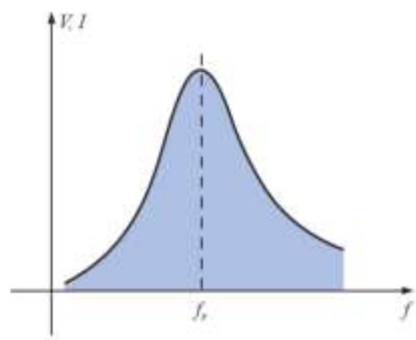
Module Title: Fundamental of Electrical Engineering (AC)

Module Code:	**UOMU024021**

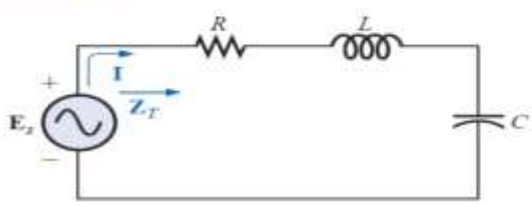
Week 11

Resonance Circuits

The resonant circuit is a combination of R, L, and C elements having a frequency response characteristic similar to the one appearing in Figure below. Note in the figure that the response is a maximum for the frequency fr, decreasing to the right and left of this frequency.



1-SERIES RESONANT CIRCUIT



$$\mathbf{Z}_T = R + j X_L - j X_C = R + j (X_L - X_C)$$

$$X_L = X_C$$

$$\mathbf{Z}_{T_s} = R$$

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$$\omega L = \frac{1}{\omega C} \quad \text{and} \quad \omega^2 = \frac{1}{LC}$$

$$\omega_s = \frac{1}{\sqrt{LC}}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

f = hertz (Hz)
L = henries (H)
C = farads (F)

$$V_{L_s} = V_{C_s}$$

$$F_p = \cos\theta = \frac{P}{S}$$

$$F_{p_s} = 1$$

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THE QUALITY FACTOR (*Q*)

The quality factor Q of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance; that is,

$$Q_s = \frac{\text{reactive power}}{\text{average power}}$$

$$Q_s = \frac{I^2 X_L}{I^2 R}$$

$$Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R}$$

$$Q_s = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi}{R}\left(\frac{1}{2\pi\sqrt{LC}}\right) L$$

$$= \frac{L}{R}\left(\frac{1}{\sqrt{LC}}\right) = \left(\frac{\sqrt{L}}{\sqrt{L}}\right)\frac{L}{R\sqrt{LC}}$$

$$Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$V_{L_s} = Q_s E$$

$$V_{C_s} = Q_s E$$

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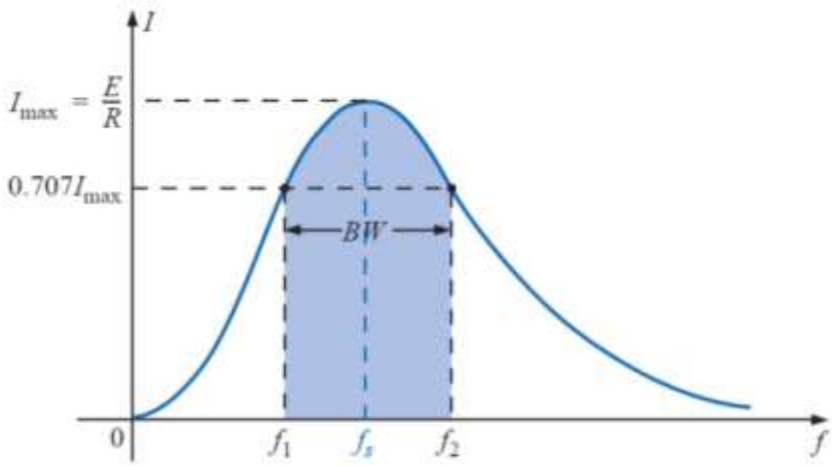
SELECTIVITY

There is a definite range of frequencies at which the current is near its maximum value and the impedance is at a minimum. Those frequencies corresponding to 0.707 of the maximum current are called the **band frequencies**, **cutoff frequencies**, or **half-power frequencies**. They are indicated by f1 and f2 in Figure below. The range of frequencies between the two is referred to as the bandwidth (abbreviated BW) of the resonant circuit.



