Ministry of Higher Education and Scientific Research
Almustaqbal University, College of Engineering
And Engineering Technologies
Computer Technology Engineering Department

Three week :

Series and Parallel AC Circuits

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Stage : One
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Module Code : UOMU022012

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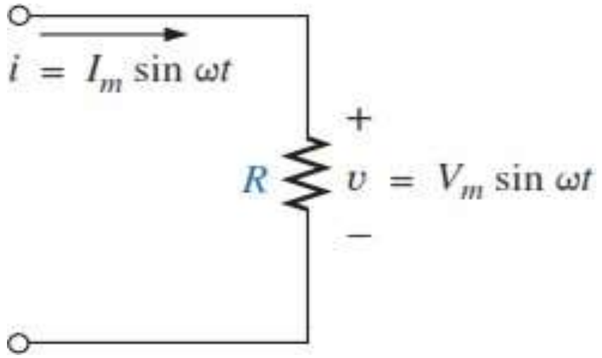
SERIES AND PARALLEL AC CIRCUITS

15.2 IMPEDANCE AND THE PHASOR DIAGRAM

Resistive Elements



For the purely resistive circuit, v and *i are in-phase,* and the magnitude

$$I_m = \frac{V_m}{R} \quad \text{or} \quad V_m = I_m R$$

In phasor form,

$$v = V_m \sin \omega t \Rightarrow \mathbf{V} = V \angle 0^\circ$$

where $V = 0.707V_m$.

Applying Ohm's law and using phasor algebra, we have

$$\mathbf{I} = \frac{V \angle 0^\circ}{R \angle \theta_R} = \frac{V}{R} \angle 0^\circ - \theta_R$$

Since *i and v are in-phase, the angle associated with i also must be 0°. To* satisfy this condition, θ_R must equal 0°.

$$\mathbf{I} = \frac{V \angle 0^\circ}{R \angle 0^\circ} = \frac{V}{R} \angle 0^\circ - 0^\circ = \frac{V}{R} \angle 0^\circ$$

so that in the time domain

$$i = \sqrt{2}\left(\frac{V}{R}\right) \sin \omega t$$

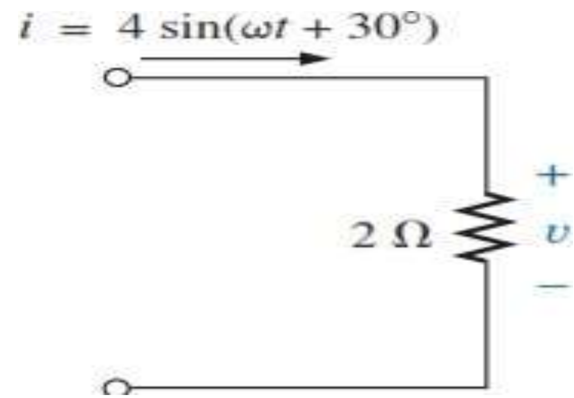
$$\mathbf{Z}_R = R \angle 0^\circ$$

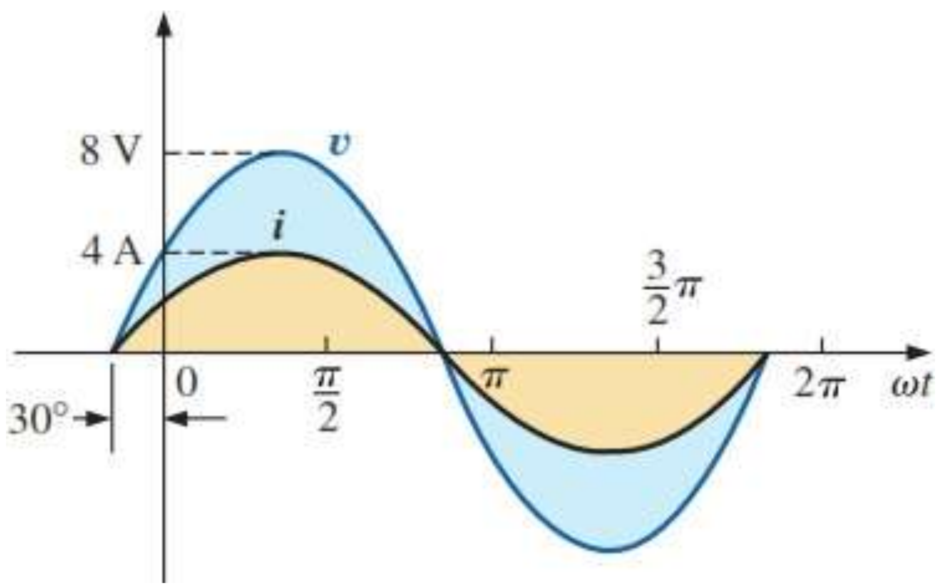
EXAMPLE 15.2 Using complex algebra, find the voltage v for the circuit in Fig. Sketch the waveforms of v and i.

$$i = 4\sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 2.828 \text{ A} \angle 30°$$

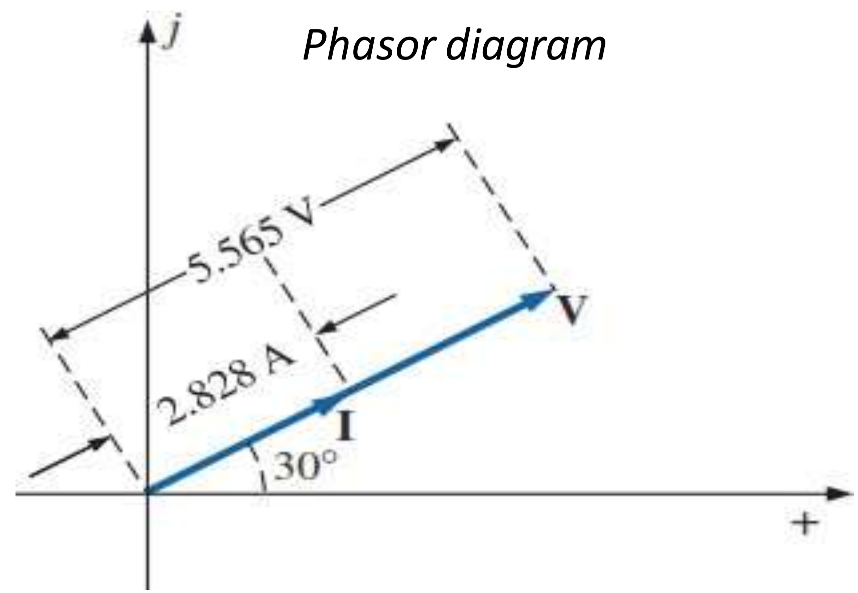
$$\mathbf{V} = \mathbf{I}\mathbf{Z}_R = (I\angle\theta)(R\angle 0°) = (2.828 \text{ A}\angle 30°)(2\ \Omega\angle 0°)$$
$$= 5.656 \text{ V}\angle 30°$$

$$v = \sqrt{2}\,(5.656)\sin(\omega t + 30°) = \mathbf{8.0\sin(\omega t + 30°)}$$



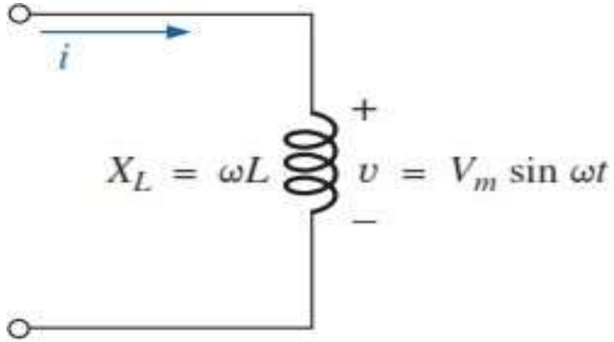


Phasor diagram



Inductive Reactance

For the pure inductor, the voltage leads the current by 90° and that the reactance of the coil X is determined by ωL.



