Chapter Four

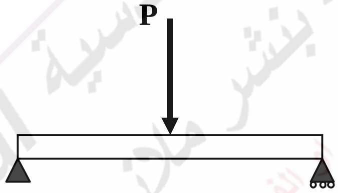
Shear and Moment in Beams

A beam may be defined as bar of material upon which are acting transverse loads and reactions .In buildings , bridges and such structures , beams are usually horizontal and the loads are usually weights .

In machines and beams may have any direction and the loads are often pressures from other parts of the machine or from other outside bodies .

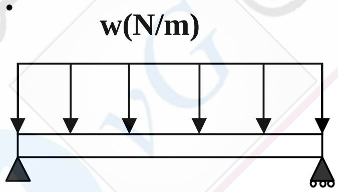
Loads are classified as :-

1- Concentrated load .



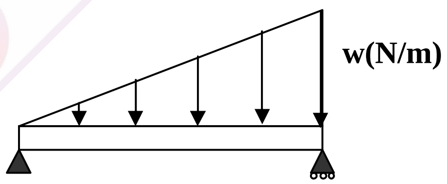
1- Concentrated Load

2- Uniformly distributed load .



2- Uniformly Distributed Load

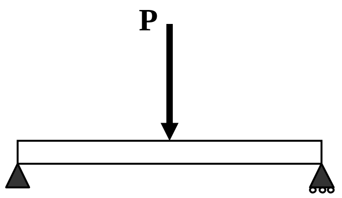
3- Varyingly distributed loads .



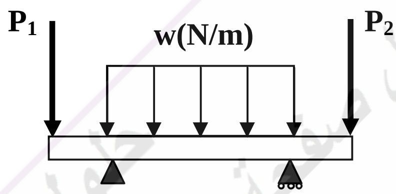
3- Varying Distributed Load

There are different types of beams supporting such as :

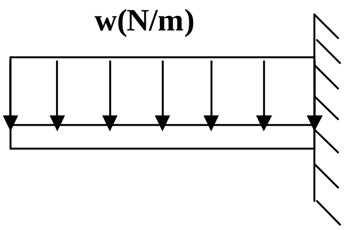
1- Simply supporting beam :



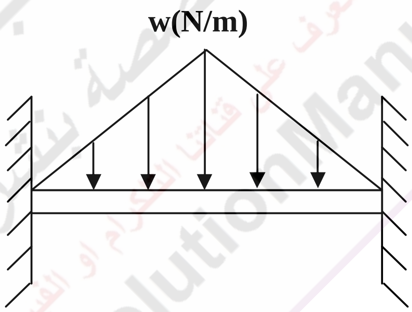
2- Overhanging beam :



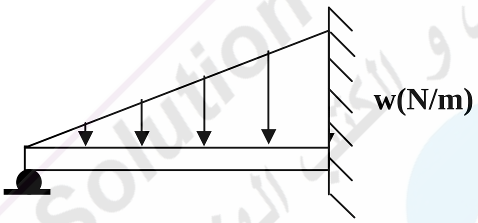
3- Cantilever beam :



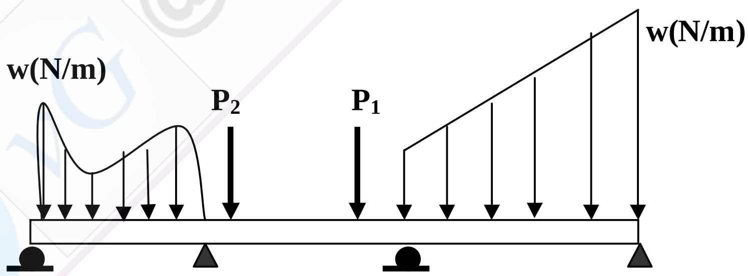
4- Fixed-end beam :



5- Propped beam :



6- Continues beam :



