Al-Mustaqbal University
Department: Medical Instrumentation Techniques Engineering
Class: 1st
Subject: Mechanics
Lecturer: Lec. Hameed Nida Al-Faris
1st term / Lecture: Force Resultants

Engineering Mechanics (Statics)

First Part

FORCE RESULTANTS

المحاضرة 3

Addition of a System of Coplanar Forces جمع نظام قوى مستوية

When a force is resolved into two components along the x and y axes, the components are then called ***rectangular components***. For analytical work we can represent these components in one of two ways, using either scalar or Cartesian vector notation.

عند تحليل قوة الى مركبتين باتجاه المحاور x و y فان المركبتين حينها تسمى مركبات المستطيل ولغرض التحليل يمكن تمثيل المركبتين باحدى طريقتين هما بيان المتجه الكمي او الكارتيزي:

Scalar Notation. The rectangular components of force **F** shown in Fig. 2–15*a* are found using the parallelogram law, so that $\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$. Because these components form a right triangle, they can be determined from

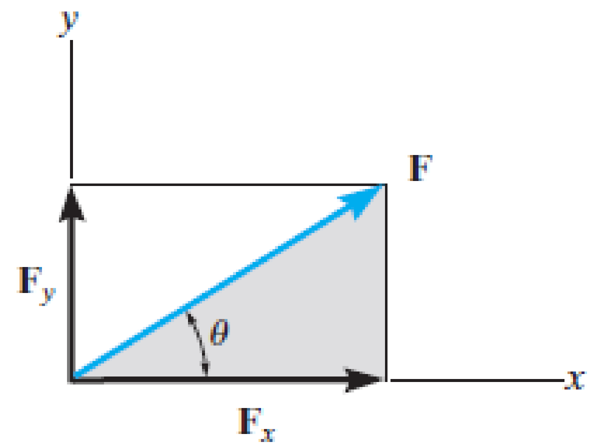
$$F_x = F\cos\theta \qquad \text{and} \qquad F_y = F\sin\theta$$

البيان الكمي : مركبات المستطيل للقوة F الموضحة في الشكل (2-15a) يتم ايجادها باستخدام قانون متوازي المستطيلات لذلك فان :

$$F = F_x + F_y$$

لان هذه المركبات تشكل المثلث الايمن ويمكن حسابهما من :

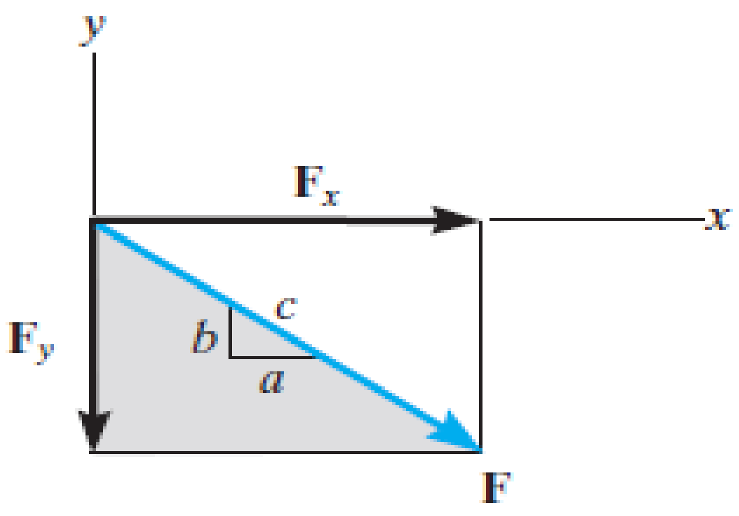
$$F_x = F\cos\theta \quad , \quad F_y = F\sin\theta$$



(a)

Instead of using the angle θ, however, the direction of **F** can also be defined using a small "slope" triangle, as in the example shown in Fig. 2–15*b*. Since this triangle and the larger shaded triangle are similar, the proportional length of the sides gives

وبدلا من استخدام الزاوية θ فانه يمكن تحديد اتجاه القوة F باستخدام مثلث الميل الصغير كما في الشكل (b)
وبما ان هذا المثلث والمثلث المظلل الاكبر متشابهان فان تناسب الاطوال يكون :



(b)

$$\frac{F_x}{F} = \frac{a}{c}$$

or

$$F_x = F\left(\frac{a}{c}\right)$$

$$\frac{F_y}{F} = \frac{b}{c}$$

or

$$F_y = -F\left(\frac{b}{c}\right)$$

Here the y component is a *negative scalar* since $\mathbf{F}_y$ is directed along the negative y axis.

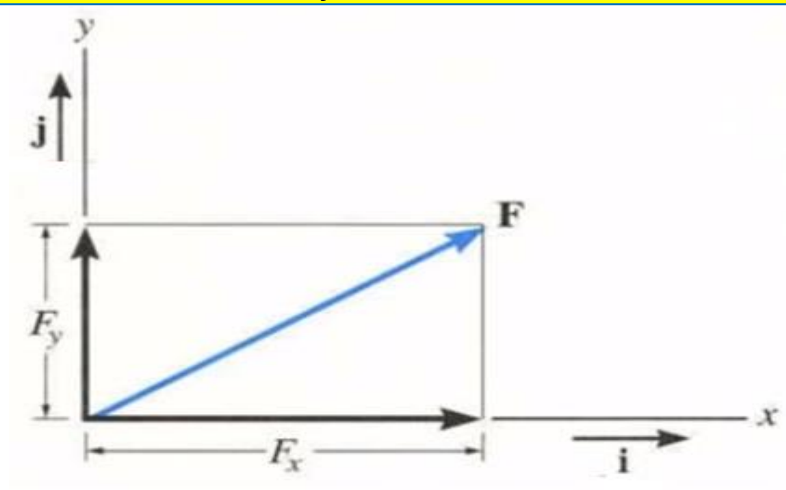
هنا المركبة y كمية سالبة لان اتجاه F_y باتجاه محور y السالب

Cartesian Vector Notation. It is also possible to represent the x and y components of a force in terms of Cartesian unit vectors **i** and **j**. They are called unit vectors because they have a dimensionless magnitude of 1, and so they can be used to designate the *directions* of the x and y axes, respectively, Fig. 2–16.*

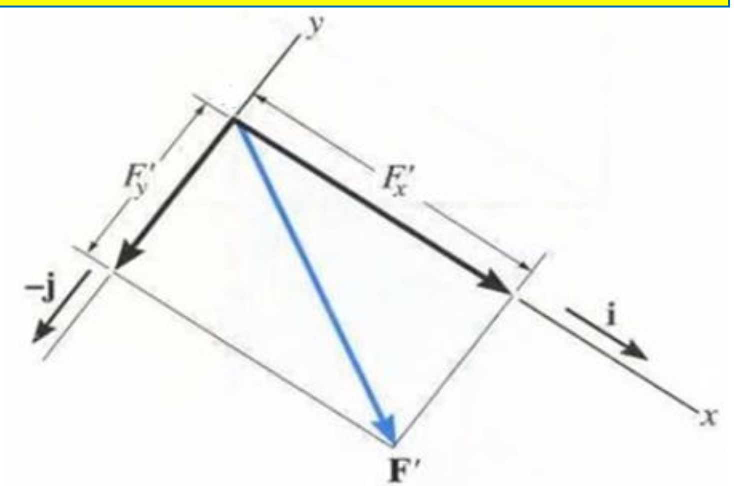
Since the *magnitude* of each component of **F** is *always a positive quantity*, which is represented by the (positive) scalars F_x and F_y, then we can express **F** as a *Cartesian vector*,

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$

بيان المتجه الكارتيزي: كما يمكن تمثيل المركبات x و y للقوة بتعبير متجهات الوحدة الكارتيزية i و j وقد سميت كذلك لان لها قيمة 1 بدون وحدات ، وتكون F كمتجه كارتيزي: $F = F_x\,i + F_y\,j$



