Chapter Two

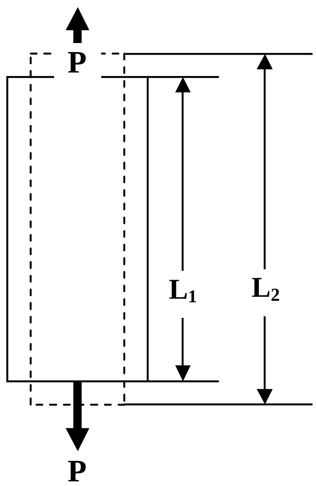
Simple Strain

Strain : Is a measure of the deformation of the material which is subjected to an external load , and its non-dimensional .

The strain may divided into : 1) Normal strain . 2) Shear strain

1) Normal Strain : It is occur due to normal stresses (tensile causes +ve strain and compressive stress causes –ve strain).

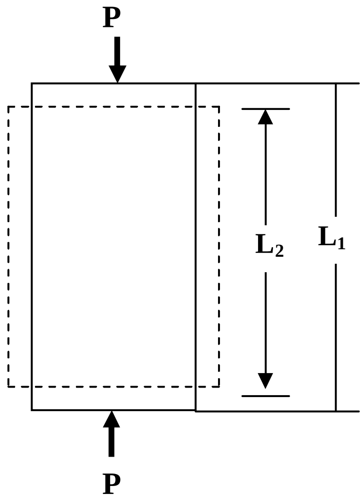
$$\varepsilon = \frac{\Delta L}{L} = \frac{L_2 - L_1}{L_1}$$



In tension :

$$\varepsilon_t = \frac{\Delta L}{L_1} = \frac{L_2 - L_1}{L_1} \quad \textbf{(+ve strain)}$$

In compression :

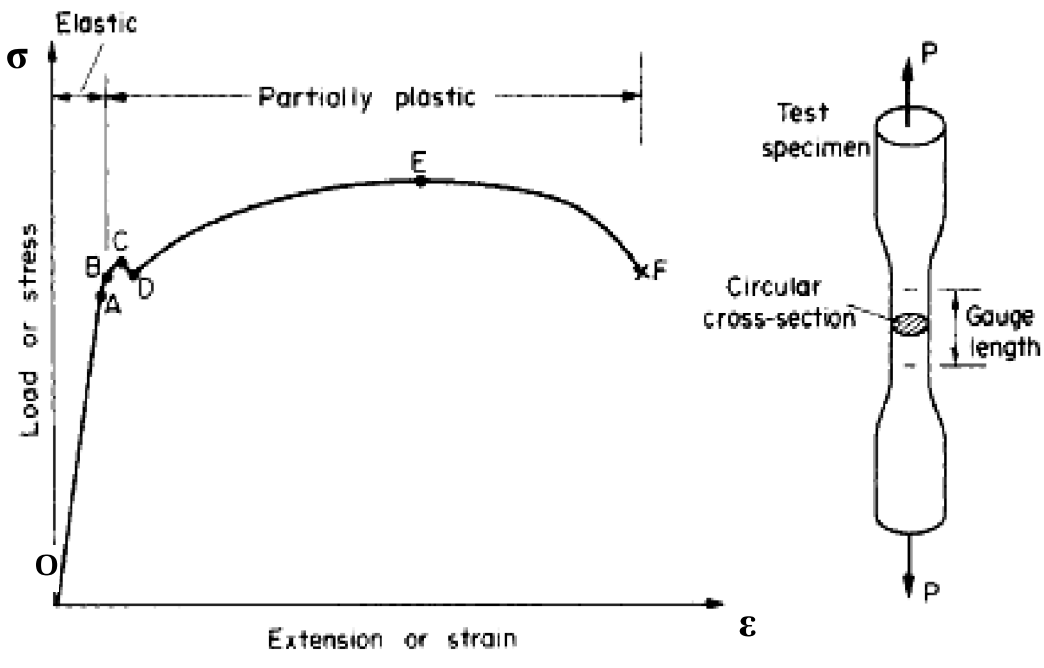


$$\varepsilon_c = \frac{\Delta L}{L_1} = \frac{L_2 - L_1}{L_1} \quad \textbf{(-ve strain)}$$

as L_1 **larger than** L_2

Stress-Strain Diagram :

In order to compare the strength of various materials to select the better material for using , it is necessary to carry out some standard form of tests to establish their relative properties . The most important of these tests is (tensile tests) , in which a circular steel bar of uniform cross-sectional area is subjected to gradually increasing tensile load until failure occurs , and measuring the change in selected length (gauge length) of bar simultaneously , and then plot the relation between the tensile stress and tensile strain for bar graph .