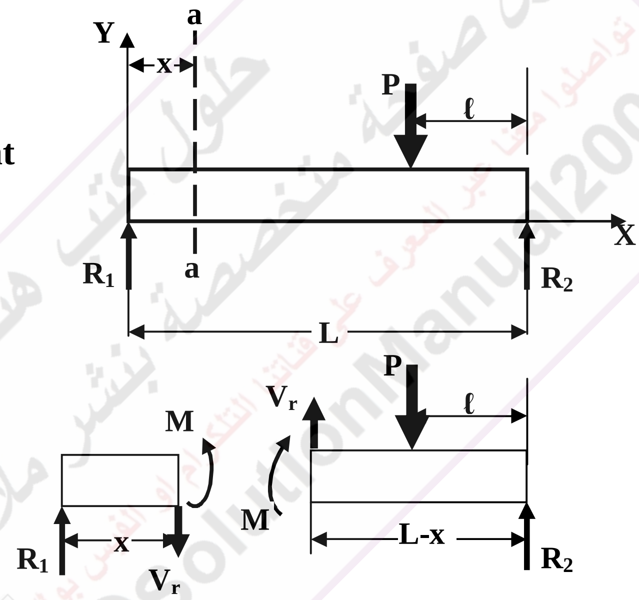
Generally two kind of stresses act over the transverse section of beams :

1) Shearing stresses : which varies directly with the shear force (V).

2) Bending stress : which varies directly with the bending moment (M) .

Shear and Moment :

A simple beam shown in figure carries a concentrated load (P) and is held in equilibrium by the reaction (R_1) and (R_2) .



Neglect the mass of beam itself and consider the effect of the load (P) only . Equilibrium of segments to left and right of any exploratory section (a-a).

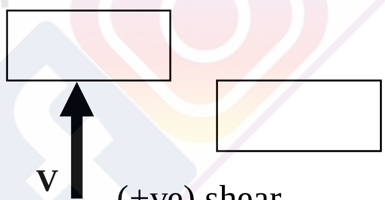
For the left segment and to maintain equilibrium :-

$$\sum F_y = 0 \Rightarrow V_r = R_1$$
$$\sum M = 0 \Rightarrow R_1 * x = M_r$$

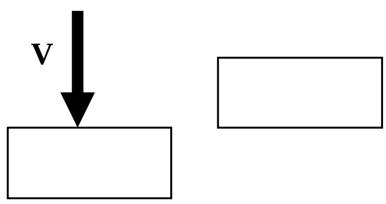
For the right segment :-

$$\sum F_y = 0 \Rightarrow V_r + R_2 = P$$
$$\sum M = 0 \Rightarrow M_r = R_2(L-x) - P(L-x-a)$$

In computing shear force (V) , upward acting force are usually considered to be positive as shown in figure where the positive shearing force tends to move the left segment upward with respect to the right segment and vise-versa .

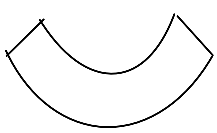


(+ve) shear

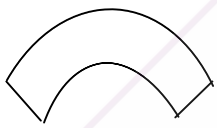


(-ve) shear

Bending moment is positive if it produces bending of the beam concave upward and vise-versa as shown in figure .



(+ve) bending



(-ve) bending

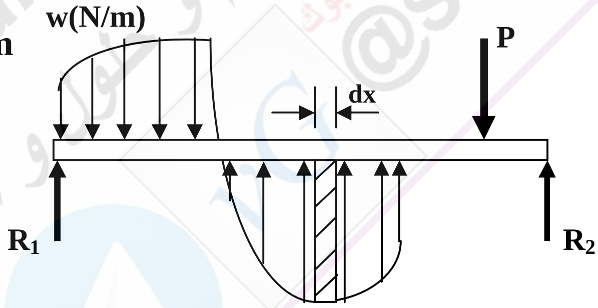
Upward acting forces generally causes (+ve) bending regardless to weather they act to the left or to right of exploratory section .

Relation between load , shear and moment :-

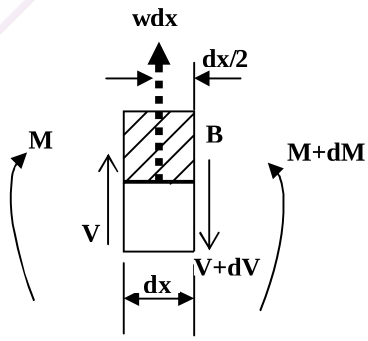
We will discuss the relations existing between the loads, shears and bending moments in any beam . Theses relations provide a method of constructing shear and moment diagrams without writing shear and moment equations .

The beam shown in figure (1) is assumed to carry any general loading .

The free body diagram of segment of this beam of length (dx) is shown in figure (2) .