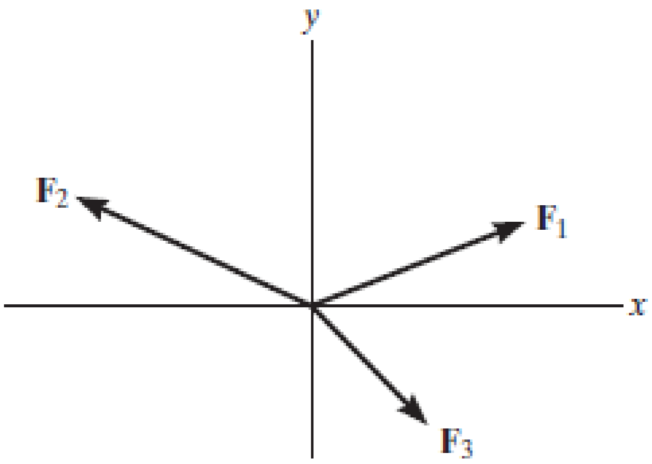
$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$



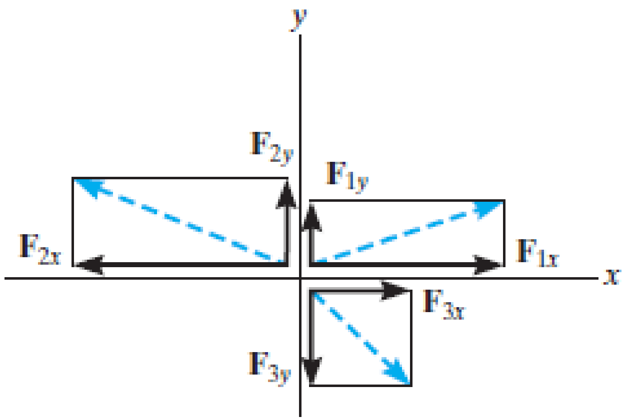
$$\mathbf{F}' = F'_x\mathbf{i} + F'_y(-\mathbf{j})$$

$$\mathbf{F}' = F'_x\mathbf{i} - F'_y\mathbf{j}$$

► Coplanar Force Resultants.   محصلة قوى مستوية ◄



(a)



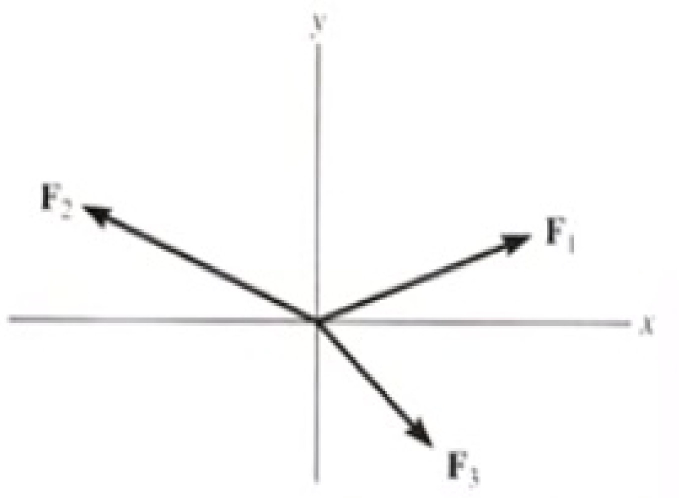
(b)

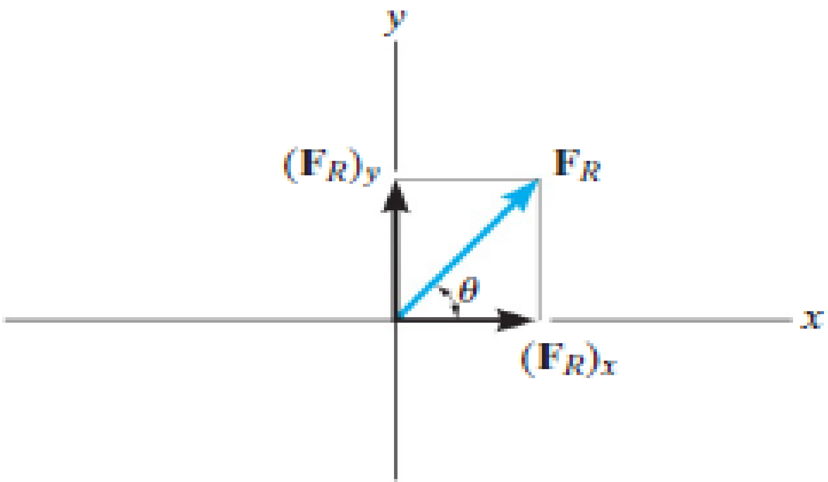
$$\mathbf{F}_1 = F_{1x}\,\mathbf{i} + F_{1y}\,\mathbf{j}$$
$$\mathbf{F}_2 = -F_{2x}\,\mathbf{i} + F_{2y}\,\mathbf{j}$$
$$\mathbf{F}_3 = F_{3x}\,\mathbf{i} - F_{3y}\,\mathbf{j}$$

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3\\ &= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j}\\ &= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j}\\ &= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}\end{aligned}$$

$$(F_R)_x = F_{1x} - F_{2x} + F_{3x}$$
$$(F_R)_y = F_{1y} + F_{2y} - F_{3y}$$

$$\begin{aligned}(\overset{+}{\rightarrow})\quad & \boxed{(F_R)_x = \Sigma F_x}\\ (+\uparrow)\quad & \boxed{(F_R)_y = \Sigma F_y}\end{aligned}$$





(c)

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$\theta = \tan^{-1}\left|\frac{(F_R)_y}{(F_R)_x}\right|$$

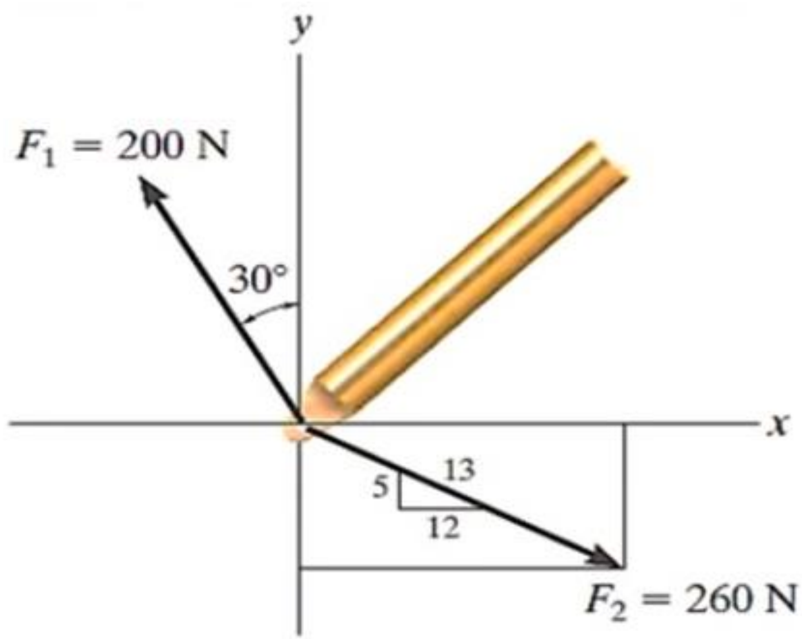
Examples أمثلة

Example (1):

Determine the X and y components of F1 and F2 acting on the boom shown. Express each force as Cartesian vector.

مثال(1):

احسب مركبات القوتين (F2, F1) اللتين تؤثران على الذراع المبين ، عبر عن كل قوة كمتجه كارتيزي



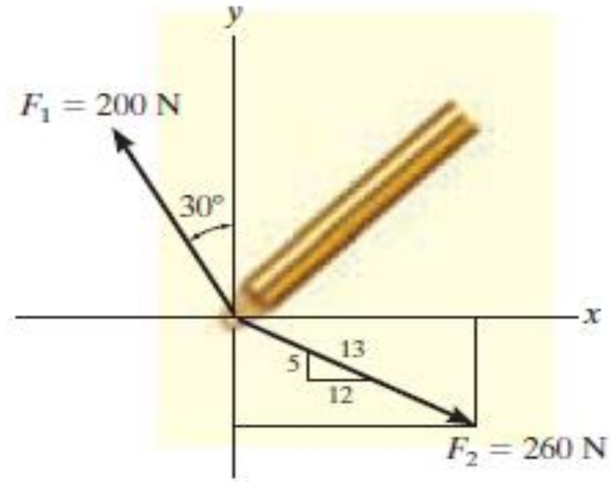
Scalar Notation.

$$F_{1x} = -200 \sin 30° \text{ N} = -100 \text{ N} = 100 \text{ N} \leftarrow$$

$$F_{1y} = 200 \cos 30° \text{ N} = 173 \text{ N} = 173 \text{ N} \uparrow$$

$$\frac{F_{2x}}{260 \text{ N}} = \frac{12}{13} \qquad F_{2x} = 260 \text{ N}\left(\frac{12}{13}\right) = 240 \text{ N} \rightarrow$$