$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0^\circ$$

By Ohm's law, $\mathbf{I} = \dfrac{V \angle 0^\circ}{X_L \angle \theta_L} = \dfrac{V}{X_L} \angle 0^\circ - \theta_L$

Since v leads i by 90°, i must have an angle of -90° associated with it. To satisfy this condition, θ_L must equal +90°. Substituting θ_L = 90° we obtain

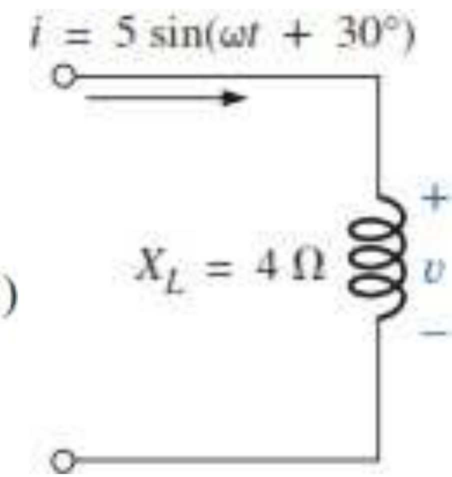
$$\mathbf{I} = \frac{V \angle 0^\circ}{X_L \angle 90^\circ} = \frac{V}{X_L} \angle 0^\circ - 90^\circ = \frac{V}{X_L} \angle -90^\circ$$

so that in the time domain

$$i = \sqrt{2}\left(\frac{V}{X_L}\right) \sin(\omega t - 90^\circ)$$

$$\mathbf{Z}_L = X_L \angle 90^\circ$$

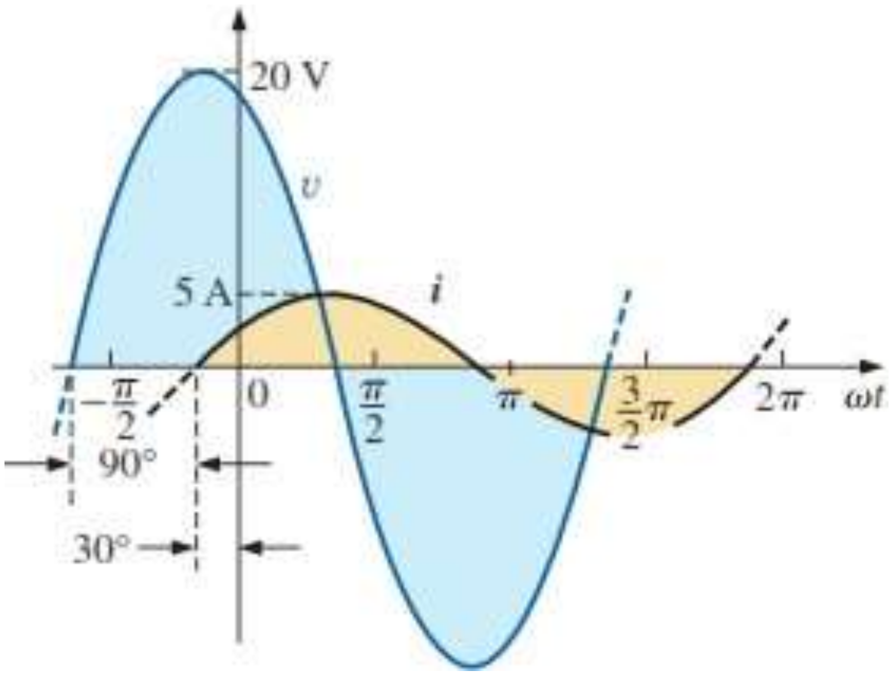
EXAMPLE 15.4 Using complex algebra, find the voltage v for the circuit in Fig. Sketch the v and i curves.



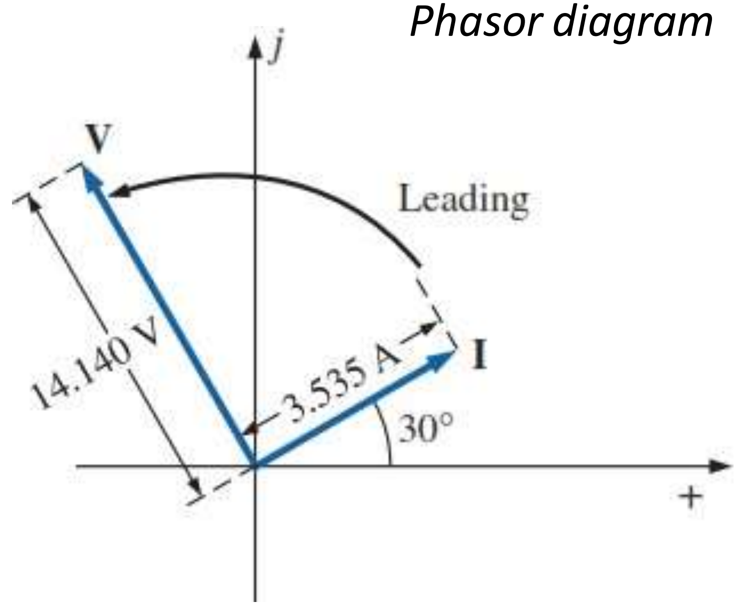
$$i = 5 \sin(\omega t + 30°) \Rightarrow \text{phasor form } \mathbf{I} = 3.535\text{ A} \angle 30°$$

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_L = (I \angle \theta)(X_L \angle 90°) = (3.535\text{ A} \angle 30°)(4\ \Omega \angle +90°)$$
$$= 14.140\text{ V} \angle 120°$$

and $\quad v = \sqrt{2}(14.140) \sin(\omega t + 120°) = \mathbf{20 \sin(\omega t + 120°)}$

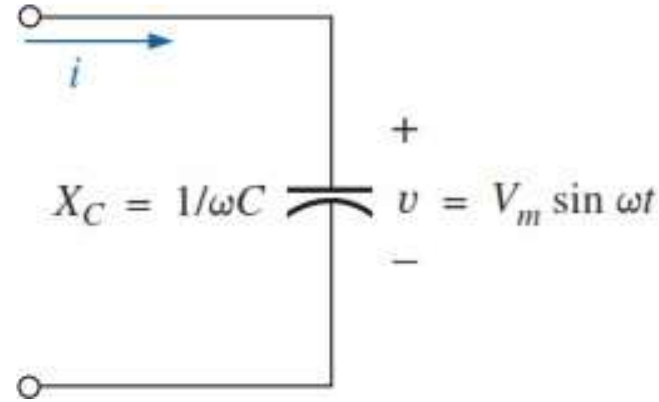


Phasor diagram



Capacitive Reactance

For the pure capacitor in Fig., the current leads the voltage by 90° and that the reactance of the capacitor Xc is determined by 1/ωC.



$$v = V_m \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = V \angle 0°$$

Applying Ohm's law and using phasor algebra, we find

$$\mathbf{I} = \frac{V \angle 0°}{X_C \angle \theta_C} = \frac{V}{X_C} \underline{/0° - \theta_C}$$

Since i leads v by 90°, i must have an angle of +90° associated with it. To satisfy this condition, θ_C must equal −90°. Substituting $\theta_C = -90°$ yields

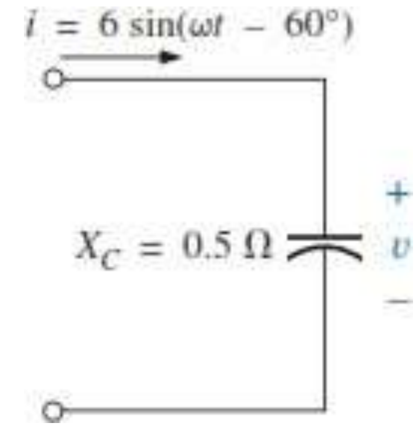
$$\mathbf{I} = \frac{V \angle 0°}{X_C \angle -90°} = \frac{V}{X_C} \underline{/0° - (-90°)} = \frac{V}{X_C} \angle 90°$$

so, in the time domain,

$$i = \sqrt{2}\left(\frac{V}{X_C}\right) \sin(\omega t + 90°)$$

$$\mathbf{Z}_C = X_C \angle -90°$$

EXAMPLE 15.6 Using complex algebra, find the voltage y for the circuit in Fig. Sketch the v and i curves.

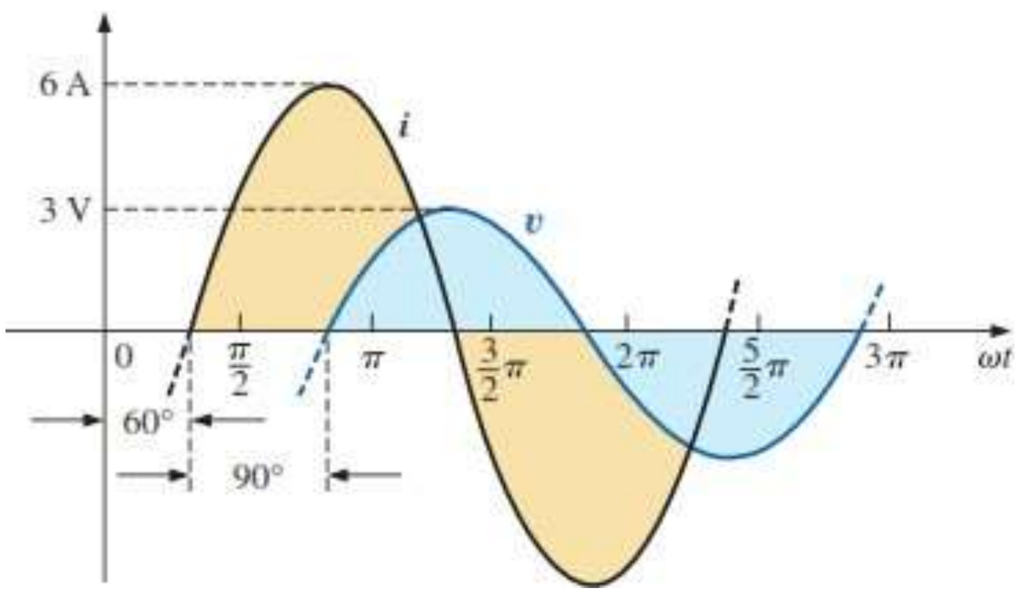


$$i = 6 \sin(\omega t - 60°) \Rightarrow \text{phasor notation } \mathbf{I} = 4.242 \text{ A} \angle -60°$$

