Chapter Nine

The Columns

Definition: The Column is a long slender bar subjected to axial compression forces . Term (column) used to describe a vertical member while term (Trust) is used to describe the inclined member . The compression bar is generally considered to be a **column** when its unsupported length is more than (10 times) its lateral dimension .

Examples of column like aircraft structural components , structural connection between stages of boosters for space vehicle , certain members in bridge trusses , and structural framework of buildings are common examples of columns .

Columns are usually subdivided into two groups , long and intermediate and sometimes the short compression block is considered to be third group .

The distinction between the three groups is by behavior , long column fail by buckling or excessive lateral bending , intermediate columns fail by combination of crushing and buckling , and short compression blocks is fail by crushing .

The Critical load of column :-

The critical load (P_{cr}) of slender bar subjected to axial compression is the value of axial force that is just sufficient to keep the bar in slightly deflected configuration as shown in figure (1) .

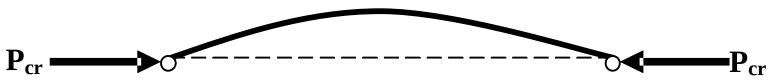


Fig.-1- Pin-ended bar

The ratio of length of the column to the minimum radius of gyration of the cross-sectional area is termed (***Slenderness ratio***) of the bar . This ratio is dimensionless

(Slenderness ratio $= \frac{L}{r}$) -----------(1)

where : L ----- length of bar

r **----- radius of gyration**

Radius of Gyration : If the moment of an area (A) then the radius of gyration is defined by :

$$r = \sqrt{\frac{I(m^4)}{A(m^2)}} \text{ --------(2)}$$

If the long slender bar of constant cross section is pinned at each end and subjected to axial compression , the load (P_{cr}) that will cause buckling is given by :-

$$P_{cr} = \frac{\pi^2 EI}{L^2} \text{ --------(3)}$$

E-----Modulus of Elasticity

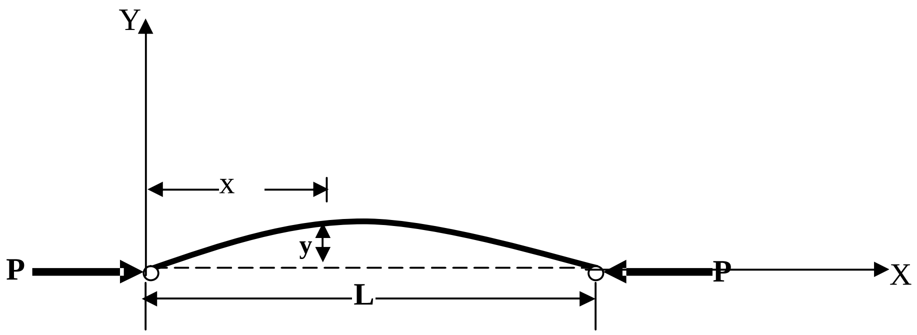


Fig.-2- Deflected shape of bar

$$EI\frac{d^2y}{dx^2} = M = -Py \text{ ----------(4)}$$

Let $\left(\frac{P}{EI} = k^2\right)$ -----------(5)

$$\Rightarrow \frac{d^2y}{dx^2} + ky = 0 \text{ ----------(6)}$$

This equation is readily solved by standard technique of differential equations as shown :

$$Y = A\sin(kx) + B\cos(kx) \text{ ----------(7)}$$

from boundary condition of bar :

***Y*=0 at x=0** ▪▪▪▪▶ ***B*=0**

Y=0 at x=L ▪▪▪▪▶ **kL=n π where (n=1,2,3,4,……,)**

$$\therefore\ kL = n\pi \text{ --------(8)}$$

Sub equ.(5) in equ.(8)