Stress-Strain diagram for mild steel

1-(O----A) : Straight line where the strain is linear proportional with stress $(\sigma \propto \varepsilon)$

$$\therefore \sigma = (Constant) * \varepsilon$$

$$\therefore \sigma = E * \varepsilon \qquad \textbf{(Hook's law)}$$

***E*-----Young modulus (modulus of elasticity) .**

$$\sigma = E * \varepsilon \ , \quad n\sigma = \frac{P}{A} \quad \textbf{and} \quad \varepsilon = \frac{\Delta L}{L} = \frac{\delta}{L}$$

$$\therefore \frac{P}{A} = E * \frac{\delta}{L} \qquad \Rightarrow \delta = \frac{P * L}{A * E}$$

A-----Refer to proportional limit .

2- (A----B) : The material may still be elastic but Hook's law not valid .

B-----Elastic limit

3-(B----C) : Beyond point (B) strain are not removed (permanent strain).

C-----Upper yield point .

4-(C----D) : Increasing in deformation without increasing in load .

D-----Lower yield point .

5- (D----E) :The reduction in cross-sectional area of specimen will occur (nicked specimen) .

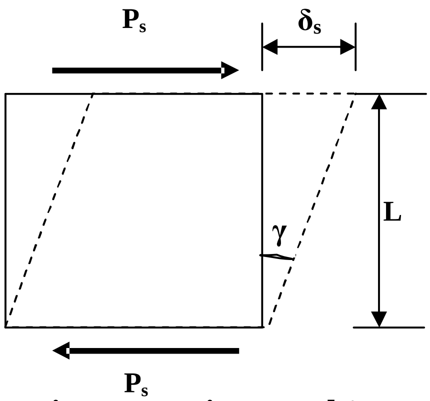
E-----Ultimate strength .

6- (E----F) : The failure will occur in specimen .

F----- Failure point .

<u>2) Shear Strain :</u> It is occur due to shear stress and its represented by (γ) in radians , and its is defined as ″ *The angular change between two faces of differential element* ″ . The shearing force cause shearing deformation , the element that subjected to shear dose not change in length of its sides , but undergoes change in shape from rectangle to parallelogram .

The average shearing strain is found by :



$$\tan(\gamma) = \frac{\delta_s}{L}$$

since the angle (γ) is usually very small , then

tan (γ) $\approx$ (γ)

$$\therefore \gamma = \frac{\delta_s}{L}$$

The relation between shearing stress and shearing strain , assuming Hook's law apply to shear .

$$\therefore \tau = Constant * \gamma$$
$$\therefore \tau = G * \gamma$$

G------Modulus of elasticity in shear (Modulus of rigidity)

Shearing deformation is expressed as :

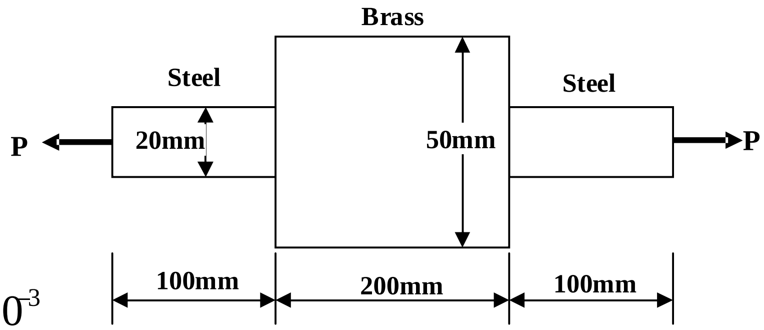
$$\tau = \frac{V}{A} \quad , \quad \tau = G * \gamma \quad , \quad \gamma = \frac{\delta_s}{L}$$

$$\therefore \frac{V}{A} = G * \frac{\delta_s}{L}$$

$$\therefore \delta_s = \frac{V * L}{A * G}$$

Ex :-6- During a tensile test of the bar shown in figure , the overall extension is (0.15mm) . Find the stress in each part of the bar .