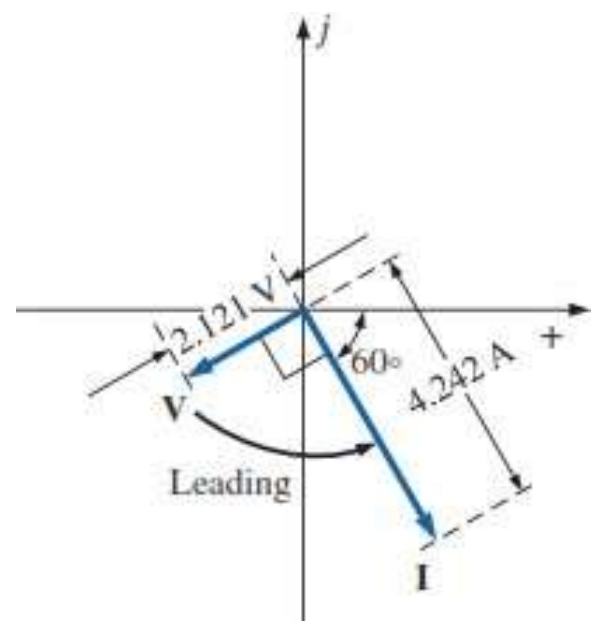
$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = (I \angle \theta)(X_C \angle -90°) = (4.242 \text{ A} \angle -60°)(0.5 \ \Omega \angle -90°)$$
$$= 2.121 \text{ V} \angle -150°$$

and

$$v = \sqrt{2}(2.121) \sin(\omega t - 150°) = \mathbf{3.0 \sin(\omega t - 150°)}$$



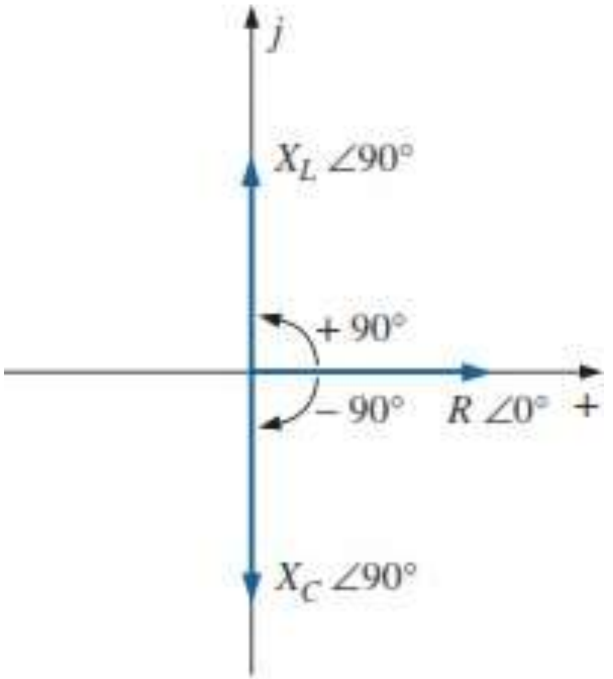
Phasor diagram



Impedance Diagram

For any network, the resistance will always appear on the positive real axis, the inductive reactance on the positive imaginary axis, and the capacitive reactance on the negative imaginary axis.

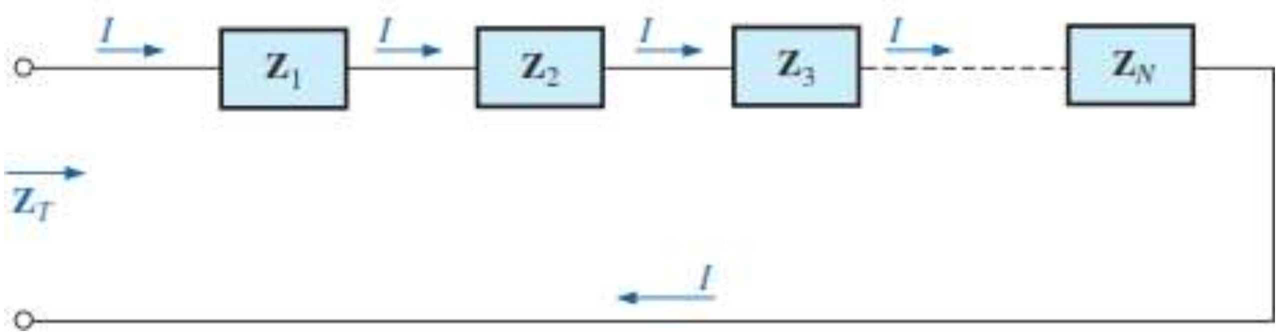
For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total impedance is the angle by which the applied voltage leads the source current. For inductive networks, θ_T will be positive, whereas for capacitive networks, θ_T will be negative.



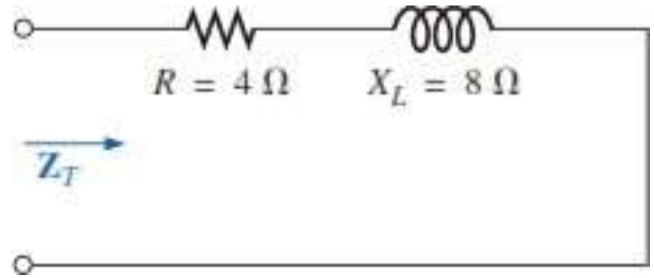
Impedance diagram.

15.3 SERIES CONFIGURATION

$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \cdots + \mathbf{Z}_N$$

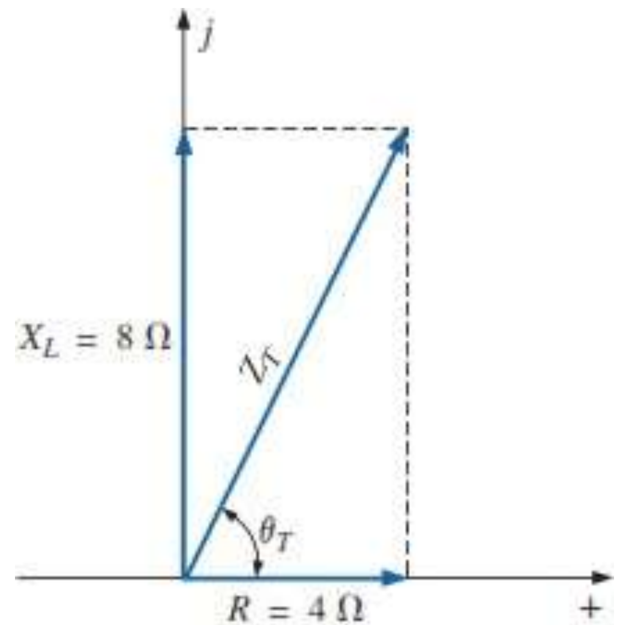


EXAMPLE 15.7 Draw the impedance diagram for the circuit in Fig., and find the total impedance.



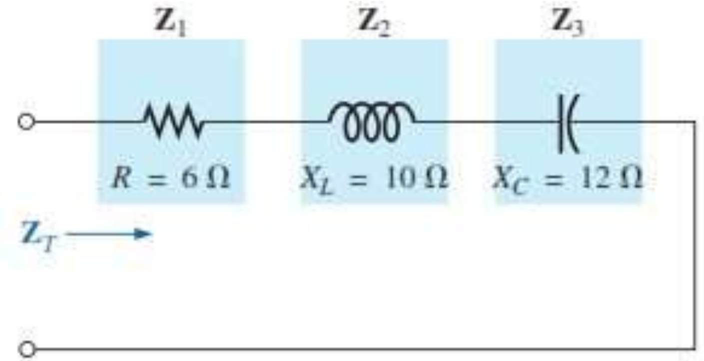
As indicated by Fig., the input impedance can be found graphically from the impedance diagram by properly scaling the real and imaginary axes and finding the length of the resultant vector Z_T and angle $\theta_{T.}$ Or, by using vector algebra, we obtain

$$
\begin{aligned}
\mathbf{Z}_T &= \mathbf{Z}_1 + \mathbf{Z}_2 \\
&= R \angle 0^\circ + X_L \angle 90^\circ \\
&= R + jX_L = 4\ \Omega + j\,8\ \Omega \\
\mathbf{Z}_T &= \mathbf{8.94\ \Omega \angle 63.43^\circ}
\end{aligned}
$$

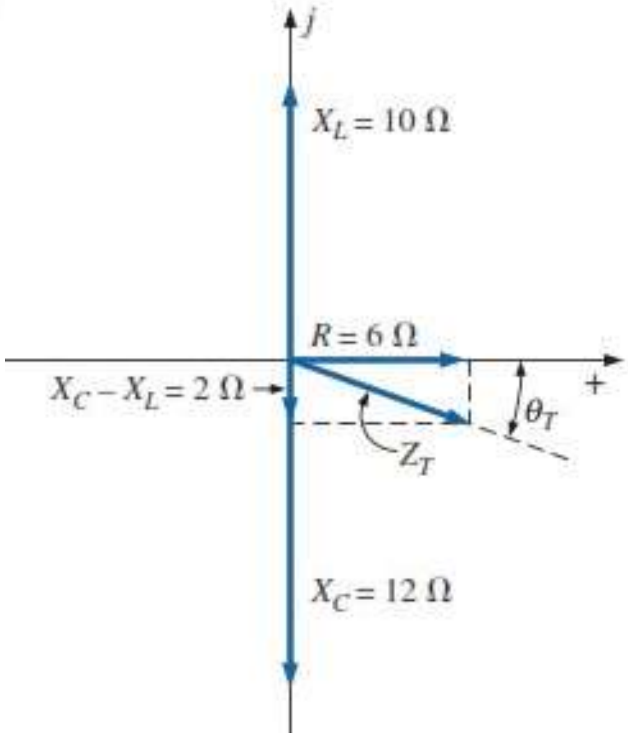


Impedance diagram.