Similarly, $$F_{2y} = 260 \text{ N}\left(\frac{5}{13}\right) = 100 \text{ N} \downarrow$$

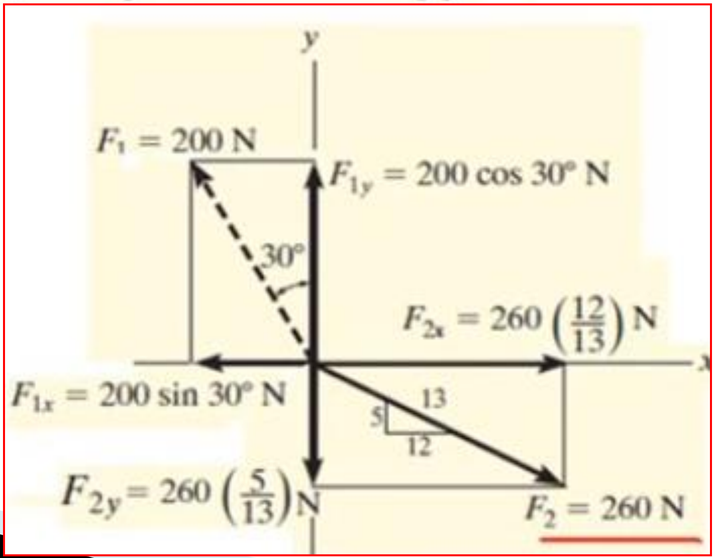


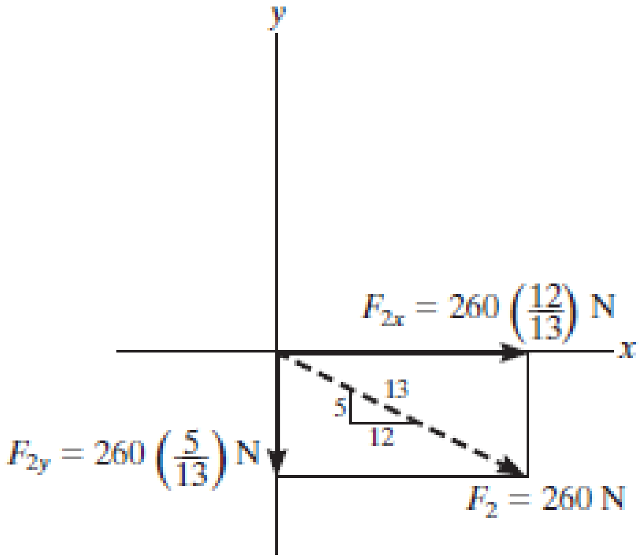
(a)

Cartesian Vector Notation.

$$\mathbf{F}_1 = \{-100\mathbf{i} + 173\mathbf{j}\} \text{ N}$$

$$\mathbf{F}_2 = \{240\mathbf{i} - 100\mathbf{j}\} \text{ N}$$





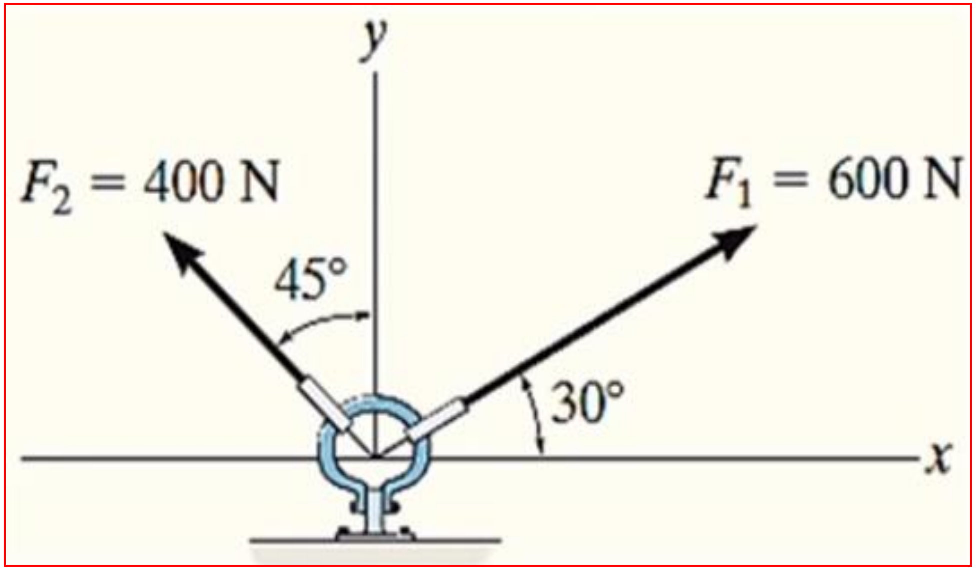
(c)

Example (2):

The shown screw eye in Figure is subjected to two Forces, F1 and F2. Determine the magnitude and direction of the resultant Force.

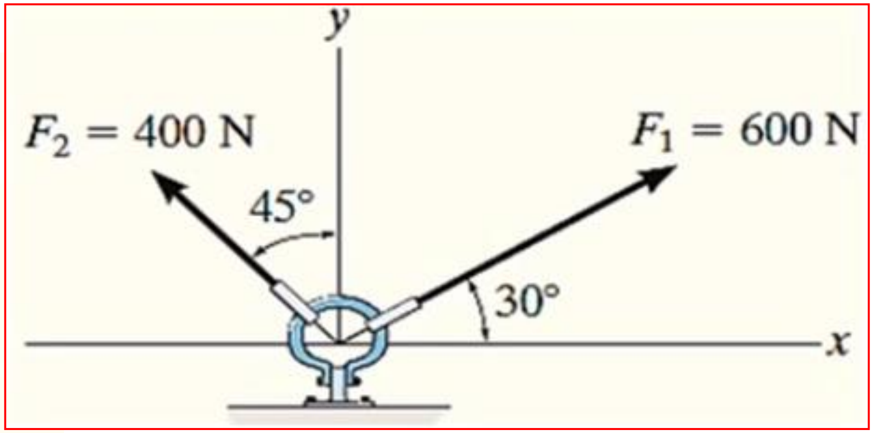
مثال(2):

احسب محصلة القوتان (F2,F1) اللتان توثران على رأس المسمار المبين بالشكل .



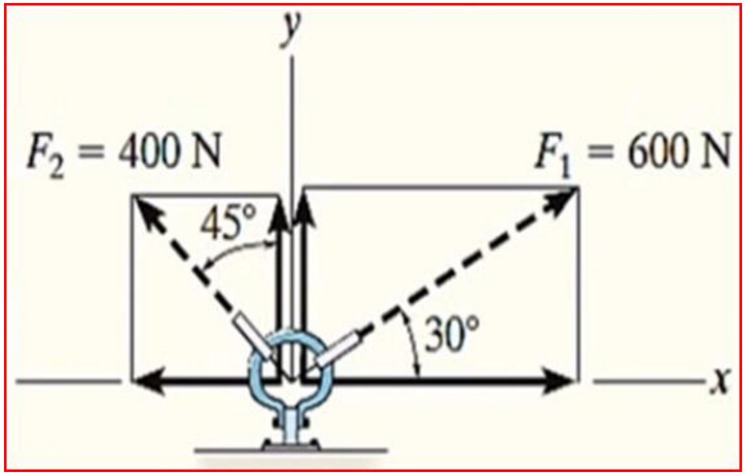
SOLUTION I

Scalar Notation.



$$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x$$

$$F_{Rx} = 600 \cos 30° \text{ N} - 400 \sin 45° \text{ N} = 236.8 \text{ N} \rightarrow$$