E_{st}=210GPa , E_{Br}=120GPa



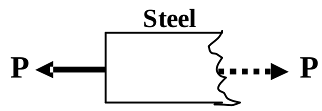
Sol:

$$2\delta_{st} + \delta_{Br} = \delta_{Total}$$

$$2\delta_{st} + \delta_{Br} = 0.15 * 10^{-3}$$

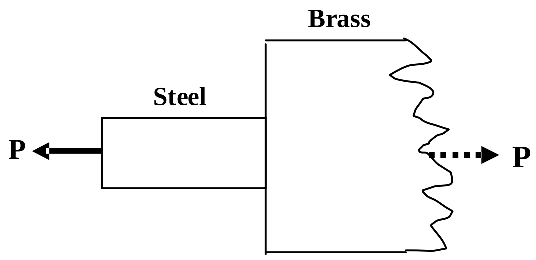
$$\therefore 2\frac{P_{st} * L_{st}}{A_{st} * E_{st}} + \frac{P_{Br} * L_{Br}}{A_{Br} * E_{Br}} = 0.15*10^{-3}$$

$$\therefore P_{st} = P_{Br} = P'$$



$$\therefore 2\frac{P_{st} * 0.1}{\frac{\pi}{4}(0.02)^2 * 210*10^9} + \frac{P_{Br} * 0.2}{\frac{\pi}{4}(0.05)^2 * 120*10^9} = 0.15*10^{-3}$$

$$\therefore P_{st} = P_{Br} = P' = 38.66kN$$

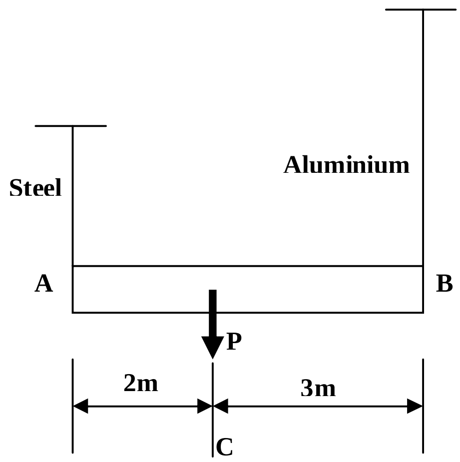


$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{38.66*10^3}{\frac{\pi}{4}(0.02)^2} = 196.6MPa$$

$$\sigma_{Br} = \frac{P_{Br}}{A_{Br}} = \frac{38.66*10^3}{\frac{\pi}{4}(0.05)^2} = 123MPa$$

Ex:-7- The rigid bar (AB) attached to two vertical rods as shown in figure , horizontally before the load (P) applied . If the load (P=50kN) , determine its vertical movement .

	St.	**Al.**
L (m)	**3**	**4**
Area (mm^2)	**300**	**500**
E (GPa)	**200**	**70**

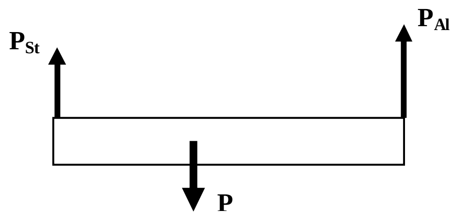


Sol :

From F.B.D

$$\therefore \sum F_y = 0 \Rightarrow P_{St} + P_{Al} = P = 50kN \quad \text{------(1)}$$

$$\therefore \sum M_C = 0 \Rightarrow 2 * P_{St} = 3 * P_{Al} \Rightarrow P_{St} = 1.5 P_{Al} \quad \text{-----(2)}$$



F.B.D

Sub. (1) into (2)

$$\therefore 1.5 P_{Al} + P_{Al} = 50kN \Rightarrow P_{Al} = 20kN$$

$$\therefore P_{St} = 30kN$$

$$\therefore \delta_{St} = \frac{P_{St} * L_{St}}{A_{St} * E_{St}} = \frac{30 * 10^3 * 3}{300 * 10^{-6} * 200 * 10^9} = 1.5 * 10^{-3} m = 1.5 mm$$