EXAMPLE 15.8 Determine the input impedance to the series network in Fig. Draw the impedance diagram.



$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3$$
$$= R\angle 0^\circ + X_L\angle 90^\circ + X_C\angle -90^\circ$$
$$= R + jX_L - jX_C$$
$$= R + j(X_L - X_C) = 6\ \Omega + j(10\ \Omega - 12\ \Omega) = 6\ \Omega - j2\ \Omega$$
$$\mathbf{Z}_T = \mathbf{6.32\ \Omega\ \angle -18.43^\circ}$$



Note that in this example, the series inductive and capacitive reactances are in direct opposition.

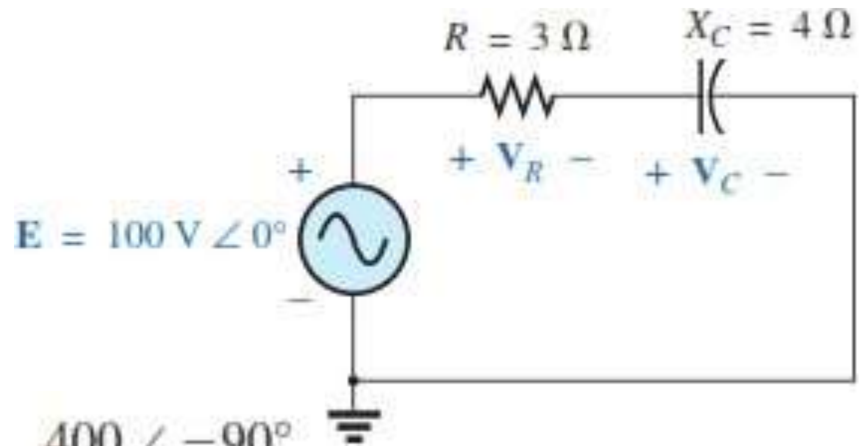
But if the inductive reactance were equal to the capacitive reactance, the input impedance would be purely resistive.

15.4 VOLTAGE DIVIDER RULE

The basic format for the **voltage divider** rule in ac circuits is exactly the same as that for dc circuits:

$$\mathbf{V}_x = \frac{\mathbf{Z}_x\mathbf{E}}{\mathbf{Z}_T}$$

EXAMPLE 15.9 Using the voltage divider rule, find the voltage across each element of the circuit in Fig.



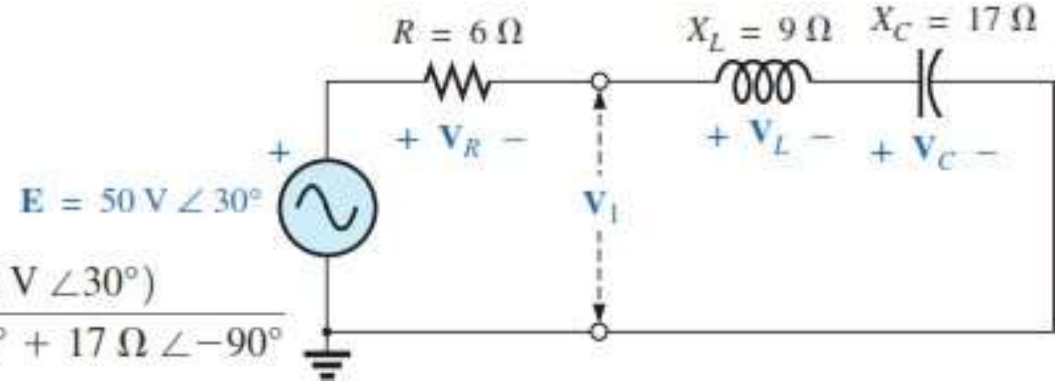
$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(4\ \Omega\ \angle -90^\circ)(100\ \text{V}\ \angle 0^\circ)}{4\ \Omega\ \angle -90^\circ + 3\ \Omega\ \angle 0^\circ} = \frac{400\ \angle -90^\circ}{3 - j4}$$

$$= \frac{400\ \angle -90^\circ}{5\ \angle -53.13^\circ} = \mathbf{80\ V\ \angle\ -36.87^\circ}$$

$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_C + \mathbf{Z}_R} = \frac{(3\ \Omega\ \angle 0^\circ)(100\ \text{V}\ \angle 0^\circ)}{5\ \Omega\ \angle -53.13^\circ} = \frac{300\ \angle 0^\circ}{5\ \angle -53.13^\circ}$$

$$= \mathbf{60\ V\ \angle\ +53.13^\circ}$$

EXAMPLE 15.10 Using the voltage divider rule, find the unknown voltages V_R, V_L, V_C and V_1 for the circuit shown.



$$\mathbf{V}_R = \frac{\mathbf{Z}_R\mathbf{E}}{\mathbf{Z}_R + \mathbf{Z}_L + \mathbf{Z}_C} = \frac{(6\ \Omega\ \angle 0°)(50\ \text{V}\ \angle 30°)}{6\ \Omega\ \angle 0° + 9\ \Omega\ \angle 90° + 17\ \Omega\ \angle -90°}$$

$$= \frac{300\ \angle 30°}{6 + j\,9 - j\,17} = \frac{300\ \angle 30°}{6 - j\,8}$$

$$= \frac{300\ \angle 30°}{10\ \angle -53.13°} = \mathbf{30\ V\ \angle 83.13°}$$

$$\mathbf{V}_L = \frac{\mathbf{Z}_L\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90°)(50\ \text{V}\ \angle 30°)}{10\ \Omega\ \angle -53.13°} = \frac{450\ \text{V}\ \angle 120°}{10\ \angle -53.13°}$$

$$= \mathbf{45\ V \angle 173.13°}$$

$$\mathbf{V}_C = \frac{\mathbf{Z}_C\mathbf{E}}{\mathbf{Z}_T} = \frac{(17\ \Omega\ \angle -90°)(50\ \text{V}\ \angle 30°)}{10\ \Omega\ \angle -53.13°} = \frac{850\ \text{V}\ \angle -60°}{10\ \angle -53°}$$

$$= \mathbf{85\ V\ \angle\ -6.87°}$$

$$\mathbf{V}_1 = \frac{(\mathbf{Z}_L + \mathbf{Z}_C)\mathbf{E}}{\mathbf{Z}_T} = \frac{(9\ \Omega\ \angle 90° + 17\ \Omega\ \angle -90°)(50\ \text{V}\ \angle 30°)}{10\ \Omega\ \angle -53.13°}$$

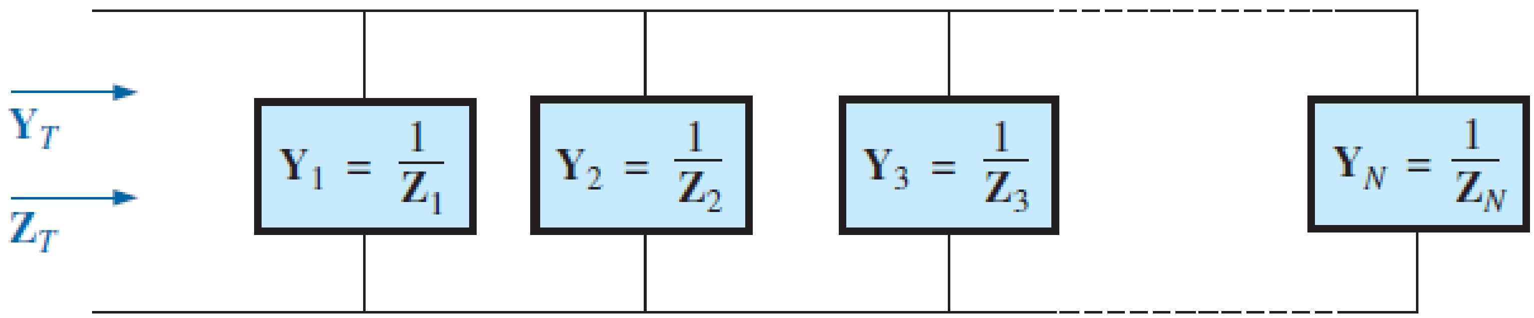
$$= \frac{(8\ \angle -90°)(50\ \angle 30°)}{10\ \angle -53.13°} = \frac{400\ \angle -60°}{10\ \angle -53.13°} = \mathbf{40\ V\ \angle -6.87°}$$

PARALLEL ac CIRCUITS

15.7 ADMITTANCE AND SUSCEPTANCE

In ac circuits, we define **admittance (Y) as being equal to 1/Z.** The unit of measure for admittance as defined by the SI system is ***siemens,*** which has the symbol **S**.