(1)



(2)

$$\sum F_y = 0 \Rightarrow V + wdx - (V + dV) = 0$$

w---------Intensity of load (N/m)

$$\therefore\ dV = wdx$$

$$\sum M_B = 0 \Rightarrow M + Vdx + (wdx)\frac{dx}{2} - (M + dM) = 0$$

where : $w\dfrac{(dx)^2}{2} \approx zero$

$$\therefore\ dM = Vdx$$

for ($dV = wdx$) we can integrating

$$\therefore \int_{V_1}^{V_2} dV = \int_{x_1}^{x_2} w dx$$

but $\int_{x_1}^{x_2} w dx \Rightarrow$ is the summation of area between (x_1 and x_2)

$$\therefore V_2 - V_1 = \Delta V = (area)_{load}$$

similarly

$$\int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V dx$$

$$M_2 - M_1 = \Delta M = (area)_{shear}$$

$$\therefore w = \frac{dV}{dx} \qquad \text{(slope of shear diagram)}$$

$$\therefore V = \frac{dM}{dx} \qquad \text{(slope of moment diagram)}$$

Basic steps for construction of shear and moment diagrams :

1- Compute the reactions .

2- Compute the values of shear at the change of load points using either

$(V = \sum F_y)_L$ **or** $(\Delta V = (area)_{load})$

3- Sketch the shear diagram , determining the shape (the intensity of the load ordinate equals the slope at corresponding ordinate of the shear diagram) .

4- Locate the points of *zero shear* .

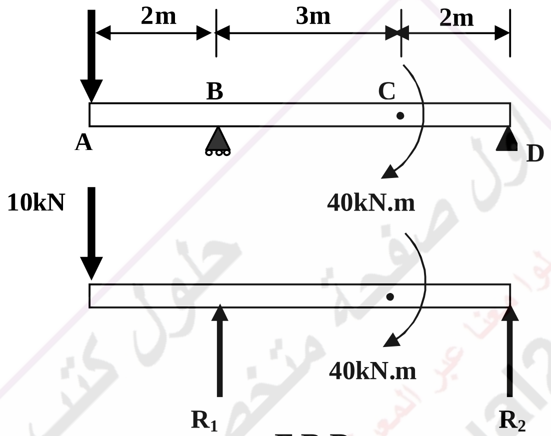
5- **Compute the values of bending moment at the change of load points and at the points of zero shear usinge ither**

$$\left(M = \sum M_L = \sum M_R\right) or \left(\Delta M = (area)_{shear}\right)$$

whichever more convenient .

6- Sketch the moment diagram depending on step (5) .

Ex:-1- Draw shear force diagram and bending moment diagram for the beam shown in figure .



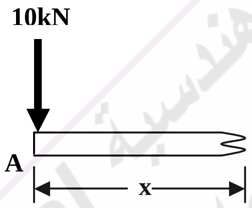
F.B.D

Sol:-

Neglect the horizontal reactions (R_x=0)

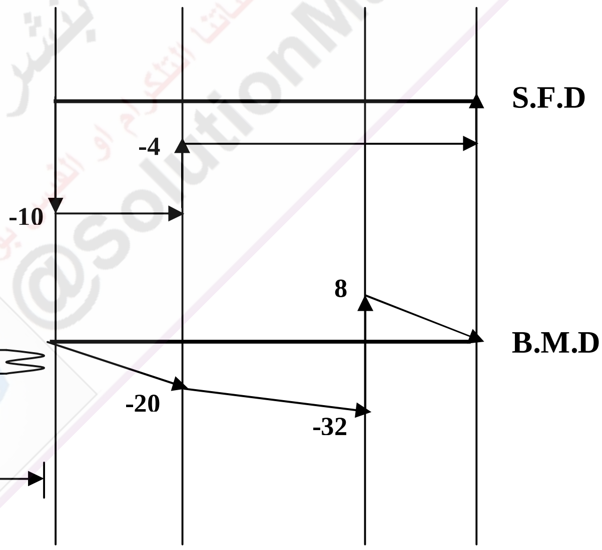
$$\sum F_y = 0 \Rightarrow R_1 + R_2 = 10kN$$
$$\sum M_D = 0 \Rightarrow 10*7 = 5*R_1 + 40$$
$$\therefore R_1 = 6kN \Rightarrow R_2 = 4kN$$

