Subject Name: Calculus I2

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Lecture No. 5

Lecture Title: Hyperbolic Functions

Hyperbolic Functions:

1. $\sinh(x) = \frac{e^x - e^{-x}}{2}$

2. $\cosh(x) = \frac{e^x + e^{-x}}{2}$

3. $\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

4. $\coth(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

5. $sech(x) = \frac{2}{e^x + e^{-x}}$

6. $csch(x) = \frac{2}{e^x - e^{-x}}$

Remarks:

- $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$

- $\coth(x) = \frac{1}{\tanh(x)} = \frac{\cosh(x)}{\sinh(x)}$

- $sec(x) = \frac{1}{\cosh(x)}$

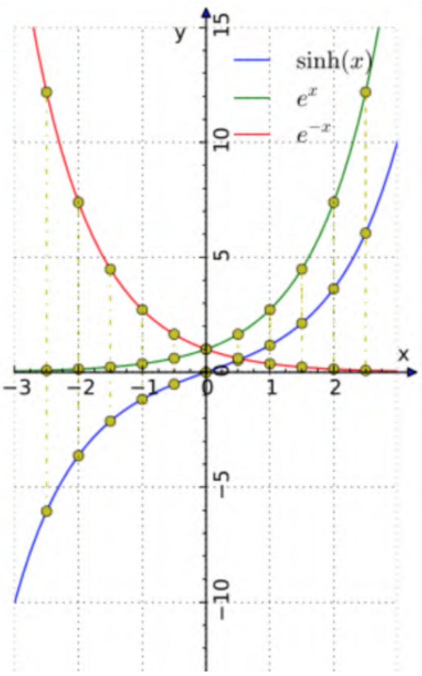
- $csc(x) = \frac{1}{\sinh(x)}$

The Graph of Hyperbolic Functions:

1. $y = \sinh(x)$

   Domain $:= \mathbb{R}$

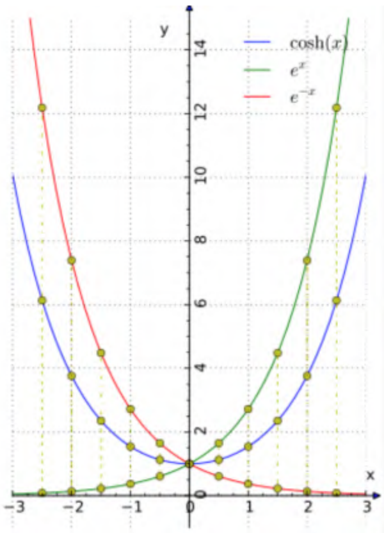
   Range $:= \mathbb{R}$



2. $y = \cosh(x)$

   Domain $:= \mathbb{R}$

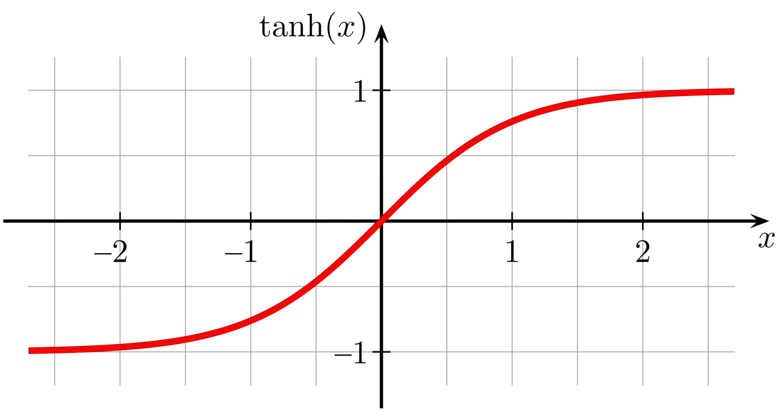
   Range $:= [1, \infty)$



3. $y = \tanh(x)$

   Domain $:= \mathbb{R}$

   Range $:= (-1, 1)$



Some Facts about Hyperbolic Functions:

1. $\cosh^2(x) - \sinh(x) = 1$

2. $1 - \tanh^2(x) = sech^2(x)$

3. $\coth^2(x) - 1 = csch^2(x)$

4. $\cosh(-x) = \cosh(x)$ " Even Function",

   $\sinh(-x) = -\sinh(x)$ " Odd Function",

   $\tan(-x) = -\tan(x)$" Odd Function"

5. $\cosh(x) + \sinh(x) = e^x$,

   $\cosh(x) - \sinh(x) = e^{-x}$

6. $\cosh(x + y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$,

   $\sinh(x + y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$,

   $\tanh(x + y) = \frac{\tanh(x)+\tanh(y)}{1-\tanh(x)\tanh(y)}$

7. $\cosh^2(x) = \frac{1}{2}(\cosh(2x) + 1)$,

   $\sinh^2(x) = \frac{1}{2}(\cosh(2x) - 1)$

8. $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$,

   $\sinh(2x) = 2\sinh(x)\cosh(x)$

The Derivative of Hyperbolic Functions:

Let u be a function of x, then:

1. $\frac{d}{dx}\Big(sinh(u)\Big) = cosh(u).\frac{du}{dx}$

2. $\frac{d}{dx}\Big(cosh(u)\Big) = sinh(u).\frac{du}{dx}$

3. $\frac{d}{dx}\Big(tanh(u)\Big) = sech^2(u).\frac{du}{dx}$

4. $\frac{d}{dx}\Big(coth(u)\Big) = -csch^2(u).\frac{du}{dx}$

5. $\frac{d}{dx}\Big(sech(u)\Big) = -sech(u).tanh(u).\frac{du}{dx}$

6. $\frac{d}{dx}\Big(csch(u)\Big) = -csch(u).coth(u).\frac{du}{dx}$

Examples: Find the derivatives of the following functions:

- $\sinh(3x)$

  $\Longrightarrow\ y' = 3\cosh(3x)$

- $y = \cosh^2(5x)$

  $\Longrightarrow\ y' = 2\cosh(5x).\sinh(5x).5$

- $\tanh(2x)$

  $\Longrightarrow\ y' = sech^2(2x).2$

- $y = \coth(\tan(x))$

  $\Longrightarrow \; y' = -csch^2(\tan x).\sec^2 x$

- $y = sech^3 x$

  $\Longrightarrow \; y' = 3sech^2(x).(-sech(x)\tanh(x).1)$

- $y = 4csch(\frac{x}{4})$

  $\Longrightarrow \; y' = 4.(-csch(\frac{x}{4})).\coth(\frac{x}{4}).\frac{1}{4}$

Problems (6.3): Find y' of the following:

1. $y = \frac{\cosh(x)}{x}$
2. $y = e^w.\cosh(w)$
3. $y = \tanh(\frac{4t+1}{5})$
4. $y = \tanh^{-1}(\frac{1}{x})$
5. $y = \coth(\frac{1}{\theta})$
6. $y = \cosh^2(5x) - \sinh^2(5x)$
7. $\sinh(y) = \tanh(x)$
8. $y = \sinh^2(3w)$
9. $\sin^{-1}(x) = sech(y)$
10. $\tan(x) = \tanh^2(y)$
11. $\sinh(y) = \sec(x)$
12. $y^2 + x\cosh y + \sinh^2 x = 50$
13. $y = csch^3(\sqrt{2x})$
14. $x = \cosh(\cos(y))$