$$\mathbf{Y}_T = \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{Y}_3 + \cdots + \mathbf{Y}_N$$



since $\mathbf{Z} = 1/\mathbf{Y}$,

$$\frac{1}{\mathbf{Z}_T} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \cdots + \frac{1}{\mathbf{Z}_N}$$

and

$$\mathbf{Z}_T = \frac{1}{\dfrac{1}{\mathbf{Z}_1} + \dfrac{1}{\mathbf{Z}_2} + \dfrac{1}{\mathbf{Z}_3} + \cdots + \dfrac{1}{\mathbf{Z}_N}}$$

Conductance is the reciprocal of resistance,

$$\mathbf{Y}_R = \frac{1}{\mathbf{Z}_R} = \frac{1}{R \angle 0^\circ} = G \angle 0^\circ$$

The reciprocal of reactance (1/X) is called **susceptance** and is a measure of how susceptible an element is to the passage of current through it. Susceptance is also measured in **Siemens** and is represented by the capital letter **B.**

For the inductor,

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{X_L \angle 90^\circ} = \frac{1}{X_L} \angle -90^\circ$$

Defining $$B_L = \frac{1}{X_L} \quad \text{(siemens, S)}$$

we have $$\mathbf{Y}_L = B_L \angle -90^\circ$$

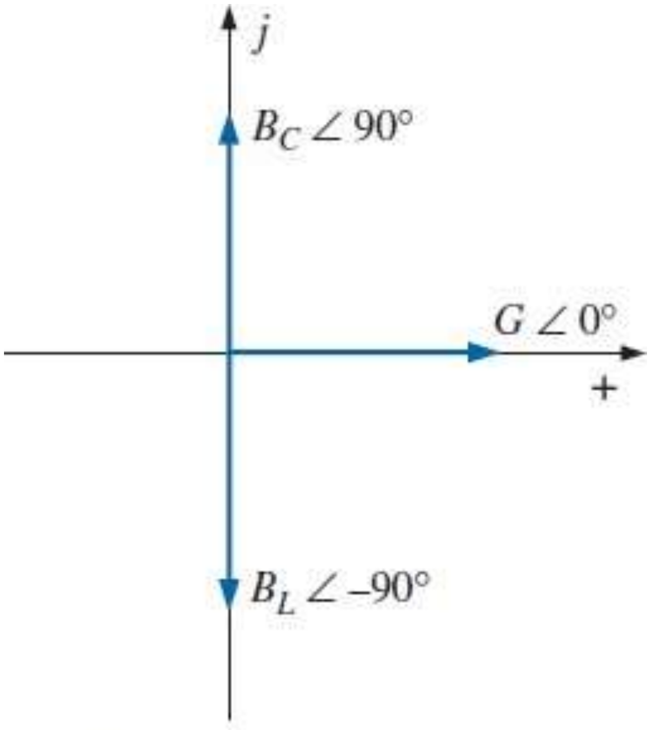
For the capacitor,

$$\mathbf{Y}_C = \frac{1}{\mathbf{Z}_C} = \frac{1}{X_C \angle -90^\circ} = \frac{1}{X_C} \angle 90^\circ$$

Defining $$B_C = \frac{1}{X_C} \quad \text{(siemens, S)}$$

we have $$\mathbf{Y}_C = B_C \angle 90^\circ$$

For any configuration (series, parallel, series-parallel, and so on), the angle associated with the total admittance is the angle by which the source current leads the applied voltage. For inductive networks, θ_T is negative, whereas for capacitive networks, θ_T is positive.



Admittance diagram.

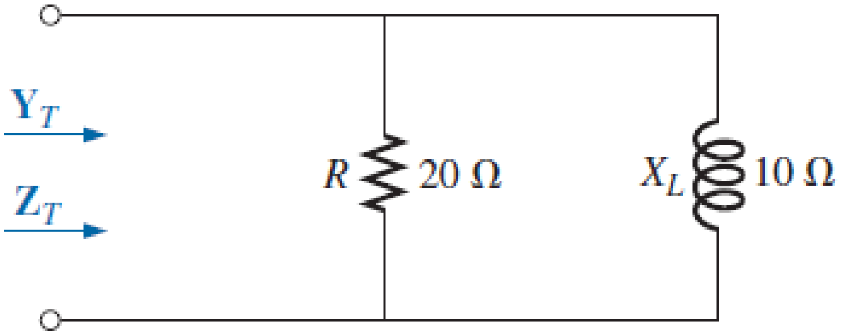
EXAMPLE 15.13 For the network in Fig. :

a. Calculate the input impedance.

b. Draw the impedance diagram.

c. Find the admittance of each parallel branch.

d.Determine the input admittance and draw the admittance diagram.

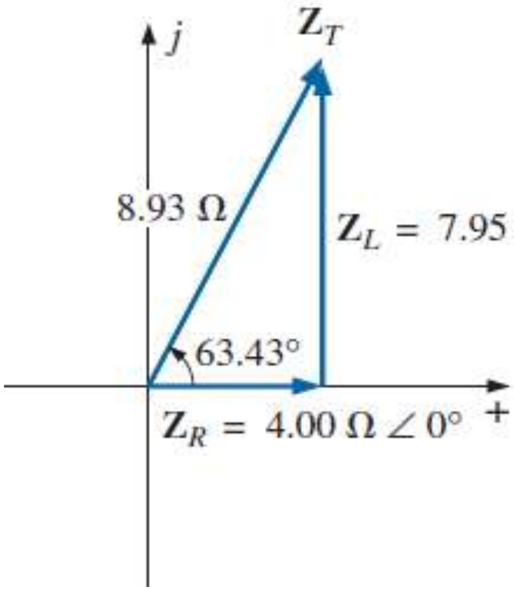


a. $$\mathbf{Z}_T = \frac{\mathbf{Z}_R\mathbf{Z}_L}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(20\ \Omega\ \angle 0^\circ)(10\ \Omega\ \angle 90^\circ)}{20\ \Omega + j\,10\ \Omega}$$

$$= \frac{200\ \Omega\ \angle 90^\circ}{22.361\ \angle 26.57^\circ} = \mathbf{8.93\ \Omega\ \angle 63.43^\circ}$$

$$= \mathbf{4.00\ \Omega} + \mathbf{j\,7.95\ \Omega} = R_T + j\,X_{L_T}$$

b. The impedance diagram appears in Fig.



c. $$\mathbf{Y}_R = G\angle 0^\circ = \frac{1}{R}\angle 0^\circ = \frac{1}{20\ \Omega}\angle 0^\circ = \mathbf{0.05\ S\ \angle 0^\circ}$$

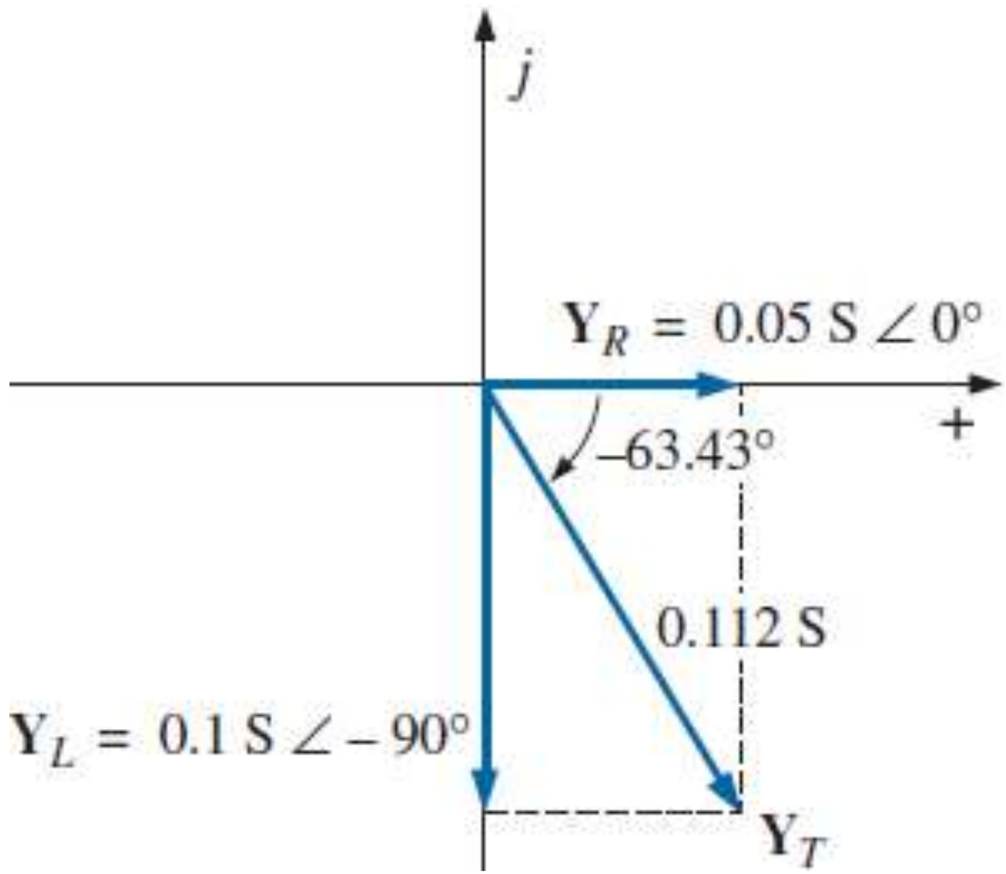
$$= \mathbf{0.05\ S} + \mathbf{j\,0}$$

$$\mathbf{Y}_L = B_L\angle -90^\circ = \frac{1}{X_L}\angle -90^\circ = \frac{1}{10\ \Omega}\angle -90^\circ$$