$$P_{HPF} = \frac{1}{2} P_{max}$$

$$BW = f_2 - f_1 = \frac{R}{2\pi L}$$

$$BW = \frac{f_s}{Q_s}$$

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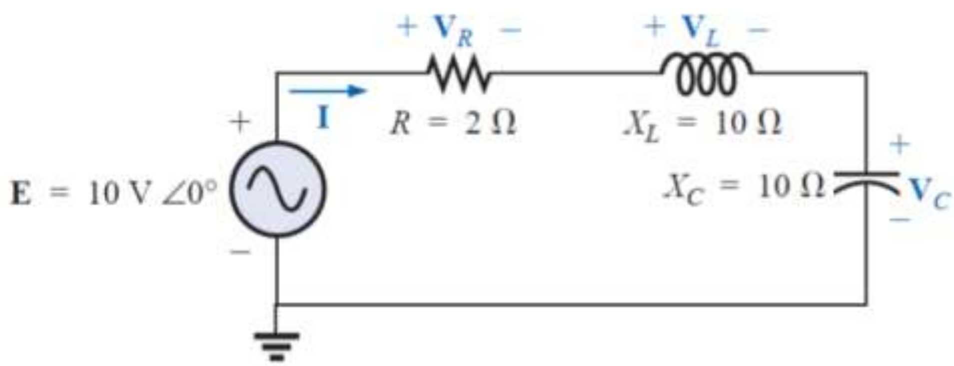
EXAMPLE:

a. For the series resonant circuit, find **I**, **V***R*, **V***L*, and **V***C* at resonance.
b. What is the *Qs* of the circuit?
c. If the resonant frequency is 5000 Hz, find the bandwidth.
d. What is the power dissipated in the circuit at the half-power frequencies?



Solutions:

a. $\mathbf{Z}_{T_s} = R = 2\,\Omega$

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_{T_s}} = \frac{10\text{ V}\angle 0^\circ}{2\,\Omega\angle 0^\circ} = \mathbf{5\text{ A}\angle 0^\circ}$$

$$\mathbf{V}_R = \mathbf{E} = 10\text{ V}\angle 0^\circ$$

$$\mathbf{V}_L = (I\angle 0^\circ)(X_L\angle 90^\circ) = (5\text{ A}\angle 0^\circ)(10\,\Omega\angle 90^\circ) = \mathbf{50\text{ V}\angle 90^\circ}$$

$$\mathbf{V}_C = (I\angle 0^\circ)(X_C\angle -90^\circ) = (5\text{ A}\angle 0^\circ)(10\,\Omega\angle -90^\circ) = \mathbf{50\text{ V}\angle -90^\circ}$$

b. $$Q_s = \frac{X_L}{R} = \frac{10\,\Omega}{2\,\Omega} = \mathbf{5}$$

c. $$BW = f_2 - f_1 = \frac{f_s}{Q_s} = \frac{5000\text{ Hz}}{5} = \mathbf{1000\text{ Hz}}$$

d. $$P_{\text{HPF}} = \frac{1}{2}P_{\max} = \frac{1}{2}I_{\max}^2 R = \left(\frac{1}{2}\right)(5\text{ A})^2(2\,\Omega) = \mathbf{25\text{ W}}$$

EXAMPLE: The bandwidth of a series resonant circuit is 400 Hz.
a. If the resonant frequency is 4000 Hz, what is the value of Q_s?
b. If $R = 10\,\Omega$, what is the value of X_L at resonance?
c. Find the inductance L and capacitance C of the circuit.

Solutions:

a. $$BW = \frac{f_s}{Q_s} \quad \text{or} \quad Q_s = \frac{f_s}{BW} = \frac{4000\text{ Hz}}{400\text{ Hz}} = \mathbf{10}$$

b. $$Q_s = \frac{X_L}{R} \quad \text{or} \quad X_L = Q_s R = (10)(10\,\Omega) = \mathbf{100\,\Omega}$$

c. $$X_L = 2\pi f_s L \quad \text{or} \quad L = \frac{X_L}{2\pi f_s} = \frac{100\,\Omega}{2\pi(4000\text{ Hz})} = \mathbf{3.98\text{ mH}}$$

$$X_C = \frac{1}{2\pi f_s C} \quad \text{or} \quad C = \frac{1}{2\pi f_s X_C} = \frac{1}{2\pi(4000\text{ Hz})(100\,\Omega)}$$
$$= \mathbf{0.398\ \mu F}$$

EXAMPLE: A series *R-L-C* circuit has a series resonant frequency of 12,000 Hz.
a. If $R = 5\ \Omega$, and if X_L at resonance is $300\ \Omega$, find the bandwidth.
b. Find the cutoff frequencies.

