Al-Mustaqbal University

College of Engineering & Technology

Biomedical Engineering Department

Subject Name: Mathematics

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Lecture No.: 2

Lecture Title: [Series]

4.7 Power Series

A power series is an expression of the form

$$\sum_{n=0}^{\infty} an\, x^n = ao + a1\, x + a2\, x^2 + \cdots \ldots \ldots .\, an\, x^n$$

an terms is constant but x is averiable whose domain of real number.

1- $if\ y = e^{ax}$. $\frac{d^n y}{dx^n} = y^n$ where n=order derivative→ $y^n = a^n e^{ax}$

Example $find\ dy^7$

1// if $y = e^{2x}$

Soltion/ $y^7 = 2^7 e^{2x} = 128e^{2x}$

2. $if\ y = \sin ax \rightarrow y^n = a^n \sin(ax + n\frac{\pi}{2})$

Example //if $y = \sin 3x$ $find\ dy^5$

Solution //$y^5 = 3^5 \sin\left(3x + 5\frac{\pi}{2}\right) = 243 \sin(3x + 5\frac{\pi}{2})$

3. $if\ y = \cos ax \rightarrow y^n = a^n \cos(ax + n\frac{\pi}{2})$

Example //if $y = 4\cos 2x \qquad find\ y^6$

Solution //$y^6 = 4(2^6\cos\left(2x + 6\frac{\pi}{2}\right) = 256\cos(2x + 6\frac{\pi}{2})$

4. $if\ y = x^a \rightarrow y^n = \frac{a!}{(a-n)!}x^{a-n}$

Example //if $y = 2x^6 \qquad find\ y^4$

Solution //$y^4 = 2\frac{6!}{(6-4)!}x^{6-4} = 2[\frac{6*5*4*3*2*1}{2*1}x^2] = 720\,x^2$

5. $if\ y = ln\ x \rightarrow y^n = (-1)^{n-1} * \frac{(n-1)!}{x^n}$

Example //if $y = ln\ 5x \qquad find\ y^6$

Solution //$y^6 = (-1)^{6-1} * \frac{(6-1)!}{x^6} =$

4.8 Taylor polynomials

the taylor polynomials generated by f(x) at x=0 is ∴

$$fn(\mathrm{x}) = f(0) + \frac{\bar{f}(0)}{1!}x + \frac{\bar{\bar{f}}(0)}{2!}x^2 + \ldots \ldots \frac{f^n(0)}{n!}x^n$$

Example //find the taylor polynomials generated by f(x)=$e^x\ \ at\ x = 0$

soltion//

$$f(x) = e^x. \quad f(0) = e^0 = 1$$

$$\bar{f}(x) = e^x. \qquad \bar{f}(0) = 1$$

$$\bar{\bar{f}}(x) = e^x. \qquad \bar{\bar{f}}(0) = 1$$

The taylor poly. is

$$fn(\mathrm{x}) = 1 + \frac{1}{1!}x + \frac{1}{2!}x^2 + \ldots \ldots \frac{1}{n!}x^n$$

Example //find the taylor polynomials for f(x)= cos x.

Soltion//

$$f(x) = \cos x. \quad f(0) = 1$$

$$\bar{f}(x) = -sin\, x. \qquad \bar{f}(0) = 0$$

$$\bar{\bar{f}}(x) = -\cos x. \qquad \bar{\bar{f}}(0) = -1$$

The taylor poly. is

$$fn(x) = 1 + 0 - \frac{1}{2!}x^2 + \ldots \ldots$$

Taylor series

the taylor series generated by f(x) at x=a is ∴

$$f(a) + \frac{\bar{f}(a)}{1!}(x-a) + \frac{\bar{\bar{f}}(a)}{2!}(x-a)^2 + \ldots \ldots \frac{f^n(a)}{n!}(x-a)^n$$

Example /find the taylor series for f(x)=cos x at x=2π.

Soltion//

$$f(x) = \cos x . \quad f(2\pi) = 1$$

$$\bar{f}(x) = -\sin x . \quad \bar{f}(2\pi) = 0$$

$$\bar{\bar{f}}(x) = -\cos x . \quad \bar{\bar{f}}(2\pi) = -1$$

The taylor series is

$$1 + 0 - \frac{1}{2!}(x - 2\pi)^2 + \ldots\ldots$$

Example /find the taylor series for f(x)= $\frac{1}{x}$ at x=2.

Soltion//

$$f(x) = \frac{1}{x} . \quad f(2) = \frac{1}{2}$$

$$\bar{f}(x) = \frac{-1}{x^2} . \quad \bar{f}(2) = \frac{-1}{4}$$

$$\bar{\bar{f}}(x) = \frac{2}{x^3} . \quad \bar{\bar{f}}(2) = \frac{2}{8} = \frac{1}{4}$$

The taylor series is

$$\frac{1}{2} - \frac{1}{4 * 1!}(x - 2) + \frac{1}{4 * 2!}(x - 2)^2 + \ldots\ldots$$