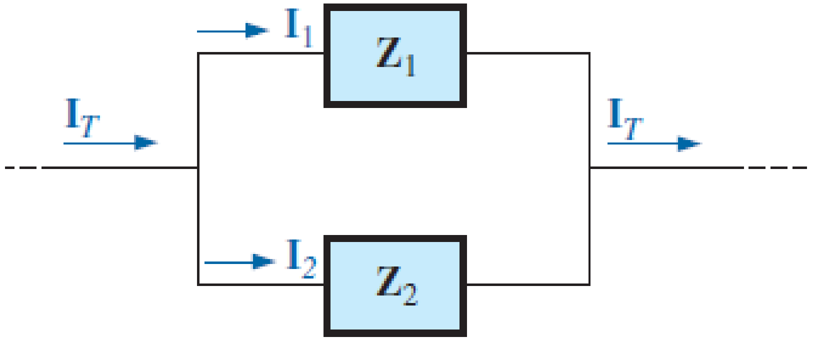
$$= \mathbf{0.1\ S\ \angle -90^\circ} = \mathbf{0} - \mathbf{j\,0.1\ S}$$

d. $\mathbf{Y}_T = \mathbf{Y}_R + \mathbf{Y}_L = (0.05\ \text{S} + j\,0) + (0 - j\,0.1\ \text{S})$
$= \mathbf{0.05\ S - j\,0.1\ S} = G - jB_L$

The admittance diagram appears in Fig.



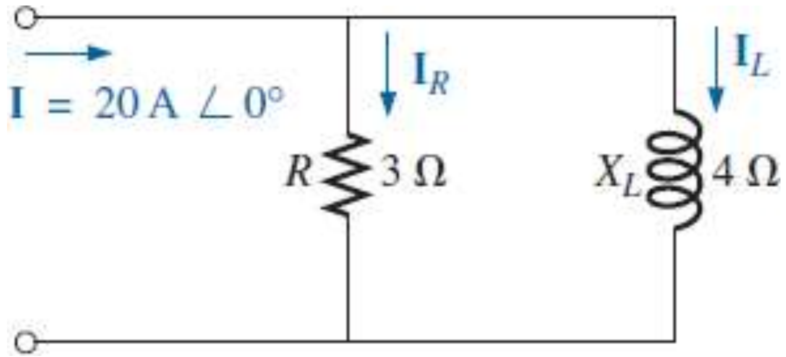
15.9 CURRENT DIVIDER RULE



The basic format for the **current divider** rule in ac circuits is exactly the same as that for dc circuits; that is, for two parallel branches with impedances **Z1 and Z2** as shown in Fig.

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2} \quad \text{or} \quad \mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}_T}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

EXAMPLE 15.16 Using the current divider rule, find the current through each impedance in Fig.

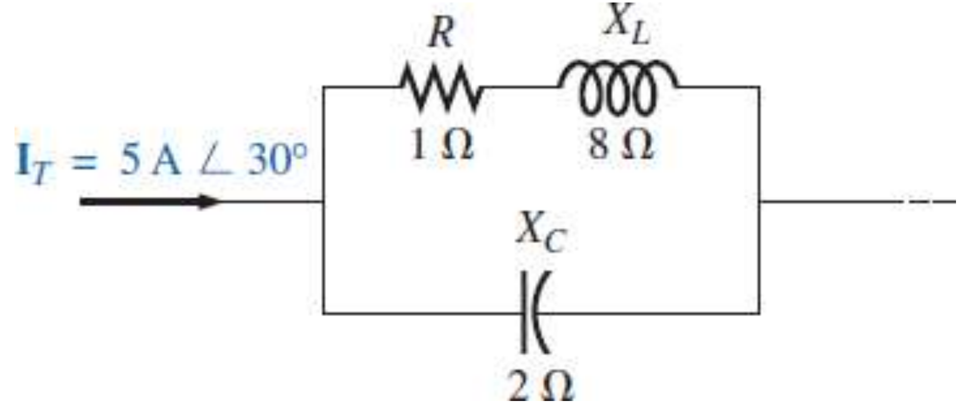


$$\mathbf{I}_R = \frac{\mathbf{Z}_L\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(4\ \Omega \angle 90^\circ)(20\ \text{A} \angle 0^\circ)}{3\ \Omega \angle 0^\circ + 4\ \Omega \angle 90^\circ} = \frac{80\ \text{A} \angle 90^\circ}{5 \angle 53.13^\circ}$$
$$= 16\ \text{A} \angle 36.87^\circ$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_R\mathbf{I}_T}{\mathbf{Z}_R + \mathbf{Z}_L} = \frac{(3\ \Omega \angle 0^\circ)(20\ \text{A} \angle 0^\circ)}{5\ \Omega \angle 53.13^\circ} = \frac{60\ \text{A} \angle 0^\circ}{5 \angle 53.13^\circ}$$
$$= 12\ \text{A} \angle -53.13^\circ$$

Draw the Current Phasor Diagram

EXAMPLE 15.17 Using the current divider rule, find the current through each parallel branch in Fig.



$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C \mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega \angle -90^\circ)(5\ \text{A} \angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\ \text{A} \angle -60^\circ}{1 + j6}$$

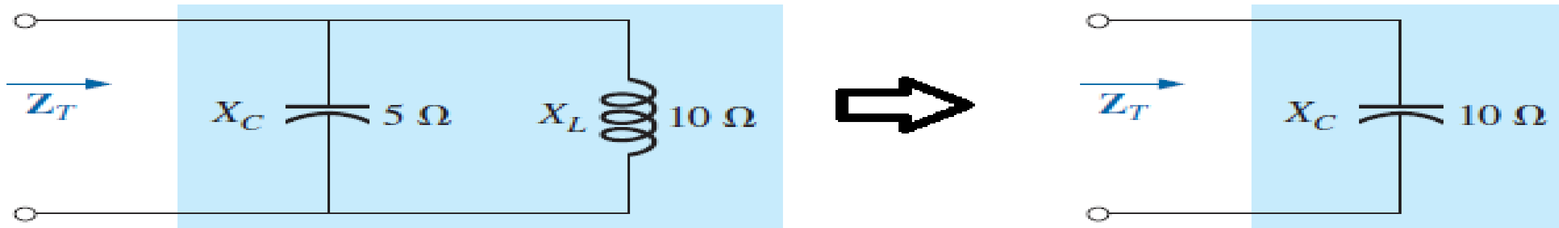
$$= \frac{10\ \text{A} \angle -60^\circ}{6.083 \angle 80.54^\circ} \cong \mathbf{1.64\ A \angle -140.54^\circ}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R\text{-}L} \mathbf{I}_T}{\mathbf{Z}_{R\text{-}L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\,8\ \Omega)(5\ \text{A} \angle 30^\circ)}{6.08\ \Omega \angle 80.54^\circ}$$

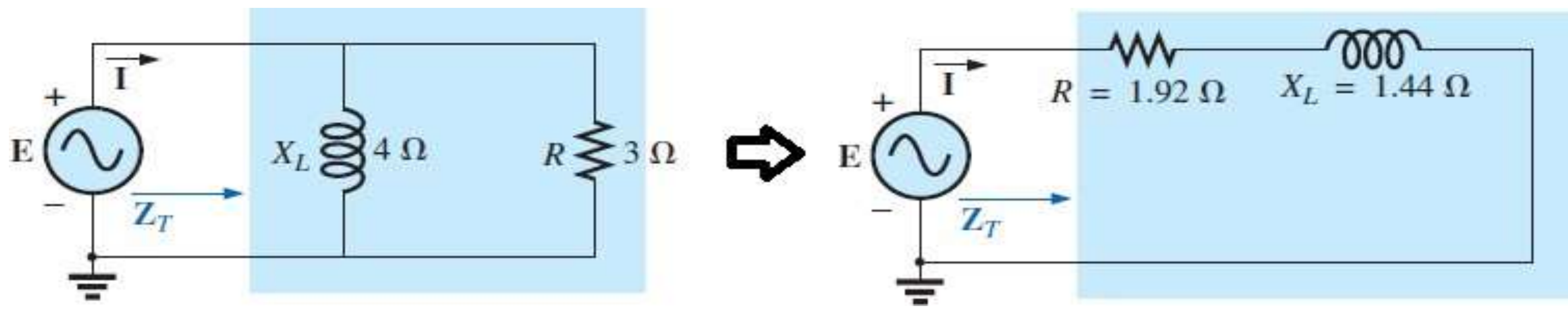
$$= \frac{(8.06 \angle 82.87^\circ)(5\ \text{A} \angle 30^\circ)}{6.08 \angle 80.54^\circ} = \frac{40.30\ \text{A} \angle 112.87^\circ}{6.083 \angle 80.54^\circ}$$