Solutions:

a. $$Q_s = \frac{X_L}{R} = \frac{300\,\Omega}{5\,\Omega} = 60$$

$$BW = \frac{f_s}{Q_s} = \frac{12{,}000\text{ Hz}}{60} = \mathbf{200\text{ Hz}}$$

b. Since $Q_s \geq 10$, the bandwidth is bisected by f_s. Therefore,

$$f_2 = f_s + \frac{BW}{2} = 12{,}000\text{ Hz} + 100\text{ Hz} = \mathbf{12{,}100\text{ Hz}}$$

and $$f_1 = 12{,}000\text{ Hz} - 100\text{ Hz} = \mathbf{11{,}900\text{ Hz}}$$

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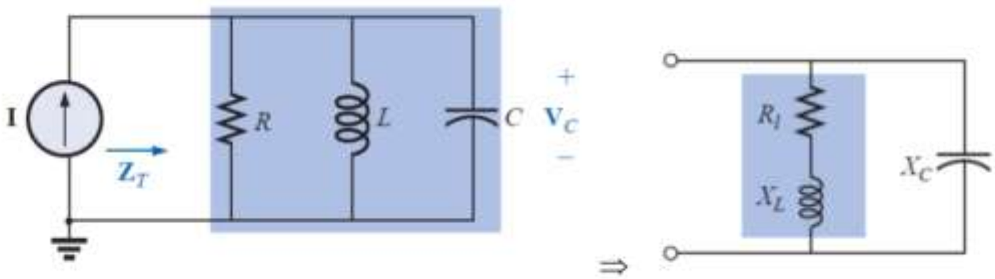
2-PARALLEL RESONANT CIRCUIT



$$R_p = \frac{R_l^2 + X_L^2}{R_l}$$

$$X_{L_p} = \frac{R_l^2 + X_L^2}{X_L}$$

$$X_{L_p} = X_C$$

$$f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}}$$

$$f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}}$$

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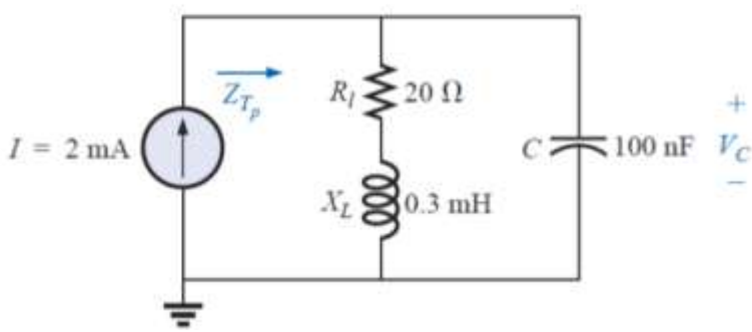
SELECTIVITY CURVE FOR PARALLEL RESONANT CIRCUITS

$$Q_p = \frac{X_L}{R_l} = Q_l$$

$$BW = f_2 - f_1 = \frac{f_r}{Q_p}$$

EXAMPLE: For the parallel resonant circuit

a. Determine *fs*, and *fp*, and compare their levels.
b. Determine the quality factor *Qp*.
c. Calculate the bandwidth.



Solutions:

a. $$f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.3\text{ mH})(100\text{ nF})}} = \mathbf{29.06\text{ kHz}}$$

$$f_p = f_s\sqrt{1 - \frac{R_l^2C}{L}} = (29.06\text{ kHz})\sqrt{1 - \left[\frac{(20\ \Omega)^2(100\text{ nF})}{0.3\text{ mH}}\right]}$$
$$= \mathbf{27.06\text{ kHz}}$$

$$Q_p = \frac{R_s \parallel R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = Q_l = \frac{X_L}{R_l}$$

$$= \frac{2\pi(27.06\text{ kHz})(0.3\text{ mH})}{20\ \Omega} = \frac{51\ \Omega}{20\ \Omega} = \mathbf{2.55}$$

c.

$$BW = \frac{f_p}{Q_p} = \frac{27.06\text{ kHz}}{2.55} = \mathbf{10.61\text{ kHz}}$$

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