For $0 \le x \le 2$

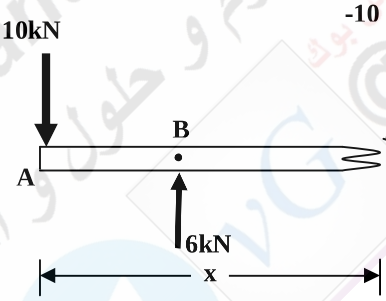


V=-10kN

M=-10x
- **x=0$\Rightarrow$M=0**
- **x=2$\Rightarrow$M=-20kN.m**



For $0 \le x \le 5$



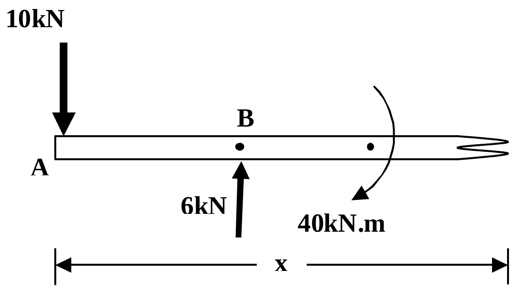
V=-10+6=-4kN

M=10x+6(x-2)

M=-10x+6x-12

M=-4x-12
- **x=2$\Rightarrow$M=-20kN.m**
- **x=5$\Rightarrow$M=-32kN.m**

For $0 \le x \le 7$

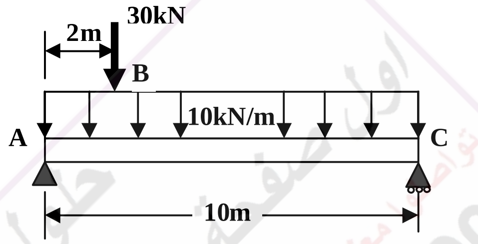


V=-10+6=-4kN

M=10x+6(x-2)+40

M=-4x+28
- **x=5$\Rightarrow$M=-32kN.m**
- **x=7$\Rightarrow$M=0**

Ex:-2- Draw shear force and bending moment diagrams for the beam shown in figure .



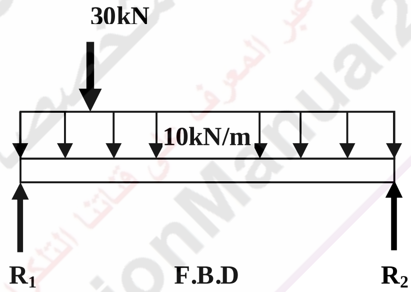
Sol:-

$$\sum F_y = 0 \Rightarrow R_1 + R_2 = 30 + 10*10$$

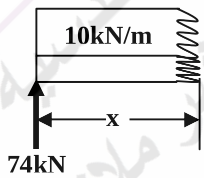
$$\sum M_c = 0 \Rightarrow 10R_1 - 30*8 - 10*10*5 = 0$$

$$\therefore R_1 = 74\,kN \;\Rightarrow\; R_2 = 56\,kN$$

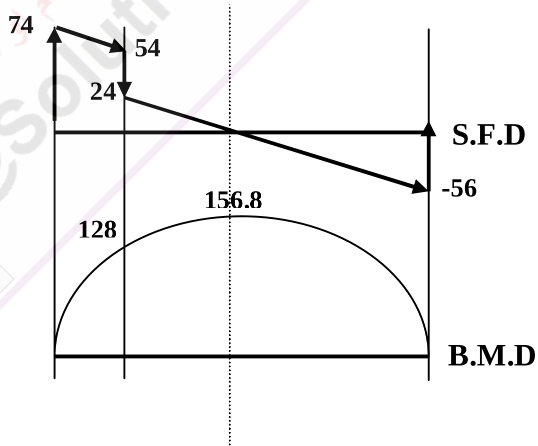
For $0 \le x \le 2$`



V=74-10x — x=0 ⇒ V=74kN ; x=2 ⇒ V=54kN



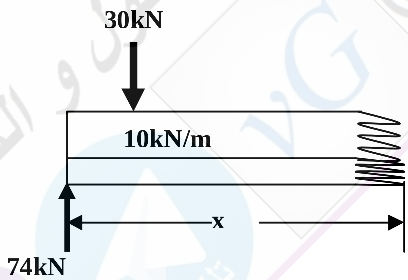
M=74x-10x*x/2



M=74x-5x^2 — x=0 ⇒ M=0 ; x=2 ⇒ M=128kN.m

For $0 \le x \le 10$

V=74-10x-30



V=44-10x — x=2 ⇒ V=24kN ; x=10 ⇒ V=-56kN

M=74x-(10x*x/2)-30(x-2) ⇒ M=-5x^2+44x=60 — x=2 ⇒ M=128kN.m ; x=10 ⇒ M=0

$$\frac{dM}{dx} = -10x + 44 = 0 \Rightarrow x = 4.4m$$

$$M_{x=4.4m} = -5(4.4)^2 + 44(4.4) + 60$$

M_{max} =156.8kN.m