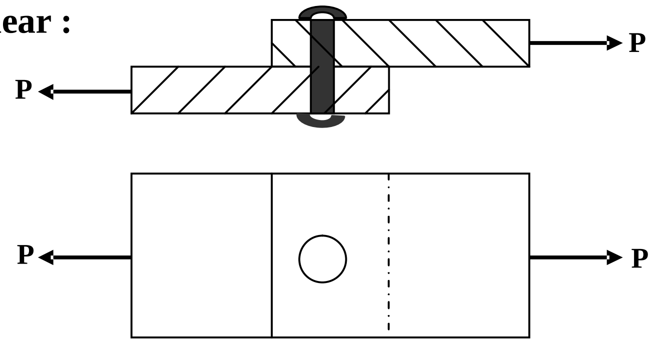
where: (V) ……. shearing load

(A_{sh}.) ……….shearing area (parallel area)

In single shear , there is single area of shear :

$$\tau = \frac{P}{\frac{\pi}{4} d^2}$$

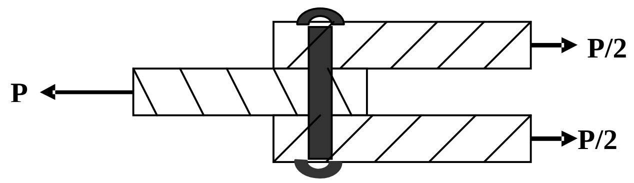
d **------- Diameter of area parallel to load direction (m) .**



(Single shear)

In double shear , there are two area of shear :

$$\tau = \frac{P}{2 * \frac{\pi}{4} d^2}$$

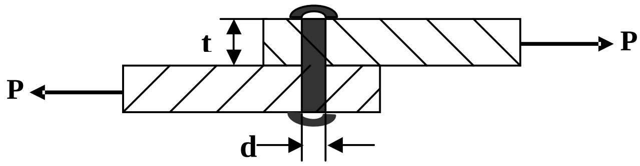


(Double shear)

3) Bearing (Crushing) Stress :

Bearing stress differs from compressive stress in that the latter is the internal stress caused by a compressive force where's the former is a contact pressure between separate bodies .

$$\sigma_{cr} = \frac{P}{A_{cr}} = \frac{P}{d * t}$$



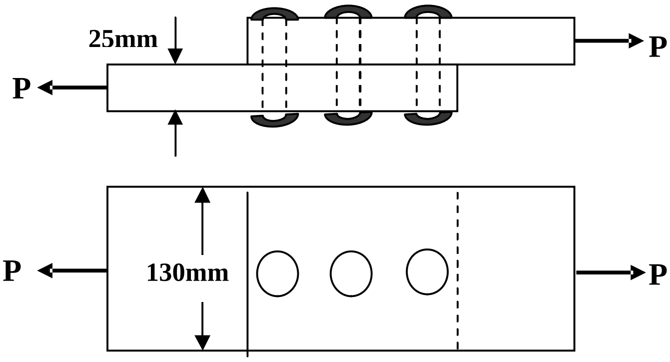
Where :

A_{cr}-------Bearing (Crushing) area

Ex:-4- The lap joint shown in figure is fastened by three (20mm) diameter rivets . Assuming that (P=50kN) applied determine : 1) The shearing stress in each rivet . 2) The bearing stress in each plate.

3) The maximum average tensile stress in each plate . Assume that the applied load (P) is distributed equally among the three rivets .

Sol:



1) $\tau = \frac{P}{A_{sh}} = \frac{P}{3A_r}$

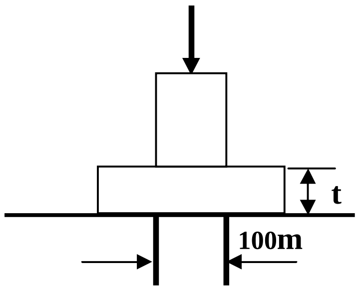
$$\tau = \frac{50*10^3}{3\left[\frac{\pi}{4}(0.020)^2\right]} = 53.1MPa$$

2) $\sigma_b = \frac{P_b}{A_b} = \frac{P}{3*d*t} = \frac{50*10^3}{3*0.02*0.025} = 33.3MPa$

3) $\sigma_t = \frac{P}{A_{plate}} = \frac{P}{(w-d)*t} = \frac{50*10^3}{(0.13-0.02)*0.025} = 18.2MPa$

Ex: -5- As shown in figure , a hole is to be punched out of a plate having an ultimate shearing stress of (300MPa) . 1)If the compressive stress in the punch is limited to (400MPa) , determine the maximum thickness of plate from which a hole (100mm)in diameter can be punched . 2)If the plate is (10mm) thick , compute the smallest diameter hole which can be punched .

<u>Sol:</u>



1) $$\sigma_{comp.} = \frac{P}{A}$$

$$P = \sigma * A = 400*10^6 * \frac{\pi}{4}(0.1)^2$$

$$P = 3141.6 kN$$

$$\tau = \frac{V}{A} \Rightarrow A = \frac{V}{\tau} \Rightarrow \pi * d * t = \frac{3141.6*10^3}{300*10^6}$$

$$t = 33.33 mm$$

2) $$\sigma_{comp.} = \frac{P}{A}$$

$$P = \sigma_{comp.} * A = 400*10^6 * \frac{\pi}{4} d^2$$

$$\tau = \frac{\sigma * A}{\pi * d * t}$$

$$300*10^6 = \frac{400*10^6 * \frac{\pi}{4} d^2}{\pi * d * 0.01}$$

$$\therefore d = 30 mm$$