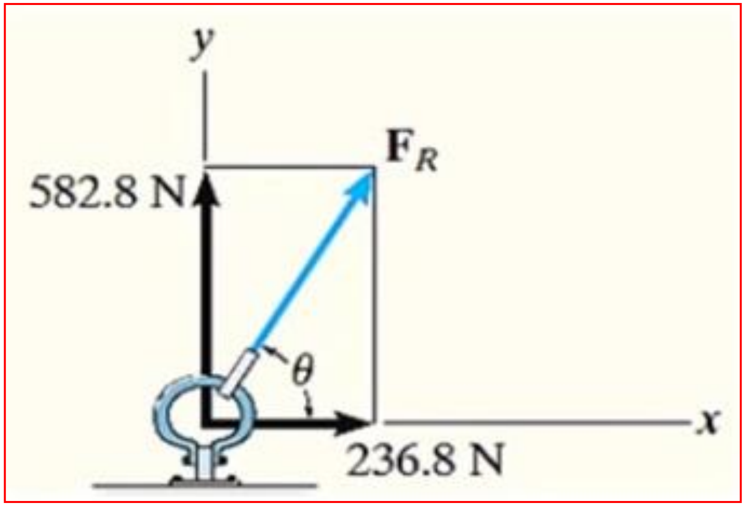


$$+\uparrow F_{Ry} = \Sigma F_y$$

$$F_{Ry} = 600 \sin 30° \text{ N} + 400 \cos 45° \text{ N} = 582.8 \text{ N} \uparrow$$

$$F_R = \sqrt{(236.8\ \text{N})^2 + (582.8\ \text{N})^2} = 629\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{582.8\ \text{N}}{236.8\ \text{N}}\right) = 67.9^\circ$$



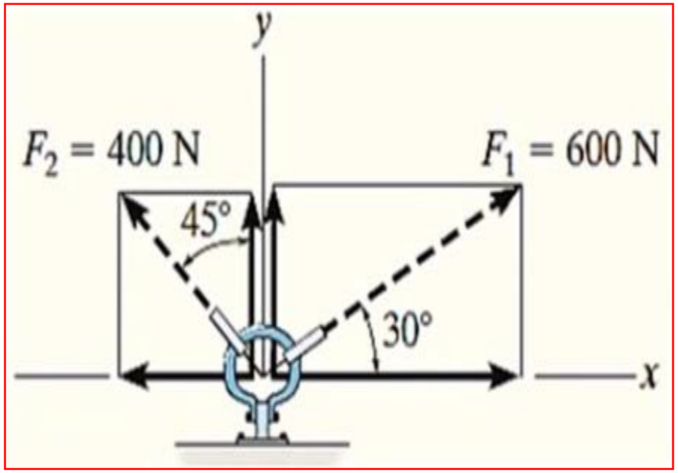
SOLUTION II

Cartesian Vector Notation.

$$\mathbf{F}_1 = \{600 \cos 30°\mathbf{i} + 600 \sin 30°\mathbf{j}\} \text{ N}$$

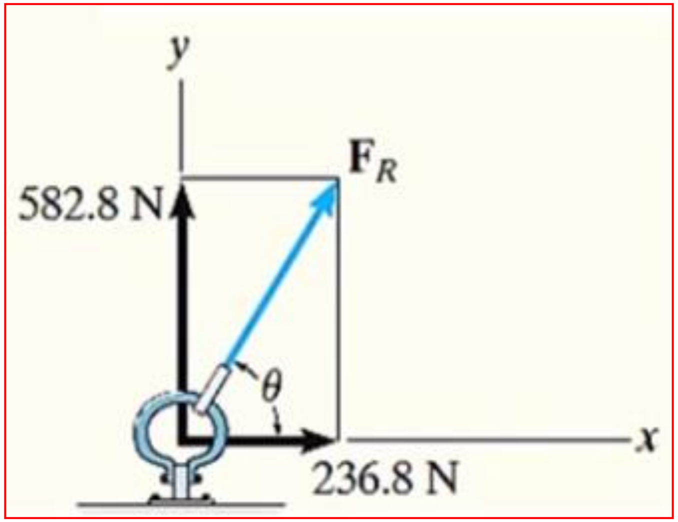
$$\mathbf{F}_2 = \{-400 \sin 45°\mathbf{i} + 400 \cos 45°\mathbf{j}\} \text{ N}$$

$$\begin{aligned} \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 &= (600 \cos 30° \text{ N} - 400 \sin 45° \text{ N})\mathbf{i} \\ &\quad + (600 \sin 30° \text{ N} + 400 \cos 45° \text{ N})\mathbf{j} \\ &= \{236.8\mathbf{i} + 582.8\mathbf{j}\} \text{ N} \end{aligned}$$



$$F_R = \sqrt{(236.8 \text{ N})^2 + (582.8 \text{ N})^2} = 629 \text{ N}$$

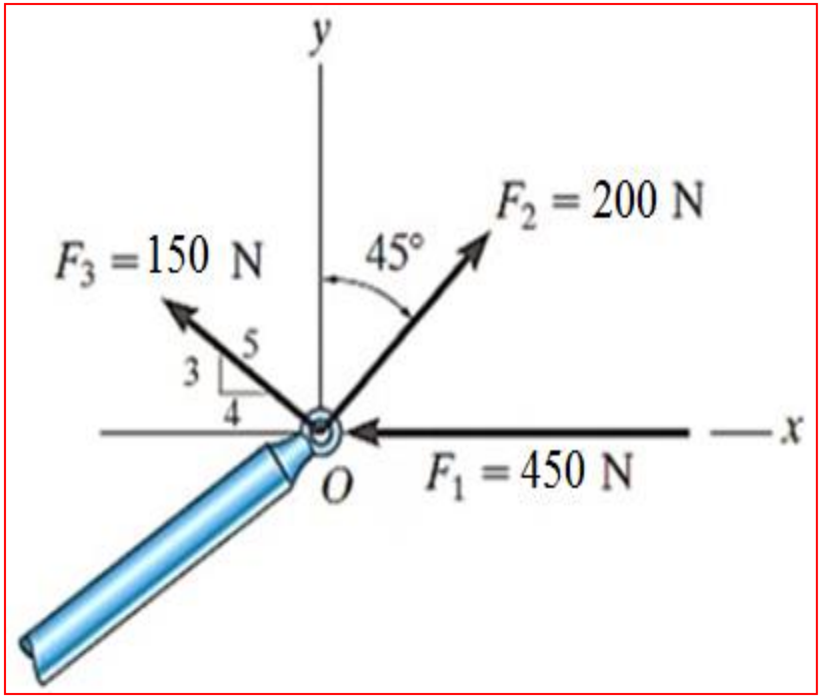
$$\theta = \tan^{-1}\left(\frac{582.8 \text{ N}}{236.8 \text{ N}}\right) = 67.9°$$



Home Work الواجب البيتي

The end of the shown boom O is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.

نقطة نهاية ذراع التطويل المبين بالشكل (O) معرض لثلاث قوى متلاقية ومستوية . احسب قيمة واتجاه المحصلة .



- Right-Handed Coordinate System ◄ قاعدة اليد اليمنى لنظام الاحداثيات

Rectangular components of a Vector *المركبات المستطيلة للمتجه*

- Cartesian Unit Vectors. ◄ متجه الوحدة الكارتيزي

Magnitude of a Cartesian Vector. قيمة المتجه الكارتيزي

Direction of a Cartesian Vector. اتجاه المتجه الكارتيزي

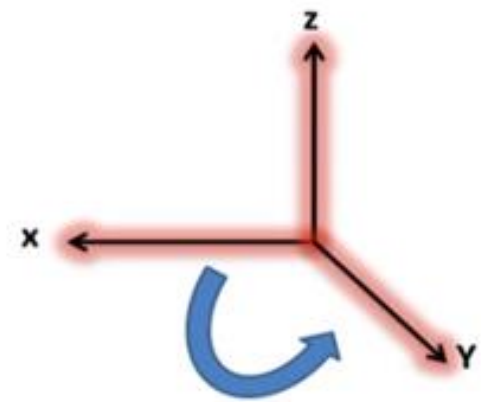
- Unit Vector. ◄ متجه الوحدة

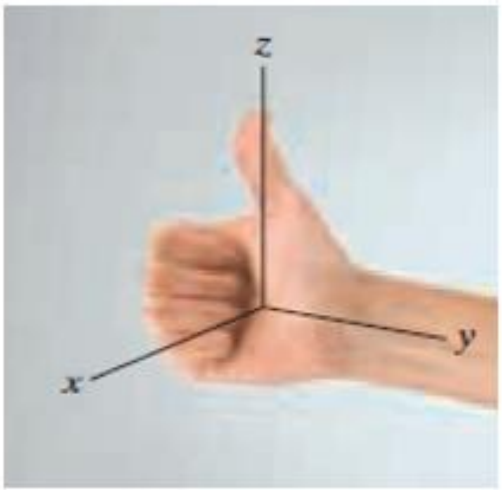
- Addition and subtraction of Cartesian Vectors ◄ جمع وطرح المتجهات الكارتيزية

► Right-Handed Coordinate System ◄ قاعدة اليد اليمنى لنظام الإحداثيات

Right-Handed Coordinate System. We will use a right-handed coordinate system to develop the theory of vector algebra that follows. A rectangular coordinate system is said to be ***right-handed*** if the thumb of the right hand points in the direction of the positive z axis when the right-hand fingers are curled about this axis and directed from the positive x towards the positive y axis, Fig. 2–21.

