Chapter Seven

Thin and Thick Cylinders

A) Thin Wall Cylinders :

The thin cylinder have a ratio of inside diameter to thickness of wall $\left(\frac{d}{t}\right)$, and in the cylinder this ratio equal to $\left(\frac{d}{t} \geq 20\right)$ while the thick cylinder have ratio of $\left(\frac{d}{t} \leq 20\right)$.

A cylindrical vessel may carrying fluid (liquid or gas) under pressure of $p\,(\frac{N}{m^2})$ and its subjected to tensile forces which resist the bursting forces developed across longitudinal and transverse section .

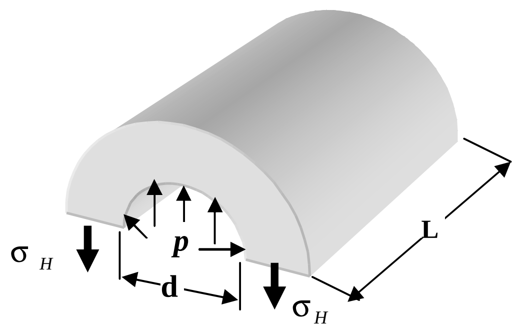
In the thin wall cylinder two types of stresses are created due to the fluid pressure :

1- Hoop stresses (circumferential or tangential stresses) : σ_H

For balancing forces on the thin cylinder :

Force due to internal pressure (F) = ***p**** Area

= ***p*** *(d *L)



Resisting force due to hoop stress (P)= 2*σ_H ***** L***** t

$$\therefore F = P$$

$$p * d * L = 2 * \sigma_H * L * t$$

$$\therefore \sigma_H = \frac{p * d}{2t}$$

hoop strain $\varepsilon_H = \frac{1}{E}\left[\sigma_H - \upsilon\sigma_L\right]$ while $\varepsilon_H = \frac{\sigma_H}{E}$

$$\therefore \varepsilon_H = \frac{p*d}{4t*E}\left[2-\upsilon\right]$$

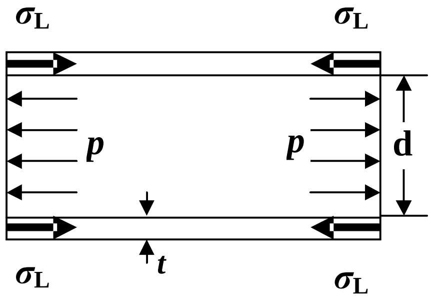
change in diameter = $\varepsilon_H * d_1 \Rightarrow d_2 = d_1(1+\varepsilon_H)$ New diameter

$$\therefore \Delta d = d_1 * \varepsilon_H$$

2- **Longitudinal stresses :** σ_L

For balancing forces on the thin cylinder :

Force due to internal pressure (F) = ***p**** Area



$$F = p*\frac{\pi}{4}d^2$$

Resisting force due to

longitudinal stress (P) = $\sigma_L * \pi * d * t$

n $F = P$

$$\therefore p*\frac{\pi}{4}*d^2 = \sigma_L * \pi * d * t$$

$$\therefore \sigma_L = \frac{p*d}{4t}$$

Longitudinal strain (ε_L)= $\frac{1}{E}(\sigma_L - \upsilon\sigma_H)$

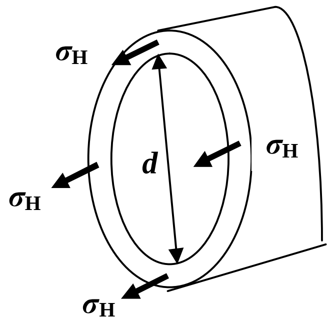
$$= \frac{p*d}{4t*E}(1-2\upsilon)$$

Change in length = $\varepsilon_L * L_1 = \Delta L$

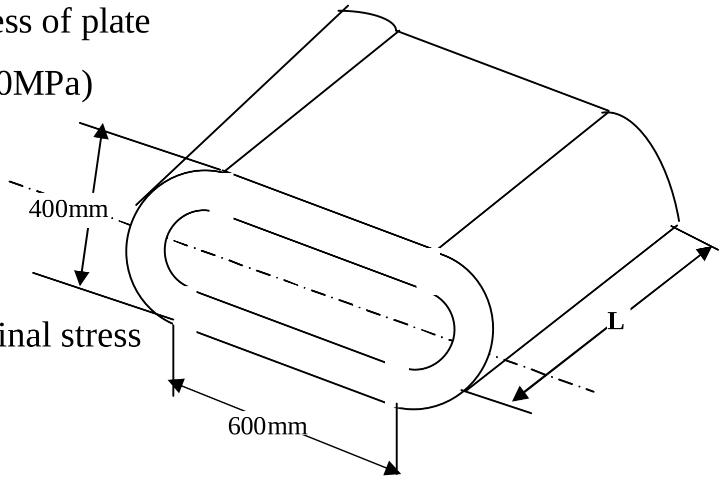
B) Thin wall Spherical vessel :

In spherical vessel there is one type of stresses

$$\therefore \sigma_H = \frac{p*d}{4t} = \sigma_L$$



Ex:- The tank shown in figure is fabricated from Steel plate . Determine the minimum thickness of plate which may be used if the stress limited to (40MPa) and internal pressure is (1.5MPa).