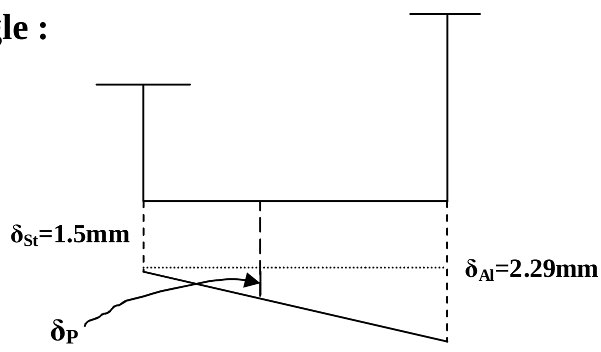
$$\therefore \delta_{Al} = \frac{P_{Al} * L_{Al}}{A_{Al} * E_{Al}} = \frac{20 * 10^3 * 4}{500 * 10^{-6} * 70 * 10^9} = 2.29 * 10^{-3} m = 2.29 mm$$

For triangle shown by similarity of triangle :

$$\therefore \frac{\delta_P - 1.5}{2} = \frac{2.29 - 1.5}{5}$$

$$\delta_P = \frac{2(0.79)}{5} + 1.5$$

$$\delta_P = 1.816 \ mm$$



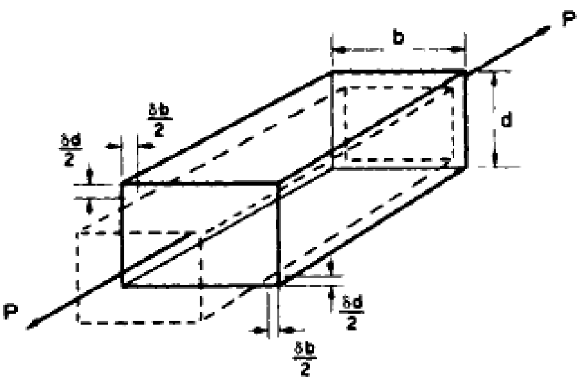
<u>Poisson's ratio (υ) :</u>

Another type of elastic deformation is the change in transverse dimensions accompanying axial tension or compression .

If the bar is extended by axial tension , there is a reduction in the transverse dimensions .The ratio of lateral strain to the longitudinal strain is called ″ *Poisson's ratio* ″ . This ratio is constant for each material like [for steel (υ)=0.25-0.3 , for concrete (υ)=0.2 , and for rubber (υ) $\leq$ 0.49] .

when the load (P) in X-direction : (P_x)

$$\upsilon = \frac{-\varepsilon_y}{\varepsilon_x} = \frac{-\varepsilon_z}{\varepsilon_x}$$



when the load (P) in Y-direction : (P_y)

$$\upsilon = \frac{-\varepsilon_x}{\varepsilon_y} = \frac{-\varepsilon_z}{\varepsilon_y}$$

when the load (P) in Z-direction : (P_z)

$$\upsilon = \frac{-\varepsilon_x}{\varepsilon_z} = \frac{-\varepsilon_y}{\varepsilon_z}$$

From above figure $\Rightarrow \varepsilon_{Lateral} = \frac{\delta d}{d}$ **(in Y-direction)**

$$\varepsilon_{Lateral} = \frac{\delta b}{b} \quad \textbf{(in Z-direction)}$$

$$\varepsilon_{Longitudinal} = \frac{\delta L}{L} \quad \textbf{(in X-direction)}$$

$$\therefore \upsilon = \frac{\varepsilon_{Lateral}}{\varepsilon_{Longitudinal}}$$

$$\upsilon \varepsilon_x = -\varepsilon_y = -\varepsilon_z$$

$$n\sigma_x = \varepsilon_x * E \Rightarrow \varepsilon_x = \frac{\sigma_x}{E}$$

$$n - \upsilon * \frac{\sigma_x}{E} = \varepsilon_y = \varepsilon_z$$ **For one axial loading**

$$\varepsilon_{X_{(1)}} = \frac{\sigma_x}{E}$$ **(due to stress along X-direction)**

$$\varepsilon_{X_{(2)}} = -\upsilon \frac{\sigma_y}{E}$$ **(due to stress along Y-direction)**