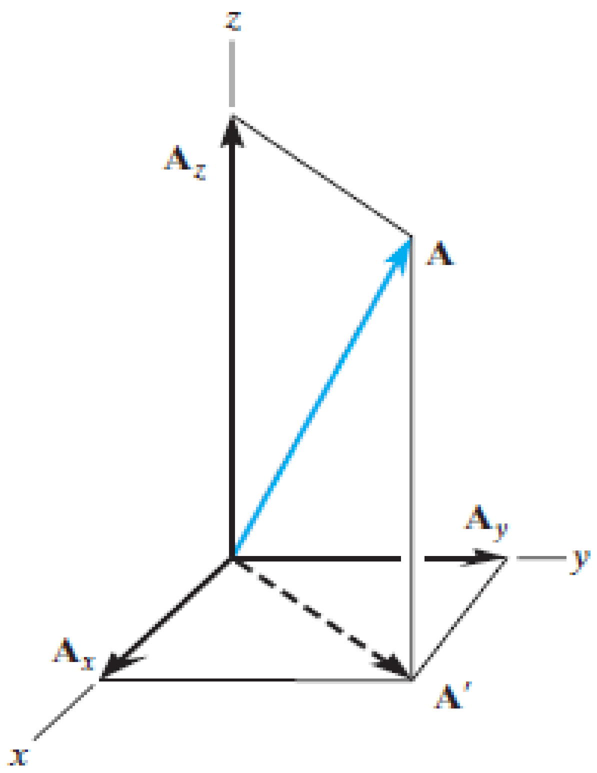
تستخدم قاعدة اليد اليمنى لانشاء نظرية جبر المتجهات التالية : يقال عن نظام احداثيات المستطيل انه يد يمنى اذا كان ابهام اليد اليمنى يشير الى الجانب الموجب من محور z عندما تكون اصابع اليد اليمنى ملتفة حول ذلك المحوروتشير من x الموجب باتجاه y الموجب





Right-handed coordinate system

► Rectangular Components of a Vector ◄ المركبات المستطيلة للمتجه

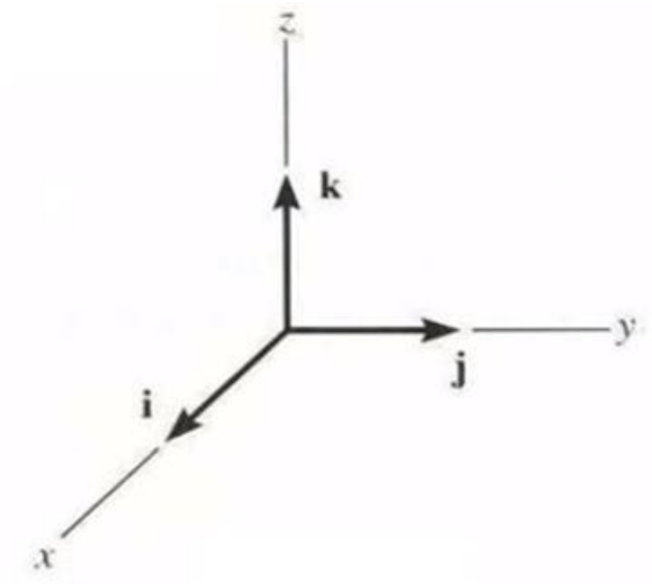


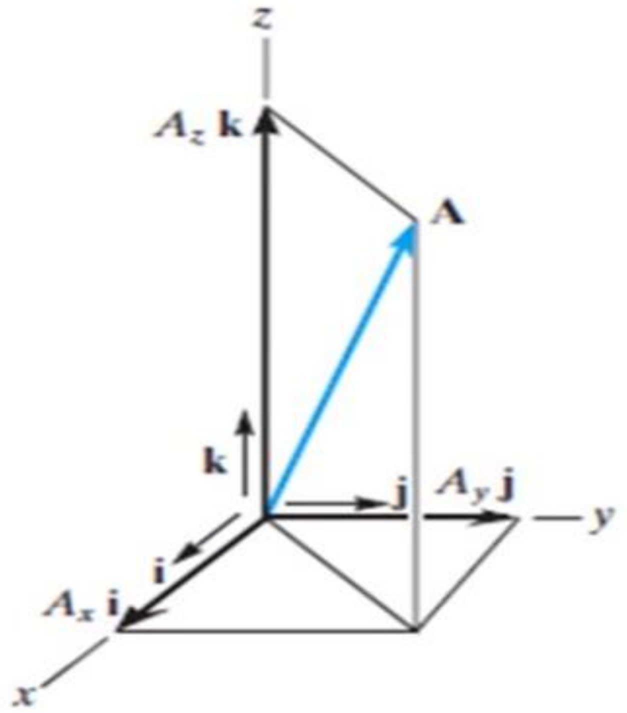
$$\mathbf{A} = \mathbf{A}' + \mathbf{A}_z$$

$$\mathbf{A}' = \mathbf{A}_x + \mathbf{A}_y$$

$$\mathbf{A} = \mathbf{A}_x + \mathbf{A}_y + \mathbf{A}_z$$

► **Cartesian Unit Vectors.** ◄ متجه الوحدة الكارتيزي





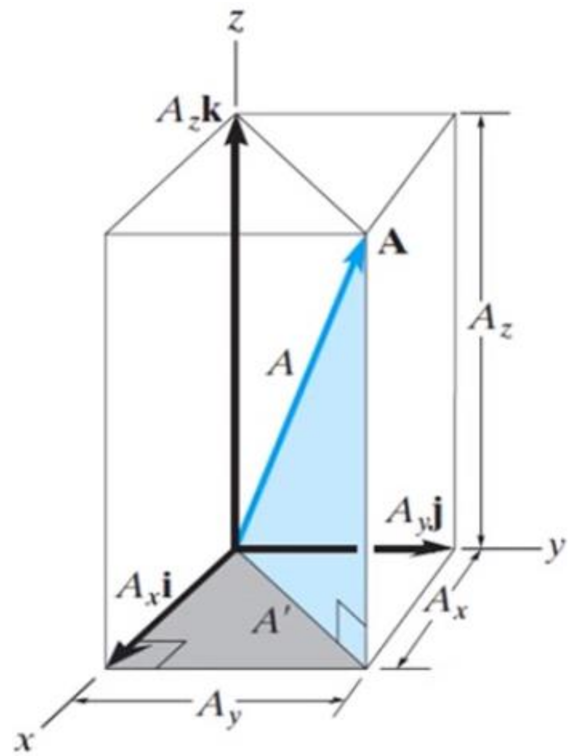
$$\boxed{\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}}$$