$$\therefore\ \sqrt{\frac{P}{EI}}L = n\pi \text{ ---------(9)}$$

$$\Rightarrow P = n^2\frac{\pi^2 EI}{L^2} \text{ ----------(10)}$$

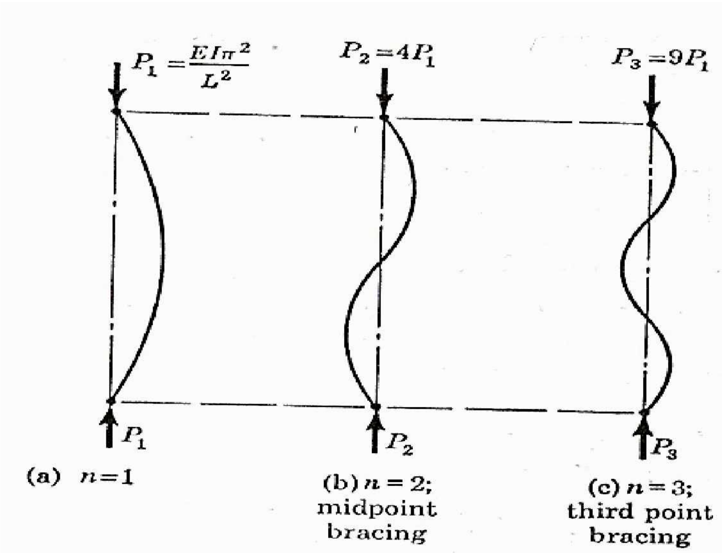


Fig.-3- Effects of (n) on loads

when n= 1 (pin-ended bar) then :- $P_{cr} = \frac{\pi^2 EI}{L^2}$ ----------(12)

and the critical stress for is define as :-

$$\sigma_{cr} = \frac{\pi^2 EI}{A * L^2} \text{ ----------(13)}$$

This is called Eulers buckling formula for load and stress of pin-ended column .

The deflection shape corresponding to this load is :

$$y = A\sin(\sqrt{\frac{P}{EI}}x) \text{ -----------(14)}$$

Substituting equ. (9) in equ.(14) and we get :

$$y = A \sin(\frac{\pi x}{L}) \text{ ---------(15)} \quad \text{when n=1 (pin-ended bar)}$$

Equation (9) may be modified to the form :-

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \text{ -----------(16)}$$

Where (KL) is an effective length of column (***L_e***) and value f (K) depend on type of fixing for the bar as shown in figure (4) .

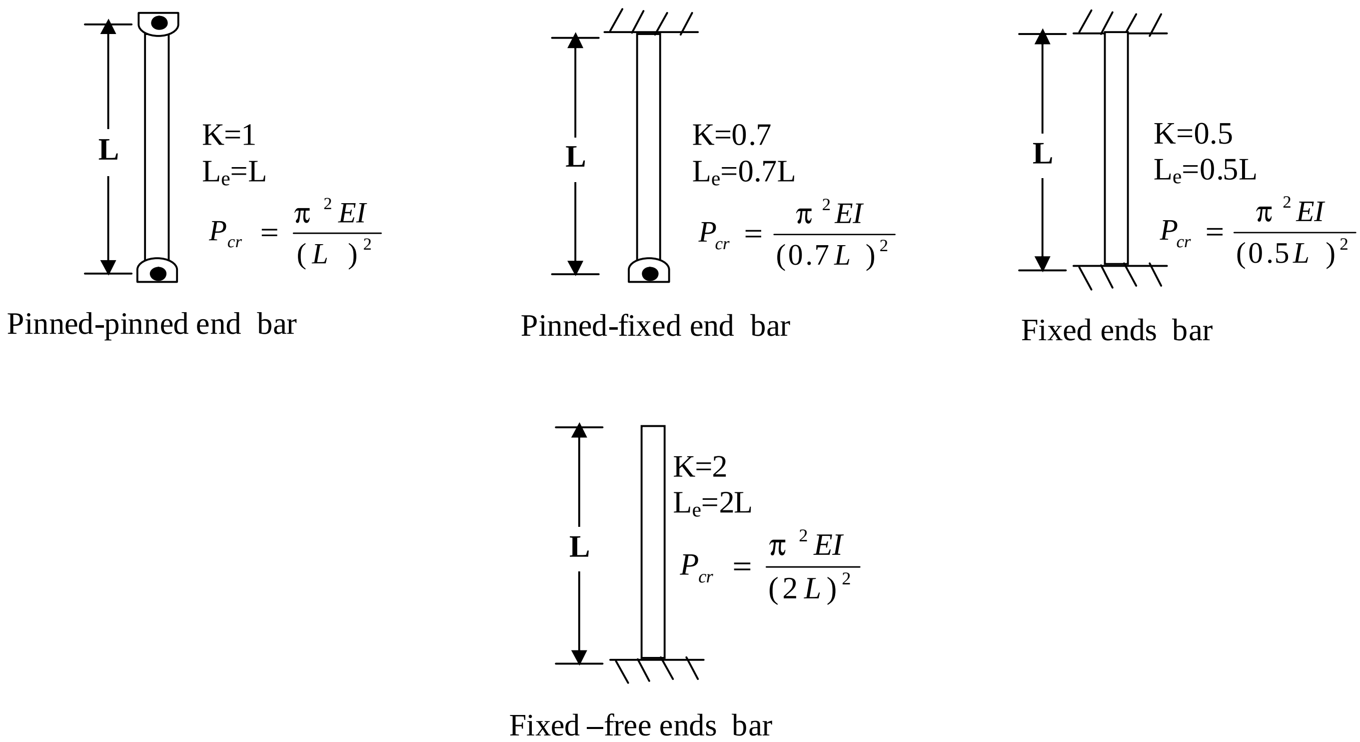
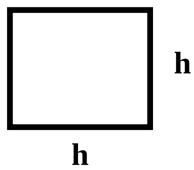