$\therefore$ **Total strain in X-direction** $(\varepsilon_{X_t}) = \frac{\sigma_x}{E} - \upsilon \frac{\sigma_y}{E}$ **----------(1)**

$\therefore$ **Total strain in Y-direction** $(\varepsilon_{Y_t}) = \frac{\sigma_y}{E} - \upsilon \frac{\sigma_x}{E}$ **-----------(2)**

From (1) and (2) :

$$\therefore \left. \begin{aligned} \sigma_x &= \frac{(\varepsilon_x + \upsilon * \varepsilon_y) E}{1 - \upsilon^2} \\ \sigma_y &= \frac{(\varepsilon_y + \upsilon * \varepsilon_x) E}{1 - \upsilon^2} \end{aligned} \right] \longrightarrow$$ **For Bi axial loading**

and for Tri axial loading :

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \upsilon(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \upsilon(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \upsilon(\sigma_x + \sigma_y)]$$

Ex: -8- A square steel plate has a tensile strains of (ε_x=2.37*10^{-7} , ε_y=3.95*10^{-4}) along X and Y directions . Find (σ_x and σ_y) when (E= 210 GPa) and (υ=0.28) .

<u>Sol:</u>

$$\varepsilon_x = \frac{\sigma_x}{E} - \upsilon \frac{\sigma_y}{E}$$

$$2.37*10^{-7} = \frac{1}{210*10^9}[\sigma_x - 0.28\sigma_y]$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \upsilon \frac{\sigma_x}{E}$$

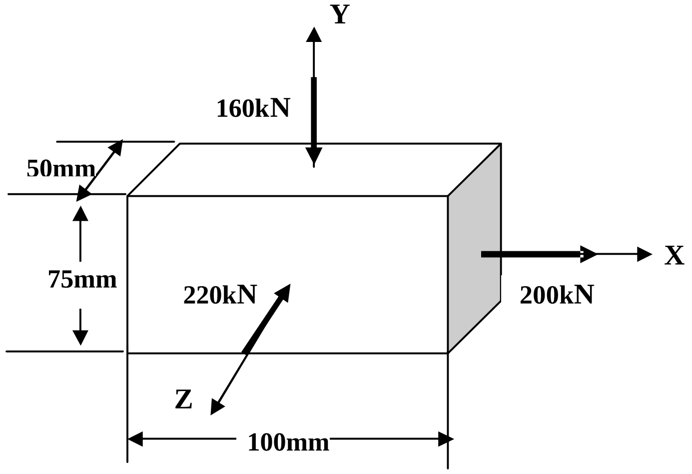
$$3.95*10^{-4} = \frac{1}{210*10^9}[\sigma_y - 0.28\sigma_x]$$

$$\therefore \sigma_x = 0.344\,MPa$$

$$\sigma_y = 1.05\,MPa$$

Ex :-9- A rectangular Aluminum block is (100mm) long in X-direction , (75mm) wide in Y-direction and (50mm) thick in Z-direction . It is subjected to tri axial loading consisting of uniformly distributed tensile force of (200kN) in the X-direction and uniformly distributed compressive forces of 160kN in Y-direction and (220kN) in Z-direction. If the Poisson's ratio (υ=0.333) and (E=70GPa) . Determine a single distributed load that must applied in X-direction that would produce the same deformation in Z-direction as original loading .

$$\varepsilon_z = \frac{\sigma_z}{E} - \upsilon\frac{\sigma_y}{E} - \upsilon\frac{\sigma_x}{E}$$



$$\sigma_x = \frac{P_x}{A} = \frac{200*10^3}{0.05*0.075} = 53.3MPa\ (tension)$$

$$\sigma_y = \frac{160*10^3}{0.05*0.1} = 32MPa\ (compression)$$

$$\sigma_z = \frac{220*10^3}{0.075*0.1} = 24.34MPa\ (compression)$$

$$\therefore \varepsilon_z = \frac{1}{70*10^9}[-24.34 - 0.333(-32 + 53.3)]*10^6$$

$$\varepsilon_z = -0.52*10^{-3}$$

$$\varepsilon_z = -\upsilon\frac{\sigma_x}{E} = -\upsilon\frac{P_x}{A*E} \Rightarrow -0.52*10^{-3} = -0.333\frac{P_x}{0.075*0.05*70*10^9}$$

$$\therefore P_x = 410\ kN\ (Tension)$$

Statically Indeterminate Members :

There are certain combinations of axially loaded members in which the equations of static equilibrium not sufficient for solution .This condition exists in structures where the reactive forces or the internal resisting forces over a cross section exceed the number of independent equation of equilibrium . Such this cases are called ″ *Statically Indeterminate* ″ and require the use of additional relations which depend upon the elastic deformation in the members .