► Magnitude of a Cartesian Vector. ◄ قيمة المتجه الكارتيزي

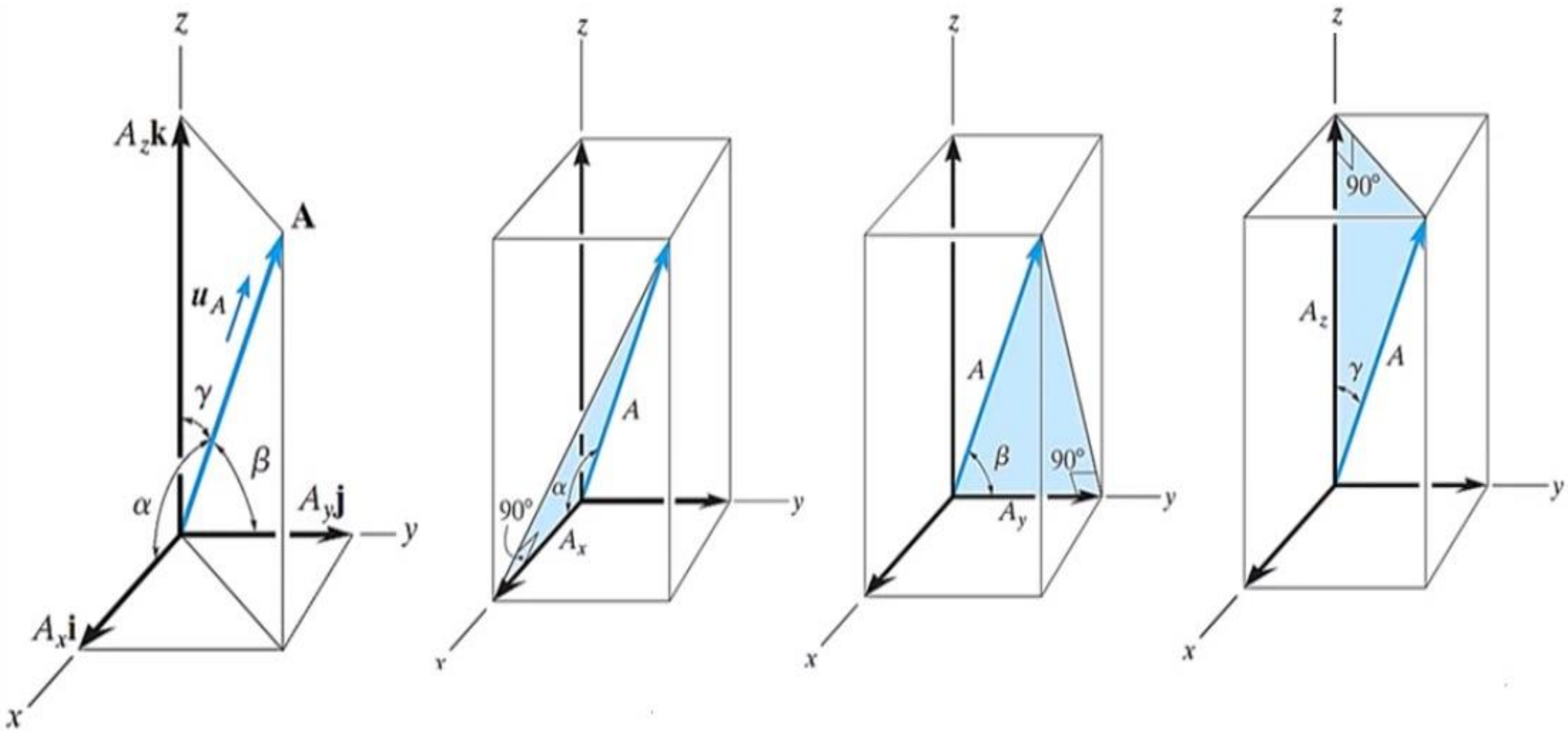
$$A = \sqrt{A'^2 + A_z^2}$$

$$A' = \sqrt{A_x^2 + A_y^2}$$

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$



► Coordinate Direction Angles احداثيات زوايا الاتجاه



$$\cos\alpha = \frac{A_x}{A} \quad \cos\beta = \frac{A_y}{A} \quad \cos\gamma = \frac{A_z}{A}$$

المحاضرة 3

► Unit Vector. ◄ متجه الوحدة

$$\boxed{\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}}$$

$$\mathbf{u}_A = \cos\alpha\,\mathbf{i} + \cos\beta\,\mathbf{j} + \cos\gamma\,\mathbf{k}$$

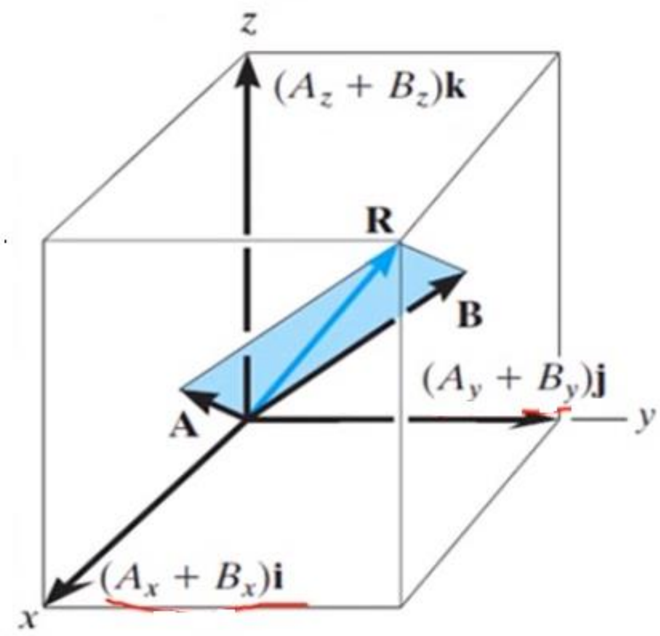
$$\boxed{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1}$$

$$\mathbf{A} = A\mathbf{u}_A$$

$$= A\cos\alpha\,\mathbf{i} + A\cos\beta\,\mathbf{j} + A\cos\gamma\,\mathbf{k}$$

$$\boxed{= A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}}$$

► Addition and subtraction of Cartesian Vectors ◄ جمع وطرح المتجهات الكارتيزية



$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k}$$

$$\mathbf{R}' = \mathbf{A} - \mathbf{B} = (A_x - B_x)\mathbf{i} + (A_y - B_y)\mathbf{j} + (A_z - B_z)\mathbf{k}$$

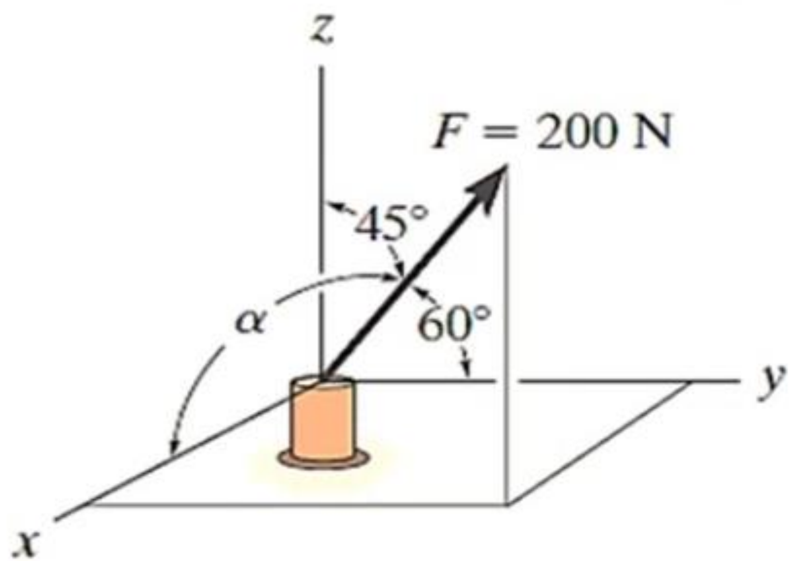
For several concurrent forces ⟹ $\boxed{\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}}$

Example (4):

Express the force F shown in the Figure as a Cartesian vector.

مثال(4):

عبر عن القوة F المبينة بالشكل كمتجه كارتيزي.



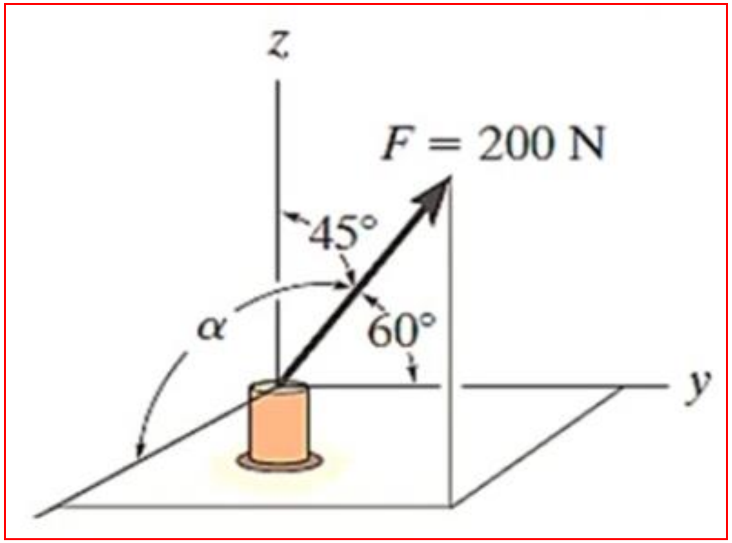
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\alpha + \cos^2 60^\circ + \cos^2 45^\circ = 1$$

$$\cos\alpha = \sqrt{1 - (0.707)^2 - (0.5)^2} = \pm 0.5$$

$$\alpha = \cos^{-1}(0.5) = 60^\circ$$

or $$\alpha = \cos^{-1}(-0.5) = 120^\circ$$



$$\mathbf{F} = F\cos\alpha\,\mathbf{i} + F\cos\beta\,\mathbf{j} + F\cos\gamma\,\mathbf{k}$$

$$= 200\cos 60^\circ\,\text{N}\,\mathbf{i} + 200\cos 60^\circ\,\text{N}\,\mathbf{j} + 200\cos 45^\circ\,\text{N}\,\mathbf{k}$$

$$= \{100.0\mathbf{i} + 100.0\mathbf{j} + 141.4\mathbf{k}\}\ \text{N}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

