Module Title: Fundamental of Electrical Engineering (AC)

Module Code:	**UOMU024021**

Week 9 and Week 10

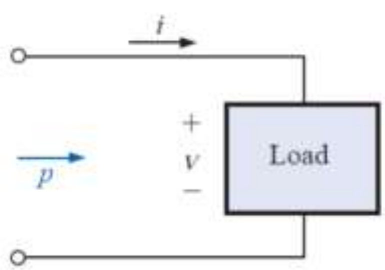
AC Power

AVERAGE POWER AND POWER FACTOR

APPARENT POWER, RESISTIVE CIRCUIT, INDUCTIVE CIRCUIT, CAPACITIVE CIRCUIT, THE POWER TRIANGLE, SOLVED PROBLEMS

1. Average Power definition:

The average power, or real power as it is sometimes called, is the power delivered to and dissipated by the load. It corresponds to the power calculations performed for DC networks.



$$p = vi$$

Since V and I are sinusoidal quantities, let us establish a general case where:

$$v = V_m \sin(\omega t + \theta)$$

$$i = I_m \sin \omega t$$

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* The angle (θ) is the phase angle between *v* and *i*.

Substituting the above equations for *v* and *i* into the power equation will result in

$$p = V_m I_m \sin \omega t \sin(\omega t + \theta)$$

If we now apply a number of trigonometric identities, the following form for the power equation will result:

$$p = VI \cos \theta(1 - \cos 2\omega t) + VI \sin \theta(\sin 2\omega t)$$

* where *V* and *I* are the **rms** values.

$$V = \frac{Vm}{\sqrt{2}} \quad \text{and} \quad I = \frac{\text{Im}}{\sqrt{2}}$$

So that the power is:

$$p = \underbrace{VI \cos \theta}_{\text{Average}} - \underbrace{VI \cos \theta}_{\text{Peak}} \underbrace{\cos 2\omega t}_{2x} + \underbrace{VI \sin \theta}_{\text{Peak}} \underbrace{\sin 2\omega t}_{2x}$$

The average power equal

$$P = VI \cos \theta$$

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* The magnitude of average power delivered is independent of whether v leads i or i leads v.

- for resistor $\theta = 0$ then $P = VI\cos 0 = VI$
- for Inductor v leads i by 90°, then $P = VI\cos 90 = 0$
- for Capacitor i leads v by 90°, then $P = VI\cos 90 = 0$

2. Power Factor (PF)

The power factor is the factor that has significant control over the delivered power level.

$$\text{Power factor} = F_p = \cos\theta$$

- Capacitive networks have leading power factors,
- Inductive networks have lagging power factors.

3. Apparent Power

It is a power rating of significant usefulness in the description and analysis of sinusoidal AC networks and in the maximum rating of a number of electrical components and systems.

$$S = VI \quad \text{(volt-amperes, VA)}$$

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$$S = I^2 Z \quad \text{(VA)}$$

$$S = \frac{V^2}{Z} \quad \text{(VA)}$$

Therefore

$$P = S \cos \theta \quad \text{(W)}$$

$$F_p = \cos \theta = \frac{P}{S}$$

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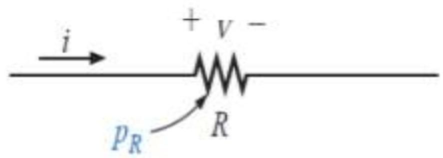
1) RESISTIVE CIRCUIT

For a purely resistive circuit, *v* and *i* are in phase, and $\theta = 0°$,



$$p_R = VI\cos(0°)(1 - \cos 2\omega t) + VI\sin(0°)\sin 2\omega t$$
$$= VI(1 - \cos 2\omega t) + 0$$

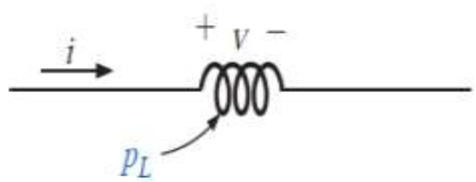
$$p_R = VI - VI\cos 2\omega t$$

The **average (real) power** is

$$P = VI = \frac{V_m I_m}{2} = I^2 R = \frac{V^2}{R} \qquad \text{(watts, W)}$$

2) INDUCTIVE CIRCUIT

For a purely inductive circuit, *v* leads *i* by 90°, $\theta = 90°$.



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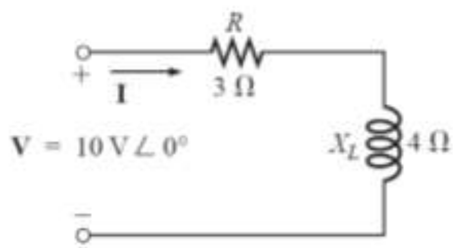
EXAMPLE: Find the total number of watts, volt-amperes reactive, and volt-amperes for the network.