$$= \mathbf{6.63\ A \angle 32.33^\circ}$$

15.12 EQUIVALENT CIRCUITS

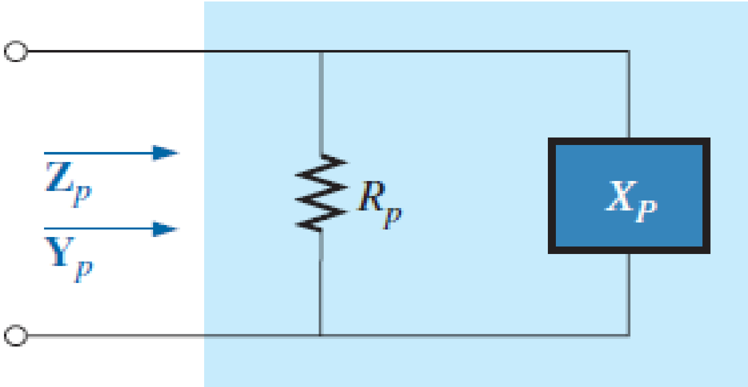


$$Z_T = \frac{Z_C Z_L}{Z_C + Z_L} = \frac{(5\ \Omega \angle -90^\circ)(10\ \Omega \angle 90^\circ)}{5\ \Omega \angle -90^\circ + 10\ \Omega \angle 90^\circ} = \frac{50 \angle 0^\circ}{5 \angle 90^\circ}$$
$$= 10\ \Omega \angle -90^\circ$$



$$Z_T = \frac{Z_L Z_R}{Z_L + Z_R} = \frac{(4\ \Omega \angle 90^\circ)(3\ \Omega \angle 0^\circ)}{4\ \Omega \angle 90^\circ + 3\ \Omega \angle 0^\circ}$$
$$= \frac{12 \angle 90^\circ}{5 \angle 53.13^\circ} = 2.40\ \Omega \angle 36.87^\circ \quad = 1.92\ \Omega + j\,1.44\ \Omega$$

$$\mathbf{Y}_p = \frac{1}{R_p} + \frac{1}{\pm jX_p} = \frac{1}{R_p} \mp j\frac{1}{X_p}$$



and

$$\mathbf{Z}_p = \frac{1}{\mathbf{Y}_p} = \frac{1}{(1/R_p) \mp j(1/X_p)}$$

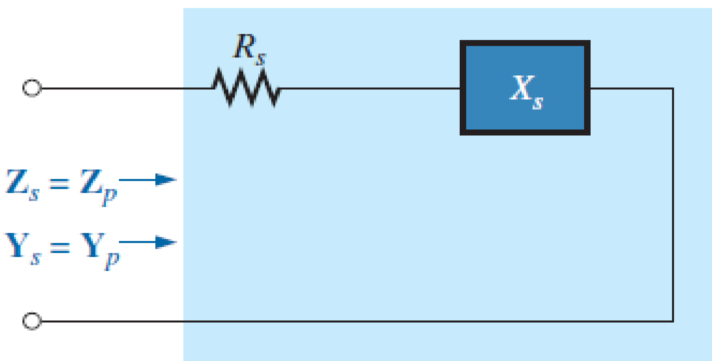
$$= \frac{1/R_p}{(1/R_p)^2 + (1/X_p)^2} \pm j\frac{1/X_p}{(1/R_p)^2 + (1/X_p)^2}$$

Multiplying the numerator and denominator of each term by $R_p^2 X_p^2$

results in $$\mathbf{Z}_p = \frac{R_p X_p^2}{X_p^2 + R_p^2} \pm j\frac{R_p^2 X_p}{X_p^2 + R_p^2}$$

$$= R_s \pm j X_s$$

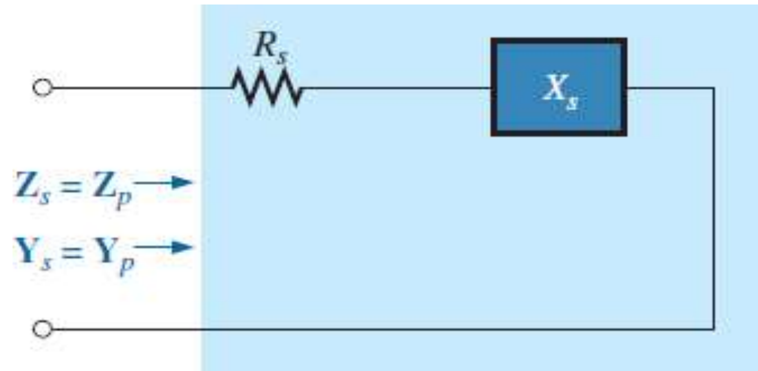
and $$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2} \qquad X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2}$$



$$\mathbf{Z}_s = R_s \pm jX_s$$

$$\mathbf{Y}_s = \frac{1}{\mathbf{Z}_s} = \frac{1}{R_s \pm jX_s} = \frac{R_s}{R_s^2 + X_s^2} \mp j\frac{X_s}{R_s^2 + X_s^2}$$

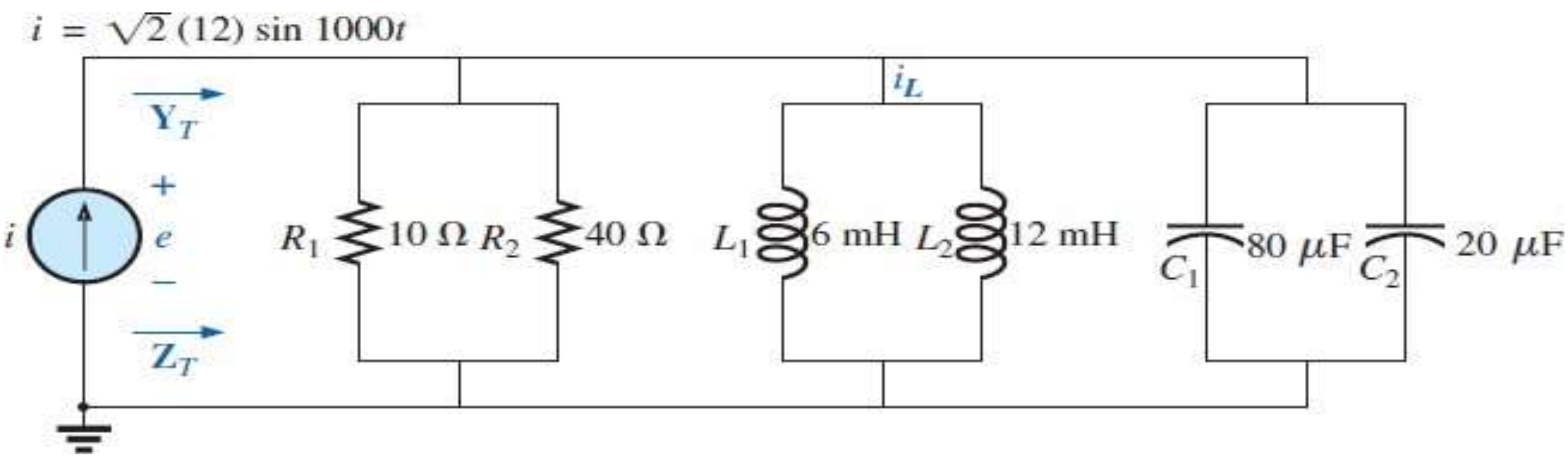
$$= G_p \mp jB_p = \frac{1}{R_p} \mp j\frac{1}{X_p}$$



or $$R_p = \frac{R_s^2 + X_s^2}{R_s} \qquad X_p = \frac{R_s^2 + X_s^2}{X_s}$$

EXAMPLE 15.19 For the network in Fig. :

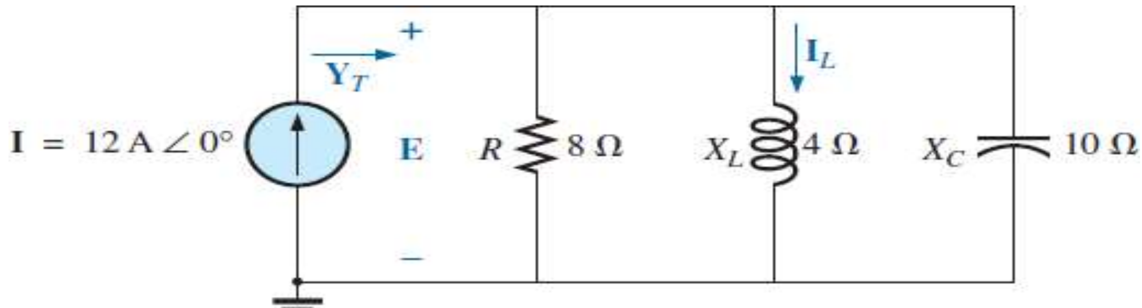
a. Determine ***Y_T* and *Z_T* .**
b. Sketch the admittance diagram.
c. Find **E and I_L .**
d. Compute the power factor of the network and the power delivered to the network.
e. Determine the equivalent series circuit.
f. Using the equivalent circuit developed in part (e), calculate **E,** and compare it with the result of part (c).
g. Determine the power delivered to the network, and compare it with the solution of part (d).
h. Determine the equivalent parallel network from the equivalent series circuit, and calculate the total admittance ***Y_T* .** Compare the result with the solution of part (a).



a. Combining common elements and finding the reactance of the inductor and capacitor, we obtain

$$R_T = 10\ \Omega \parallel 40\ \Omega = 8\ \Omega$$
$$L_T = 6\text{ mH} \parallel 12\text{ mH} = 4\text{ mH}$$
$$C_T = 80\ \mu\text{F} + 20\ \mu\text{F} = 100\ \mu\text{F}$$