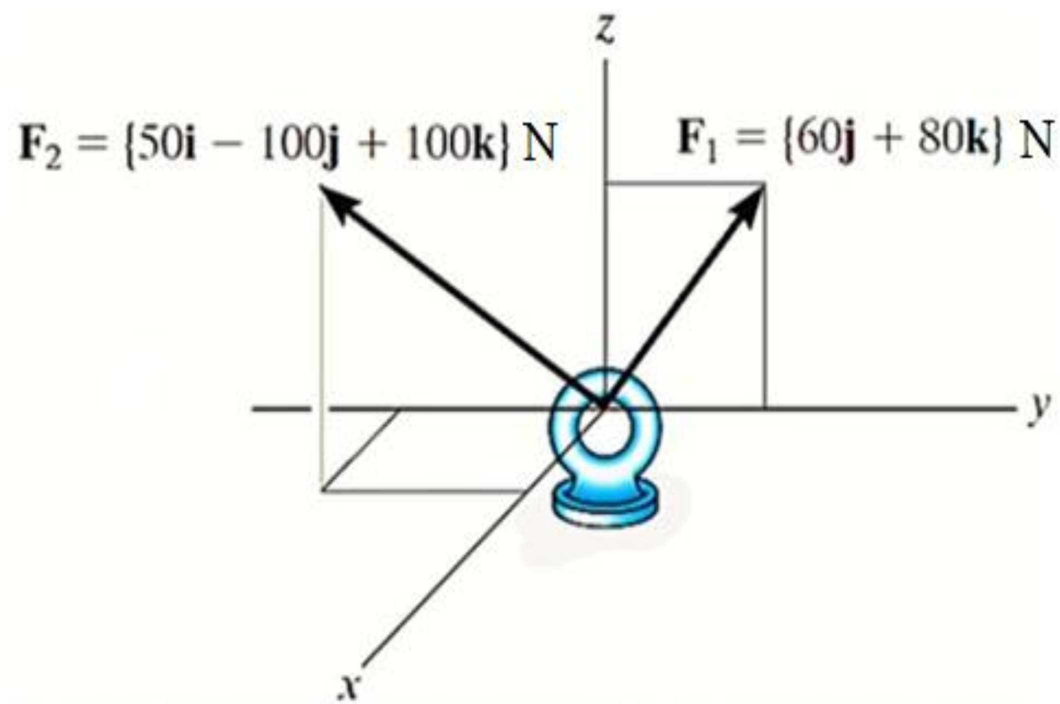
$$= \sqrt{(100.0)^2 + (100.0)^2 + (141.4)^2} = 200\ \text{N}$$

Example (5):

Determine the magnitude and the coordinate direction angles of the resultant force acting on the ring In the Figure

مثال(5):

احسب قيمة واتجاه محصلة القوتين اللتين توثران على الحلقة المبينة بالشكل .

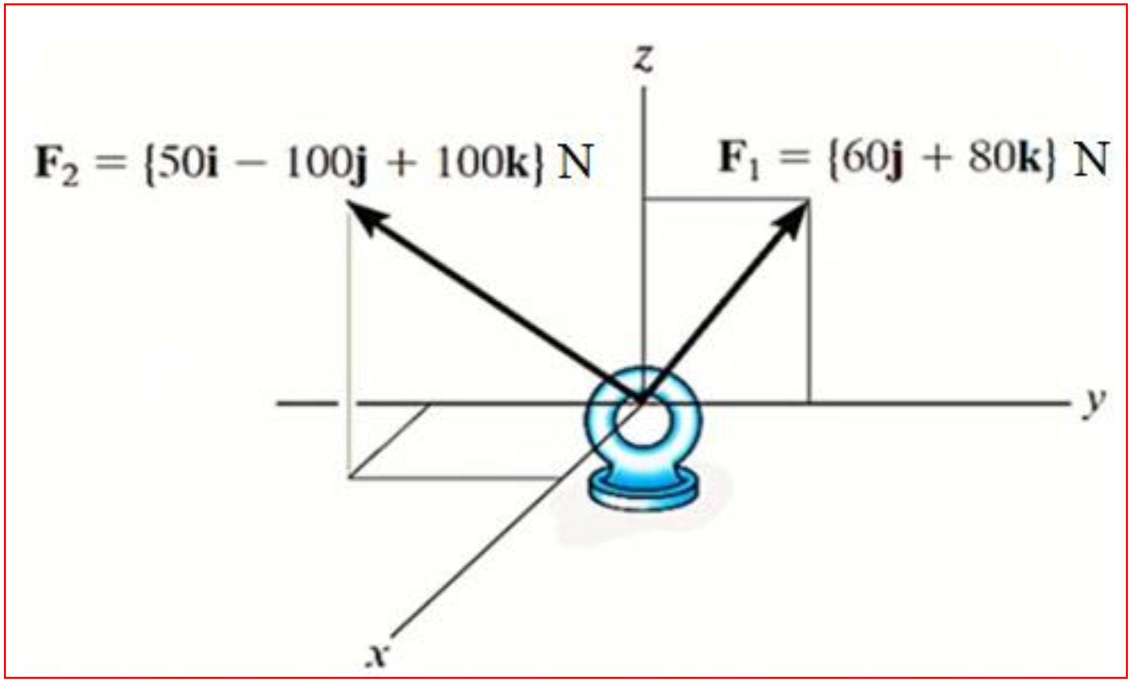


$$\mathbf{F}_R = \Sigma\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \{60\mathbf{j} + 80\mathbf{k}\}\,\text{lb} + \{50\mathbf{i} - 100\mathbf{j} + 100\mathbf{k}\}\,\text{N}$$

$$= \{50\mathbf{i} - 40\mathbf{j} + 180\mathbf{k}\}\,\text{N}$$

$$F_R = \sqrt{(50)^2 + (-40)^2 + (180)^2}$$

$$= 191.0\ \text{N}$$



$\mathbf{F_R} = 50\mathbf{i} - 40\mathbf{j} + 180\mathbf{k}$

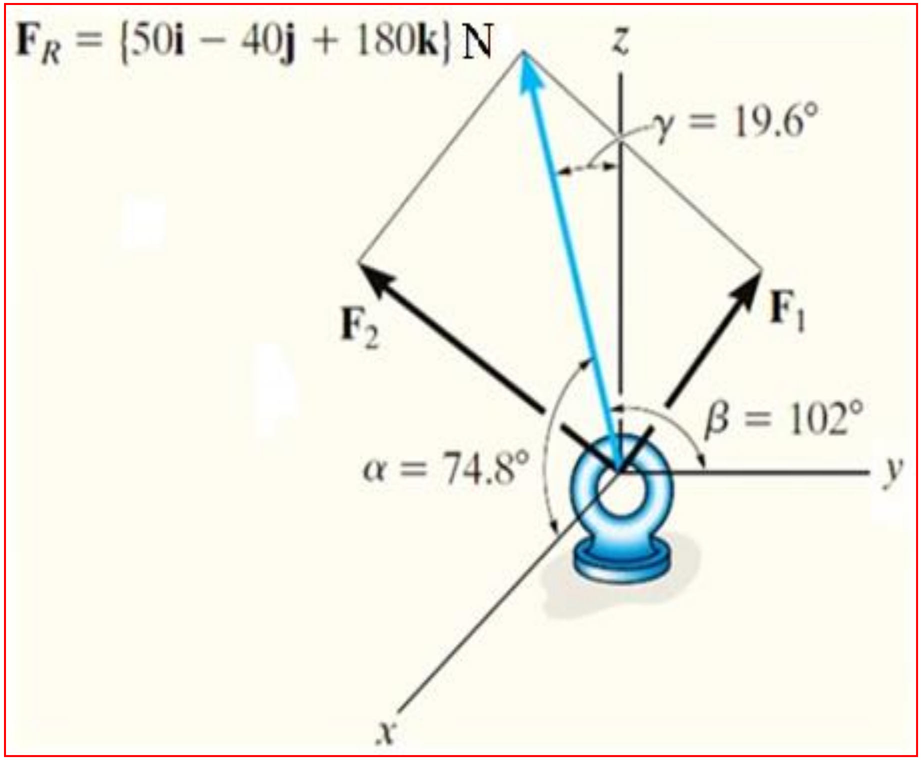
$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{50}{191.0}\mathbf{i} - \frac{40}{191.0}\mathbf{j} + \frac{180}{191.0}\mathbf{k}$$

