قسـم الامـن الـسيبراني

# Department of Cyber Security

## Subject:

## Sequences of sets

## Class:

## First

## Lecturer:

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# Lecture: (6)

## Sequences of sets

A sequence is a discrete structure used to represent an ordered list.For example,

1, 2, 3, 5, 8 is a sequence with five terms (called a *list*)

1, 3, 9, 27, 81 , . . . , 3*n*, . . . is an infinite sequence.

A *sequence* is a function from subset of the set of integers (usually either the set {0, 1, 2, . . .} or the set {1, 2, 3, . . .}) to a set *S*.. The notation *an* is used to denote the image of the integer n that called the term of the sequence and used to describe the sequence . Thus a sequence is usually denoted by *a*1, *a*2, *a*3, . . .

We describe sequences by listing the terms of the sequence in order of increasing subscripts.

**EXAMPLE 1**

Consider the sequence {*an*}, where

$$a_n = \frac{1}{n};$$

The list of the terms of this sequence, beginning with *a*1, namely,

$$a1,\ a2,\ a3,\ a4,\ .\ .\ .\ ,$$

**EXAMPLE 2**

a) The sequences {bn} with bn = $(-1)^n$

if we start at n = 0, the list of terms beg

starts with

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ \ldots\ .$$

ins with 1,−1, 1,−1, 1, . . .

b) The sequences {cn} with cn = 2 × $5^n$

if we start at n = 0, the list of terms begins with

2, 10, 50, 250, 1250, . . .

c) The sequences {dn} with dn = 6 × $(\frac{1}{3})^n$

if we start at n = 0, The list of terms begins with

$$6,\ 2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots\ldots$$

d- The sequences {bn} with bn = $2^{-n}$

if we start at n = 0, The list of terms begins with

$$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots.$$

e- The sequences {bn} with $a_n = \frac{1}{n}$

if we start at n = 1, The list of terms begins with

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots.$$

## RECURSIVELY DEFINED FUNCTIONS

A function is said to be *recursively defined* if the function definition refers to itself. In order for the definition not to be circular, the function definition must have the following two properties:

(1) There must be certain arguments, called *base values*, for which the function does not refer to itself.

(2) Each time the function does refer to itself, the argument of the function must be closer to a base value. A recursive function with these two properties is said to be *well- defined*.

## Factorial Function

The product of the positive integers from 1 to *n*, inclusive, is called ―*n* factorial‖ and is usually denoted by *n*!. That is,

$n! = n(n-1)(n-2) \cdot \cdot \cdot 3 \cdot 2 \cdot 1$

where

0! = 1, so that the function is defined for all nonnegative integers. Thus:

We have: f(0) = 0! = 1

f (1) = 1! = 1,

f (2) = 2! = 1 · 2 = 2,

f (6) = 6! = 1 · 2 · 3 · 4 · 5 · 6 = 720,

and

f (20) = 1x 2 x3x 4 x 5 x 6 x 7 x 8 x 9 x 10 x 11x 12 x 13 x 14 x 15x16 x 17 x 18 x 19 x20 = 2,432,902,008,176,640,000.

the factorial function grows extremely rapidly as *n* grows.

This is true for every positive integer $n$; that is,

$n! = n \cdot (n-1)!$

Accordingly, the factorial function may also be defined as follows:

**Definition of Factorial Function:**

(a) If $n = 0$, then $n! = 1$.

(b) If $n > 0$, then $n! = n \cdot (n-1)!$

The definition of $n!$ is recursive, since it refers to itself when it uses $(n-1)!$. However:

(1) The value of $n!$ is explicitly given when $n = 0$ (thus 0 is a base value).

(2) The value of $n!$ for arbitrary $n$ is defined in terms of a smaller value of $n$ which is closer to the base value 0.

Accordingly, the definition is not circular, or, in other words, the function is well-defined.

**EXAMPLE 7:** the 4! Can be calculated in 9 steps using the recursive definition .

(1) $4! = 4 \cdot 3!$

(2) $3! = 3 \cdot 2!$

(3) $2! = 2 \cdot 1!$

(4) $1! = 1 \cdot 0!$

(5) $0! = 1$

(6) $1! = 1 \cdot 1 = 1$

(7) $2! = 2 \cdot 1 = 2$

(8) $3! = 3 \cdot 2 = 6$

(9) $4! = 4 \cdot 6 = 24$