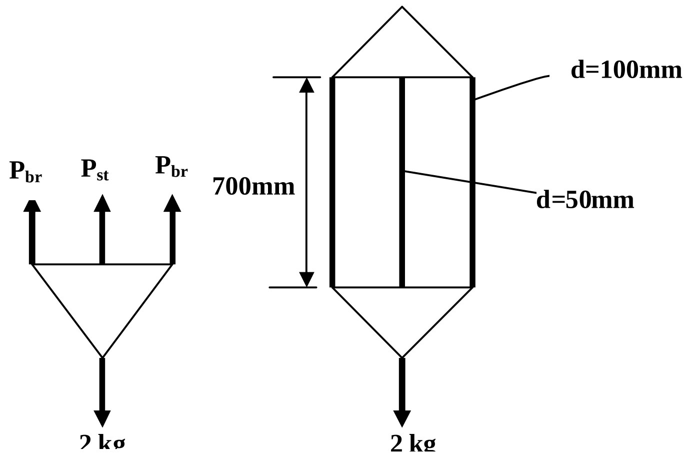
Following general principals that can be applied to solve this kind of problems :-

1- Applied the equations of static equilibrium to free body diagram of the structure or part of it .

2- Obtain additional equations from the geometric relations between the elastic deformations that produced by loads for the unknown which are more than independent equations of equilibrium .

Ex:-10- In the figure shown , a pendulum loaded with (2kg), the pendulum consist of 3-bars of equally length , if the outer are made of bronze and the meddle of steel and their diameter shown on figure . Determine the axial force in each member .

E_s=200GPa , E_b=120GPa .



F.B.D

Sol:- **From F.B.D**

$$\sum F_y = 0 \Rightarrow 2P_{br} + P_{st} = 2*9.81 \cdots\cdots (1)$$

$$\delta_{st} = \delta_{br}$$

$$\therefore \frac{P_{st} * L_{st}}{A_{st} * E_{st}} = \frac{P_{br} * L_{br}}{A_{br} * L_{br}}$$

$$\therefore P_{st} = \frac{A_{st} * P_{br} * E_{st}}{A_{br} * E_{br}}$$

$$P_{st} = \frac{\frac{\pi}{4}(0.05)^2 * P_{br} * 200 * 10^9}{\frac{\pi}{4}(0.1)^2 * 120 * 10^9} = 0.416 P_{br}$$

$$\therefore P_{st} = 0.146 P_{br}$$ **…….(2)** **sub in equ.(1)**

$$2 * P_{br} + (0.416) P_{br} = 2 * 9.81$$

$$\therefore P_{br} = 8.12 N$$

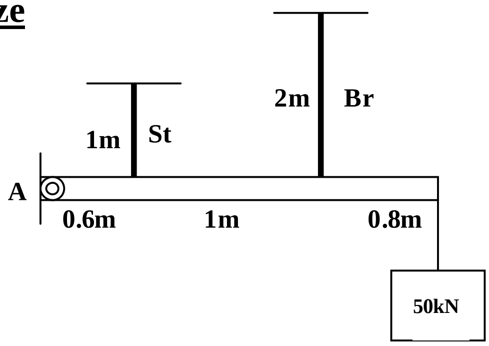
$$P_{st} = 3.37 N$$

$$\therefore \sigma_{br} = \frac{P_{br}}{A_{br}} = \frac{8.12}{\frac{\pi}{4}(0.1)^2} = 1.033 kPa$$

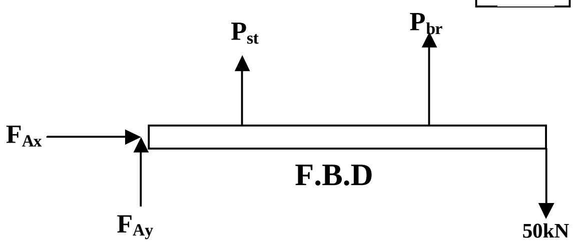
$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \frac{3.37}{\frac{\pi}{4}(0.05)^2} = 1.716 kPa$$

Ex-11- A horizontal bar of negligible mass, hanged by hinge at point (A) as shown. The bar is assumed to be rigid and is supported by vertical bronze bar of (2m) length , a steel bar of (1m) length , using the data given below , determine the stress in each bar .