$$= 0.2617\mathbf{i} - 0.2094\mathbf{j} + 0.9422\mathbf{k}$$

$\cos\alpha = 0.2617$ $\quad\quad$ $\alpha = 74.8^\circ$

$\cos\beta = -0.2094$ $\quad\quad$ $\beta = 102^\circ$

$\cos\gamma = 0.9422$ $\quad\quad$ $\gamma = 19.6^\circ$

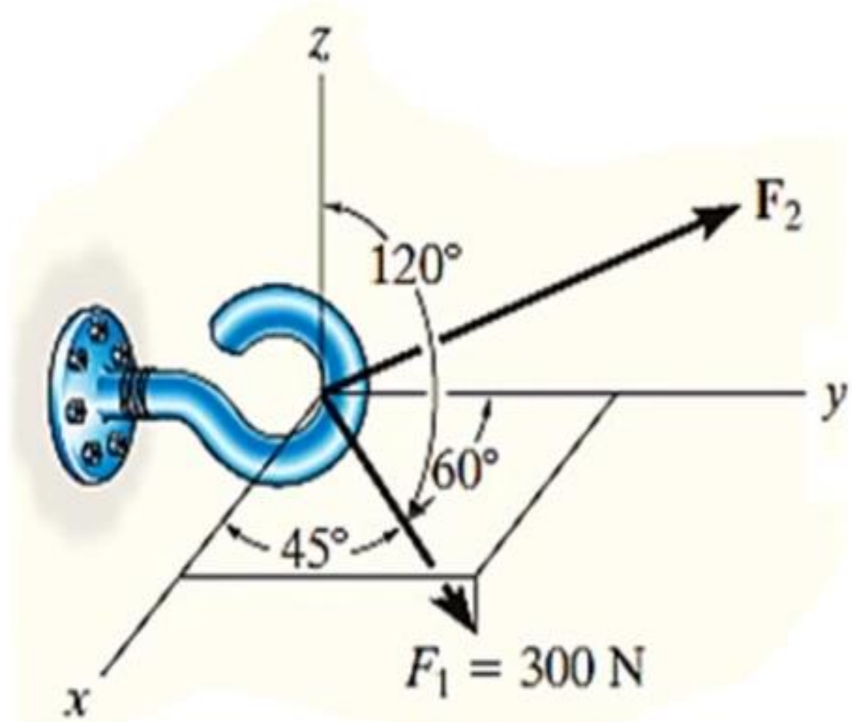


Example (6):

Two forces act on the hook shown in the Figure. Specify the magnitude of F2 and Its coordinate direction angles of F2 that the resultant force FR acts along the positive y axis and has a magnitude of 800 N.

مثال (6):

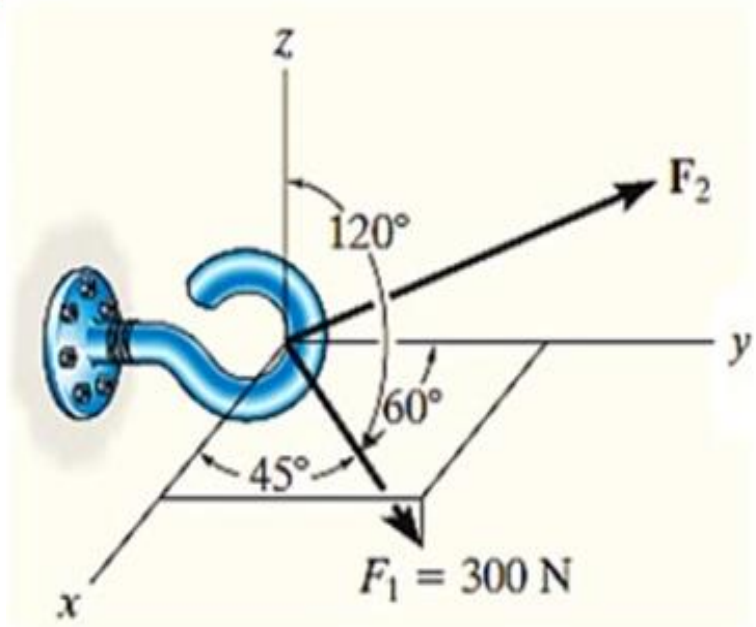
توجد قوتان توثران على الخطاف المبين بالشكل احسب قيمة واتجاه القوة (F2) التي تجعل المحصلة تؤثر في اتجاه محور (Y) الموجب وقيمتها 800 نيوتن .



$$\mathbf{F}_1 = F_1\mathbf{u}_{F_1} = F_1 \cos\alpha_1\mathbf{i} + F_1\cos\beta_1\mathbf{j} + F_1\cos\gamma_1\mathbf{k}$$
$$= 300\cos 45^\circ\,\mathbf{i} + 300\cos 60^\circ\,\mathbf{j} + 300\cos 120^\circ\,\mathbf{k}$$
$$= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\}\ \text{N}$$

$$\mathbf{F}_2 = F_2\mathbf{u}_{F_2} = F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$

$$\mathbf{F}_R = (800\ \text{N})(+\mathbf{j}) = \{800\mathbf{j}\}\ \text{N}$$



$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$800\mathbf{j} = 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$
$$800\mathbf{j} = (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}$$

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

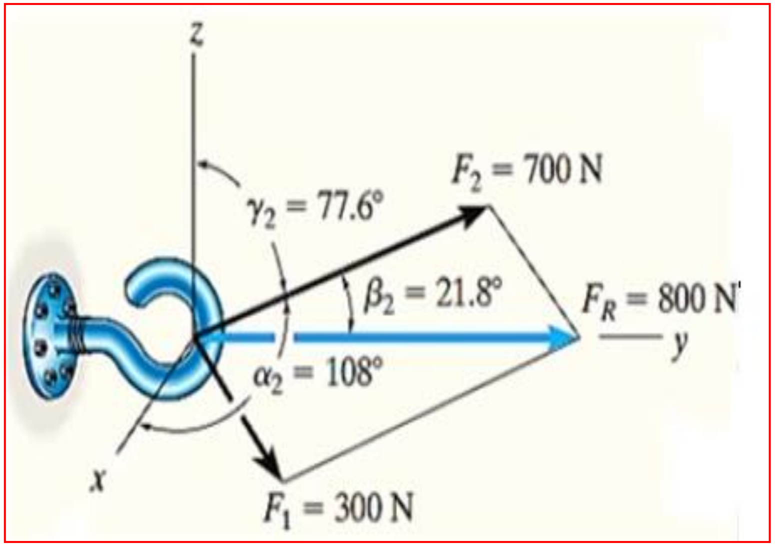
$$800\mathbf{j} = 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}$$

$$800\mathbf{j} = (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}$$

$$0 = 212.1 + F_{2x} \qquad F_{2x} = -212.1 \text{ N}$$

$$800 = 150 + F_{2y} \qquad F_{2y} = 650 \text{ N}$$

$$0 = -150 + F_{2z} \qquad F_{2z} = 150 \text{ N}$$



$$-212.1 = 700\cos\alpha_2 \quad \alpha_2 = \cos^{-1}\left(\frac{-212.1}{700}\right) = 108°$$

$$650 = 700\cos\beta_2 \quad \beta_2 = \cos^{-1}\left(\frac{650}{700}\right) = 21.8°$$

$$150 = 700\cos\gamma_2; \quad \gamma_2 = \cos^{-1}\left(\frac{150}{700}\right) = 77.6°$$

Home work

الواجب

Express the force F shown in the Figure as a Cartesian vector.

عبر عن القوة المبينة بالشكل كمتجه كارتيزي.