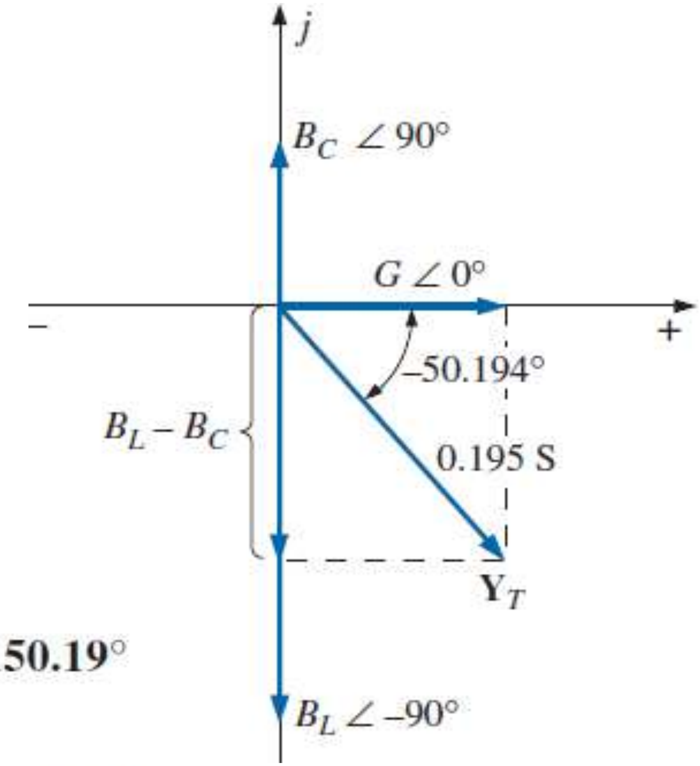
$$X_L = \omega L = (1000\text{ rad/s})(4\text{ mH}) = 4\ \Omega$$
$$X_C = \frac{1}{\omega C} = \frac{1}{(1000\text{ rad/s})(100\ \mu\text{F})} = 10\ \Omega$$



$$\begin{aligned}\mathbf{Y}_T &= \mathbf{Y}_R + \mathbf{Y}_L + \mathbf{Y}_C\\ &= G\angle 0^\circ + B_L\angle -90^\circ + B_C\angle +90^\circ\\ &= \frac{1}{8\ \Omega}\angle 0^\circ + \frac{1}{4\ \Omega}\angle -90^\circ + \frac{1}{10\ \Omega}\angle +90^\circ\\ &= 0.125\text{ S}\angle 0^\circ + 0.25\text{ S}\angle -90^\circ + 0.1\text{ S}\angle +90^\circ\\ &= 0.125\text{ S} - j\,0.25\text{ S} + j\,0.1\text{ S}\\ &= 0.125\text{ S} - j\,0.15\text{ S} = \mathbf{0.195\ S}\ \angle\mathbf{-50.194^\circ}\end{aligned}$$

$$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.195\text{ S}}\angle -50.194^\circ = \mathbf{5.13\ \Omega}\ \angle\mathbf{50.19^\circ}$$

b- Admittance diagram for the parallel R-L-C network



c. $\mathbf{E} = \mathbf{I}\mathbf{Z}_T = \dfrac{\mathbf{I}}{\mathbf{Y}_T} = \dfrac{12\ \text{A} \angle 0°}{0.195\ \text{S} \angle -50.194°} = \mathbf{61.54\ V \angle 50.19°}$

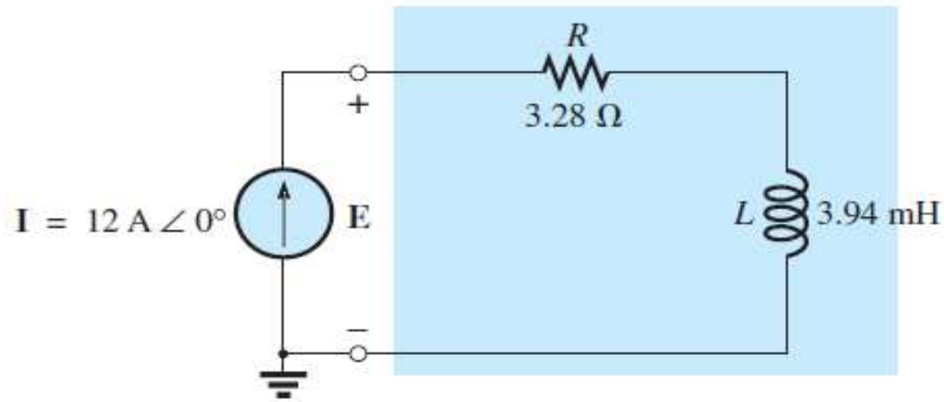
$$\mathbf{I}_L = \frac{\mathbf{V}_L}{\mathbf{Z}_L} = \frac{\mathbf{E}}{\mathbf{Z}_L} = \frac{61.538\ \text{V} \angle 50.194°}{4\ \Omega \angle 90°} = \mathbf{15.39\ A \angle -39.81°}$$

d. $F_p = \cos\theta = \dfrac{G}{Y_T} = \dfrac{0.125\ \text{S}}{0.195\ \text{S}} = \mathbf{0.641\ lagging}$ (**E** leads **I**)

$$P = EI\cos\theta = (61.538\ \text{V})(12\ \text{A})\cos 50.194°$$
$$= \mathbf{472.75\ W}$$

e. $$\mathbf{Z}_T = \frac{1}{\mathbf{Y}_T} = \frac{1}{0.195\ \text{S}\ \angle -50.194^\circ} = 5.128\ \Omega\ \angle +50.194^\circ$$
$$= 3.28\ \Omega + j\,3.94\ \Omega$$
$$= R + j\,X_L$$

$$X_L = 3.94\ \Omega = \omega L$$
$$L = \frac{3.94\ \Omega}{\omega} = \frac{3.94\ \Omega}{1000\ \text{rad/s}} = \mathbf{3.94\ mH}$$

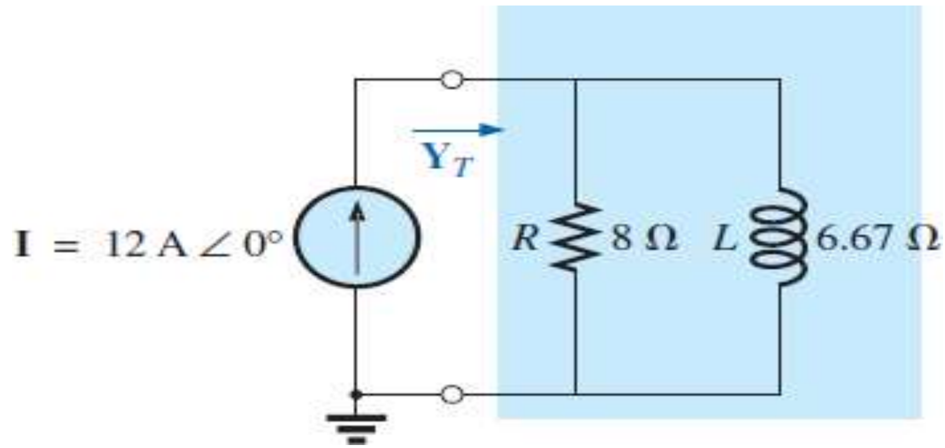


f. $$\mathbf{E} = \mathbf{I}\mathbf{Z}_T = (12\ \text{A}\ \angle 0^\circ)(5.128\ \Omega\ \angle 50.194^\circ)$$
$$= \mathbf{61.54\ V\ \angle 50.194^\circ} \quad \text{(as above)}$$

g. $$P = I^2R = (12\ \text{A})^2(3.28\ \Omega) = \mathbf{472.32\ W} \qquad \text{(as above)}$$

h. $$R_p = \frac{R_s^2 + X_s^2}{R_s} = \frac{(3.28\ \Omega)^2 + (3.94\ \Omega)^2}{3.28\ \Omega} = \mathbf{8\ \Omega}$$
$$X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{(3.28\ \Omega)^2 + (3.94\ \Omega)^2}{3.94\ \Omega} = \mathbf{6.67\ \Omega}$$

The parallel equivalent circuit



$$\mathbf{Y}_T = G\angle 0^\circ + B_L\angle -90^\circ = \frac{1}{8\ \Omega}\angle 0^\circ + \frac{1}{6.675\ \Omega}\angle -90^\circ$$
$$= 0.125\ \text{S}\angle 0^\circ + 0.15\ \text{S}\angle -90^\circ$$
$$= 0.125\ \text{S} - j\,0.15\ \text{S} = \mathbf{0.195\ S\angle -50.194^\circ} \quad \text{(as above)}$$

PROBLEMS

SECTION 15.2 Impedance and the Phasor Diagram: 1, 2

SECTION 15.3 Series Configuration: 4, 5, 6, 7, 9, 10

SECTION 15.4 Voltage Divider Rule: 15, 17, 20

SECTION 15.7 Admittance and Susceptance: 25, 26

SECTION 15.9 Current Divider Rule: 34

SECTION 15.12 Equivalent Circuits: 40, 42, 44

Thank you very much