	Steel	**Bronze**
A(mm^2)	**600**	**300**
E(GPa)	**200**	**83**

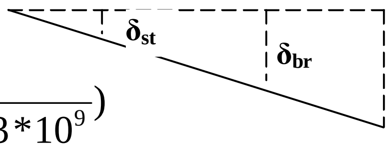


Sol:-



$$\sum M_A = 0 \Rightarrow 0.6P_{st} + 1.6P_{br} = 50*2.4 ....(1)$$

From triangles symmetry :



$$\frac{\delta_{st}}{0.6} = \frac{\delta_{br}}{1.6}$$

$$\frac{1}{0.6}\left(\frac{P_{st} * L_{st}}{A_{st} * E_{st}}\right) = \frac{1}{1.6}\left(\frac{P_{br} * L_{br}}{A_{br} * E_{br}}\right)$$

$$\frac{1}{0.6}\left(\frac{P_{st} * 1}{600*10^{-6} * 200*10^{9}}\right) = \frac{1}{1.6}\left(\frac{P_{br} * 2}{300*10^{-6} * 83*10^{9}}\right)$$

$\therefore\ P_{st} = 3.61 P_{br}$ .... (2) **Sub in (1)**

$$\therefore 0.6(3.61P_{br}) + 1.6P_{st} = 120$$

$$\therefore P_{br} = 31.9kN \Rightarrow \sigma_{br} = \frac{31.9*10^3}{300*10^{-6}} = 106MPa$$

$$P_{st} = 115kN \Rightarrow \sigma_{st} = \frac{115*10^3}{600*10^{-6}} = 191.66MPa$$

Thermal Stresses :

The change in temperature causes bodies to expand or contract , the amount of linear deformation (δ_{th}) is expressed as follows :

$$\delta_{th} = \alpha * L * \Delta T$$

where :

α-------The coefficient of linear deformation in unit of (m/m.C^o)

L-------Length of the body (m)

ΔT------Temperature change (C^o).

A general procedure for computing the loads and stresses caused when thermal deformation happened as result for temperature changing is outlines in steps :

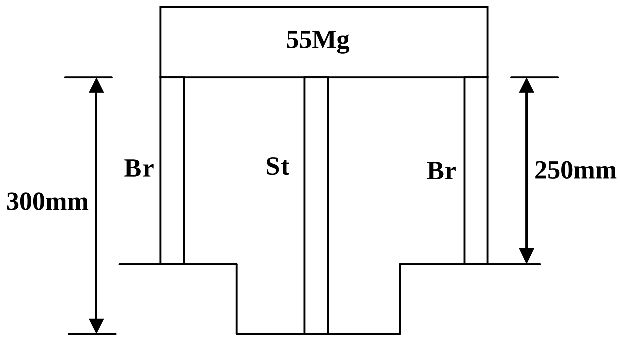
1-Assume that the body is free from all applied loads and constraints so that thermal deformations can occur freely .

2- Apply sufficient load to the body to restore it to the original condition

3- Solve to find unknowns , using equations of equilibrium and equations which are obtained from geometric relations between the temperature and load deformation .

Ex-12- At ($20C^o$) rigid slab having a mass of (55Mg) is placed upon two bronze rods and one steel rod as shown in figure . At what temperature will the mechanical stress in the steel rod be zero .