Sol:-

Load due to pressure = Load due to longitudinal stress

F = P

$$\sigma_L * A = p * A$$

$$\sigma_L * (\pi * d * t + 2t * l) = p * (\frac{\pi}{4} d^2 + d * l)$$

$$40*10^6 (\pi * 0.4 * t + 2t * 0.6) = 1.5*10^6 (\frac{\pi}{4} (0.4)^2 + 0.4 * 0.6)$$

$$\therefore 98.3t = 0.55 \Rightarrow t = 5.6mm$$

$$\sigma_H * 2 * t * L = p * d * L$$

$$40*10^6 * 2 * t * L = 1.5*10^6 * 0.4 * L$$

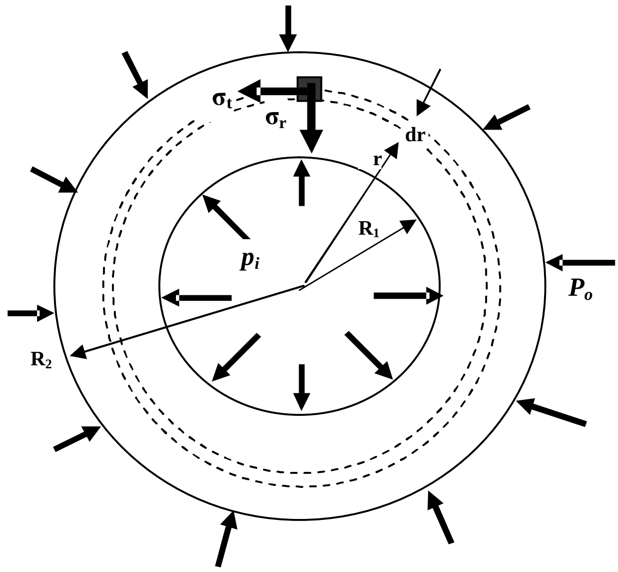
$$\therefore t = 7.5mm$$

We use the (t=7.5 mm)

B) Thick Wall Cylinder :

The cylinder may be called as ***Thick Cylinder*** when $(\frac{d}{t} \leq 20)$.

In thick cylinder we have tangential (Hoop) stress ($\sigma_t = \sigma_H$), longitudinal stress (σ_L) , and radial stress (σ_r) . But the common types of stresses that used in thick cylinder is tangential and radial stresses , while the radial stress in thin cylinder is so small so that it can be neglected .



$$\sigma_t = A + \frac{B}{r^2}$$

$$\sigma_r = A - \frac{B}{r^2}$$

where :

A , B ⟶ Constants are determined by boundary condition

For internal pressure (***p***$_i$):

$$\sigma_r = -p_i = A - \frac{B}{R_1^{\,2}} \quad \text{------------(1)}$$

For external pressure ($\boldsymbol{p_o}$)

$$\sigma_r = -p_o = A - \frac{B}{R_2^{\,2}} \quad \text{------------(2)}$$

from equ.(1) and (2) we can find : $A = \dfrac{R_1^2 p_i - R_2^2 p_o}{R_2^2 - R_1^2}$, $B = \dfrac{R_1^2 R_2^2 (p_i - p_o)}{R_2^2 - R_1^2}$

$$\therefore \sigma_t = \frac{R_1^2 p_i - R_2^2 p_o}{R_2^2 - R_1^2} + \frac{R_1^2 R_2^2 (p_i - p_o)}{(R_2^2 - R_1^2) r^2}$$

$$\therefore \sigma_r = \frac{R_1^2 p_i - R_2^2 p_o}{R_2^2 - R_1^2} - \frac{R_1^2 R_2^2 (p_i - p_o)}{(R_2^2 - R_1^2) r^2}$$

$$\therefore \sigma_L = \frac{R_1^2 p_i - R_2^2 p_o}{R_2^2 - R_1^2}$$

<u>For internal pressure only</u> : $p_{o=0}$

$$\therefore \sigma_t = \frac{R_1^2 p_i}{R_2^2 - R_1^2}\left[1 + \frac{R_2^2}{r^2}\right]$$

$$\therefore \sigma_r = \frac{R_1^2 p_i}{R_2^2 - R_1^2}\left[1 - \frac{R_2^2}{r^2}\right]$$

where :