Fig.-4- Types of bar fixing

Ex-1-: A square aluminum bar is to be supported a load of (40kN) with length of (3m) . The type of fixing bar was pin-ends , determine the dimensions of bar section when slenderness ratio of bar is (120) . G_{Al}=70GPa

Sol:-

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{(KL)^2}$$



for pin-ends bar $\Rightarrow K=1$

$$\therefore 40*10^3 = \frac{\pi^2 * 70*10^9 * I}{(3)^2}$$

$$\therefore I = 5.21*10^{-7} m^4 = \frac{b*h^3}{12} = \frac{h^4}{12}$$

$$\therefore h = 0.05\,m = 50\,mm$$

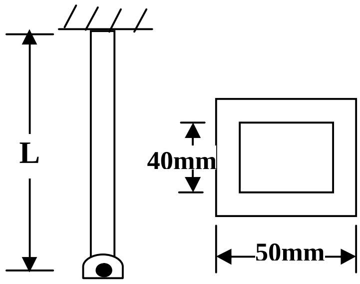
$$P_{cr} = \frac{\pi^2 E * A}{(\frac{L}{r})^2}$$

$$\therefore 40*10^3 = \frac{\pi^2 * 70*10^9 * h^2}{(120)^2}$$

h=0.029m = 29mm

We chose the dimension (h=50mm)

Ex-2- : For a square column shown , find its smallest length that is caused buckling for it when $(\sigma_{cr} = 100 MPa)$ and the column is pinned-fixed ends type , and find the slenderness ratio for this column . E=200GPa



Sol:-

$$\sigma_{cr} = \frac{P_{cr}}{A} \Rightarrow P_{cr} = \sigma_{cr} * A$$

$$\therefore P_{cr} = 100 * 10^6 * (0.05 * 0.05 - 0.04 * 0.04)$$

$$\therefore P_{cr} = 90\, kN = \frac{\pi^2 * E * I}{L_e^2}$$

$$I = \frac{b * h^3}{12} = \frac{1}{12}\left[(0.05)^4 - (0.04)^4\right] = 3 * 10^{-7} m^4$$

$$\therefore 90 * 10^3 = \frac{\pi^2 * 200 * 10^9 * 3 * 10^{-7}}{(0.7 L)^2}$$

$$\therefore L = 5.25\, m$$

Slenderness ratio $= \frac{L}{r}$

$$I = A * r^2 \Rightarrow r^2 = \frac{I}{A} \Rightarrow r = \sqrt{\frac{3*10^{-7}}{(0.05*0.05-0.04*0.04)}} = 18.5 mm$$

$$\frac{L}{r} = \frac{5250}{18.5} = 283.8$$