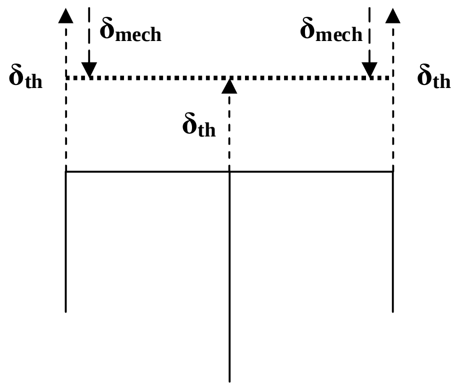
	Steel	**Bronze**
A(mm^2)	**6000**	**6000**
E(GPa)	**200**	**83**
α($1/C^o$)	**$11.7*10^{-6}$**	**$19*10^{-6}$**



Sol :-

$$\delta_{St_{total}} = \delta_{Br_{total}}$$

$$\delta_{total} = \delta_{thermal} + \delta_{mechanical}$$



when stress in steel been zero ⇒ $P_{st}=0$ ⇒ $\delta_{st}=0$

$$\therefore \delta_{St_{total}} = \delta_{St_{thermal}}$$

From figure of total deformation :

$$\therefore \delta_{St_{thermal}} = \delta_{Br_{thermal}} - \delta_{Br_{mechanical}}$$

$$\therefore \alpha_{st} * L_{st} * \Delta T = \alpha_{br} * L_{br} * \Delta T - \frac{P_{br} * L_{br}}{A_{br} * E_{br}}$$

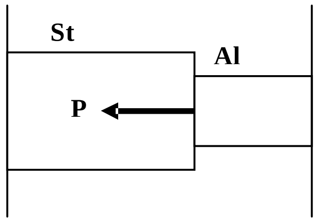
$$11.7*10^{-6} * 0.3 * \Delta T = 19*10^{-6} * 0.25 * \Delta T - \frac{\frac{55*10^3 * 9.81}{2} * 0.25}{6000*10^{-6} * 83*10^9}$$

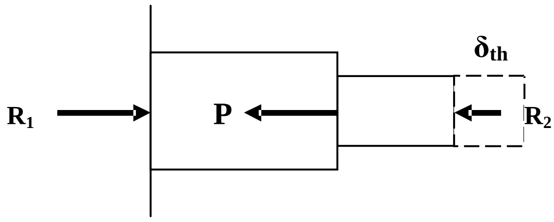
$\Delta T=109C^o$, $T_2=T_1+\Delta T$

∴ $T_2=20+109=\underline{129C^o}$

Ex-13- Composite bar shown in figure is consist of steel and aluminum bar and its supports rigidly , and the load of (P=100kN)is applied as shown in figure at temperature ($20C^o$) . Find the stresses in steel and aluminum bar when temperature reach to ($100C^o$) .

	Steel	**Aluminum**
A(mm^2)	**1200**	**900**
E(GPa)	**200**	**70**
α($1/C^o$)	**$11.7*10^{-6}$**	**$23*10^{-6}$**
L(m)	**0.5**	**0.3**





Sol :

$$\sum F_x = 0 \Rightarrow R_1 = R_2 + P ....(1)$$

n **The bars is rigidly supports**

$$\therefore \delta_{th_{total}} = \delta_{mech_{total}}$$

$$\delta_{th_{St}} + \delta_{th_{Al}} = \delta_{mech_{St}} + \delta_{mech_{Al}}$$

$$\text{n}\ \alpha_{St} * L_{St} * \Delta T + \alpha_{Al} * L_{Al} * \Delta T = \frac{P_{St} * L_{St}}{A_{St} * E_{St}} + \frac{P_{Al} * L_{Al}}{A_{Al} * E_{Al}}$$

$$11.7*10^{-6}*0.5*(100-20) + 23*10^{-6}*0.3*(100-20) = \frac{P_{St}*0.5}{1200*10^{-6}*200*10^{9}} + \frac{P_{Al}*0.3}{900*10^{-6}*10*10^{9}}$$

$$\therefore P_{St} + 2.266 P_{Al} = 485.7*10^3 ....(2)$$

when : **$P_{st}=R_1$, $P_{Al}=R_2=R_1$- P , P=100kN**

Sub (1) in (2)

$$\therefore R_1 + 2.266\left(R_1 - 100*10^3\right) = 485.7*10^3$$

$\therefore$ **R_1=218kN=P_{St}**

$$\therefore \sigma_{St} = \frac{218 * 10^3}{1200 * 10^{-6}} = 181.75\ MPa$$

$$R_2 = 218 - 100 = 118\ kN$$

$$\therefore \sigma_{Al} = \frac{118 * 10^3}{900 * 10^{-6}} = 131.1 MPa$$