$\sigma_t \longrightarrow$ **Tensile stress (Positive)**

$\sigma_r \longrightarrow$ **Compressive stress (Negative)**

The maximum stress at inside surface of cylinder at ($r=R_1$)

$$\sigma_{t_{Max.(r=R_1)}} = \left(\frac{R_2^2 + R_1^2}{R_2^2 - R_1^2}\right) p_i$$

$$\sigma_{r_{Max.(r=R_1)}} = -p_i$$

$$\therefore \tau_{Max.(r=R_1)} = \frac{\sigma_{t_{Max.}} - \sigma_{r_{Max.}}}{2} = \frac{\frac{(R_2^2 + R_1^2)}{R_2^2 - R_1^2} p_i + p_i}{2}$$

$$\therefore \tau_{Max.(r=R_1)} = \frac{R_2^2}{R_2^2 - R_1^2} p_i$$

$$\therefore \sigma_L = \frac{p_i R_1^2 - p_o R_2^2}{R_2^2 - R_1^2}$$

$$\therefore \sigma_{L_{p_o=0}} = \frac{p_i R_1^2}{R_2^2 - R_1^2}$$

For external pressure only : $\boldsymbol{p_{i}=0}$

$$\therefore \sigma_t = -\frac{R_2^2 p_o}{R_2^2 - R_1^2}\left[1 + \frac{R_1^2}{r^2}\right]$$

$$\therefore \sigma_r = -\frac{R_2^2 p_o}{R_2^2 - R_1^2}\left[1 - \frac{R_1^2}{r^2}\right]$$

In this case the (σ_t) and (σ_r) are always compressive and maximum (σ_t) occur at inner surface of cylinder while ($\sigma_r = 0$) at ($r=R_1$) .

$$\therefore \sigma_{t_{Max.}} = -\frac{2R_2^2 p_o}{R_2^2 - R_1^2}$$

$$\therefore \sigma_r = 0$$

$$\therefore \sigma_L = \frac{p_i R_1^2 - p_o R_2^2}{R_2^2 - R_1^2} = \frac{-p_o R_2^2}{R_2^2 - R_1^2}$$

Ex:- A thick cylinder of (200mm) internal diameter and (300mm) outer diameter is subjected to internal pressure of (60 MPa) and external pressure of (30 MPa) . Find the stresses at inside and outside surface of cylinder .

Sol:-

$$\sigma_t = A + \frac{B}{r^2}$$

$$\sigma_r = A - \frac{B}{r^2}$$

at $r=R_1=0.1$ m $\Rightarrow \sigma_r = -p_i = -60\,MPa$

at $r=R_2=0.15$ m $\Rightarrow \sigma_r = -p_o = -30\,MPa$

$$-p = A - \frac{B}{r^2}$$

$$-60 = A - \frac{B}{(0.1)^2} --- (1)$$

$$\pm 30 = « A \pm \frac{B}{(0.15)^2} --- (2)$$

-30 = -55.5 B $\Rightarrow$ B=0.54 $\Rightarrow$ A=-60 +100*0.54

A= -6

$$\therefore \sigma_{t_{r=0.1}} = -6 + \frac{0.54}{(0.1)^2} = 48\ MPa$$ **(Tensile)**

$$\sigma_r = -60\ MPa$$ **Compressive**

$$\therefore \sigma_{t_{r=0.15}} = -6 + \frac{0.54}{(0.15)^2} = 18\ MPa$$ **(Tensile)**

$$\sigma_r = -p_o = -30\ MPa$$ **Compressive**

$$\therefore \sigma_L = \frac{p_i R_1^2 - p_o R_2^2}{R_2^2 - R_1^2} = \frac{60(0.1)^2 - 30(0.15)^2}{(0.15)^2 - (0.1)^2} = -6\,MPa$$ **Compressive**

Ex :- A thick cylinder of (100mm) internal radius and (125mm) outer radius , its subjected to external pressure of (1.4MPa) . Find the stresses on the cylinder .

Sol:-

$$\therefore \sigma_t = -\frac{R_2^2 p_o}{R_2^2 - R_1^2}\left[1 + \frac{R_1^2}{r^2}\right]$$

$$\therefore \sigma_{t_{r=R_1}} = -\frac{R_2^2 p_o}{R_2^2 - R_1^2}\left[1 + \frac{R_1^2}{R_1^2}\right]$$

$$\therefore \sigma_{t_{Max.}} = -2\frac{R_2^2 p_o}{R_2^2 - R_1^2} = -7.8\,MPa$$

$$\therefore \sigma_{r_{Max\ (r=R_1)}} = -\frac{R_2^2 p_o}{R_2^2 - R_1^2}\left[1 - \frac{R_1^2}{r^2}\right] = 0$$

$$\therefore \sigma_L = \frac{p_i R_1^2 - p_o R_2^2}{R_2^2 - R_1^2} = \frac{-p_o R_2^2}{R_2^2 - R_1^2} = \frac{-1.4 * 10^6 * (0.125)^2}{(0.125)^2 - (0.1)^2}$$

$$\therefore \sigma_L = -3.88\,MPa$$

$$\tau_{Max.} = \frac{\sigma_{t_{Max.}} - \sigma_{r_{Max.}}}{2} = \frac{-7.8 - 0}{2} = -3.88\,MPa$$