Solutions:

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_T} = \frac{10\ \text{V}\ \angle 0^\circ}{3\ \Omega + j\,4\ \Omega} = \frac{10\ \text{V}\ \angle 0^\circ}{5\ \Omega\ \angle 53.13^\circ} = 2\ \text{A}\ \angle -53.13^\circ$$

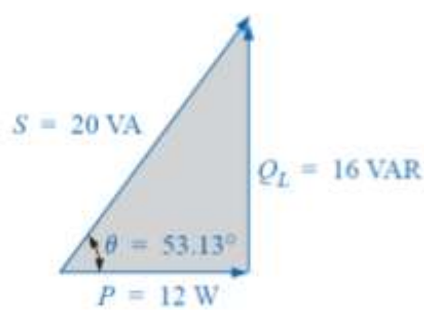
$$P = I^2R = (2\ \text{A})^2(3\ \Omega) = 12\ \text{W}$$

$$Q_L = I^2X_L = (2\ \text{A})^2(4\ \Omega) = 16\ \text{VAR}\ (L)$$

$$\mathbf{S} = P + j\,Q_L = 12\ \text{W} + j\,16\ \text{VAR}\ (L) = 20\ \text{VA}\ \angle 53.13^\circ$$

Or

$$\mathbf{S} = \mathbf{VI}^* = (10\ \text{V}\ \angle 0^\circ)(2\ \text{A}\ \angle +53.13^\circ) = 20\ \text{VA}\ \angle 53.13^\circ$$



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$$p_L = VI\cos(90°)(1 - \cos 2\omega t) + VI\sin(90°)(\sin 2\omega t)$$
$$= 0 + VI\sin 2\omega t$$

$$p_L = VI\sin 2\omega t$$

The **reactive power** is

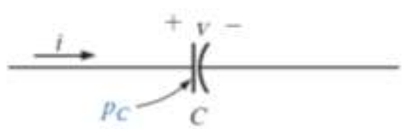
$$Q = VI\sin\theta \qquad \text{(volt-ampere reactive, VAR)}$$

$$Q_L = I^2X_L \qquad \text{(VAR)}$$

$$Q_L = \frac{V^2}{X_L} \qquad \text{(VAR)}$$

3) CAPACITIVE CIRCUIT

For a purely capacitive circuit, *i* leads *v* by 90°, θ =-90°.



$$p_C = VI\cos(-90°)(1 - \cos 2\omega t) + VI\sin(-90°)(\sin 2\omega t)$$
$$= 0 - VI\sin 2\omega t$$

$$p_C = -VI\sin 2\omega t$$

The **reactive power** is

$$Q = VI\sin\theta \qquad \text{(volt-ampere reactive, VAR)}$$

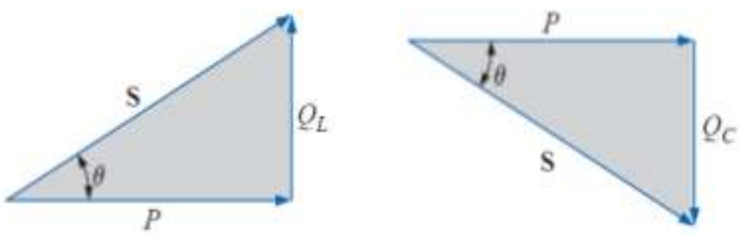
$$Q_C = I^2X_C$$

Er

$$Q_C = \frac{V^2}{X_C}$$

THE POWER TRIANGLE

The three quantities **average power (P)**, **apparent power (S)**, and **reactive power (Q)** can be related in the vector domain by



$$S = P + jQ$$
$$Q = Q_L - Q_C$$

$$S^2 = P^2 + Q^2$$

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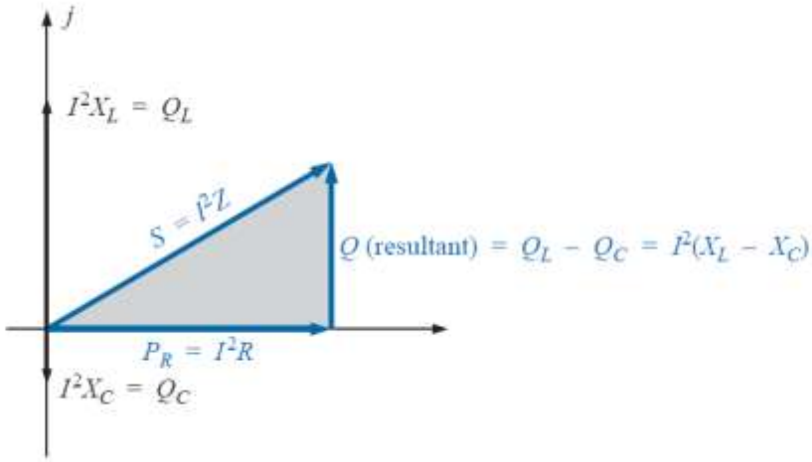


$$S = \sqrt{P^2 + Q^2}$$

$$F_p = \cos\theta = \frac{P}{S}$$

EXAMPLE: An electrical device is rated 5 kVA, 100 V at a 0.6 power-factor lag. What is the impedance of the device in rectangular coordinates?

Solutions:

$S = EI = 5000\ \text{VA}$

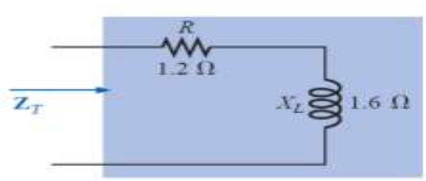
$I = \dfrac{5000\ \text{VA}}{100\ \text{V}} = 50\ \text{A}$

$\theta = \cos^{-1} 0.6 = 53.13°$

for $\mathbf{E} = 100\ \text{V} \angle 0°$,

$\mathbf{I} = 50\ \text{A} \angle -53.13°$

$$\mathbf{Z}_T = \frac{\mathbf{E}}{\mathbf{I}} = \frac{100\ \text{V} \angle 0°}{50\ \text{A} \angle -53.13°} = 2\ \Omega \angle 53.13° = \mathbf{1.2\ \Omega\ + j\,1.6\ \Omega}$$



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