Module Title: Fundamental of Electrical Engineering (AC)

Module Code:	**UOMU024021**

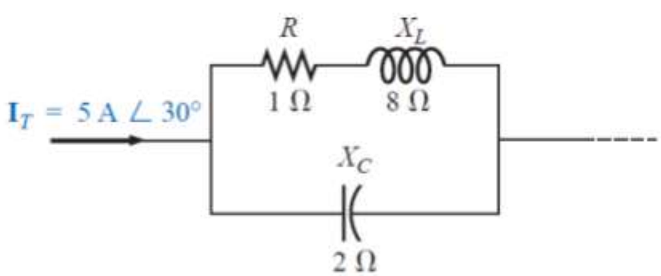
Week 7, and Week 8

Week 7:

Series-Parallel AC Network

The combination of RLC circuits contains current divider or voltage divider. The current through each parallel branch depends on the load. The voltage through each parallel branch is equal potential.

EXAMPLE 1 : Using the current divider rule, find the current through each parallel branch.



Solutions:

$$\mathbf{I}_{R\text{-}L} = \frac{\mathbf{Z}_C \mathbf{I}_T}{\mathbf{Z}_C + \mathbf{Z}_{R\text{-}L}} = \frac{(2\ \Omega \angle -90^\circ)(5\ \text{A} \angle 30^\circ)}{-j\,2\ \Omega + 1\ \Omega + j\,8\ \Omega} = \frac{10\ \text{A} \angle -60^\circ}{1 + j\,6}$$

$$= \frac{10\ \text{A} \angle -60^\circ}{6.083 \angle 80.54^\circ} \cong \mathbf{1.644\ A \angle -140.54^\circ}$$

$$\mathbf{I}_C = \frac{\mathbf{Z}_{R\text{-}L} \mathbf{I}_T}{\mathbf{Z}_{R\text{-}L} + \mathbf{Z}_C} = \frac{(1\ \Omega + j\,8\ \Omega)(5\ \text{A} \angle 30^\circ)}{6.08\ \Omega \angle 80.54^\circ}$$

$$= \frac{(8.06 \angle 82.87^\circ)(5\ \text{A} \angle 30^\circ)}{6.08 \angle 80.54^\circ} = \frac{40.30\ \text{A} \angle 112.87^\circ}{6.083 \angle 80.54^\circ}$$

$$= \mathbf{6.625\ A \angle 32.33^\circ}$$

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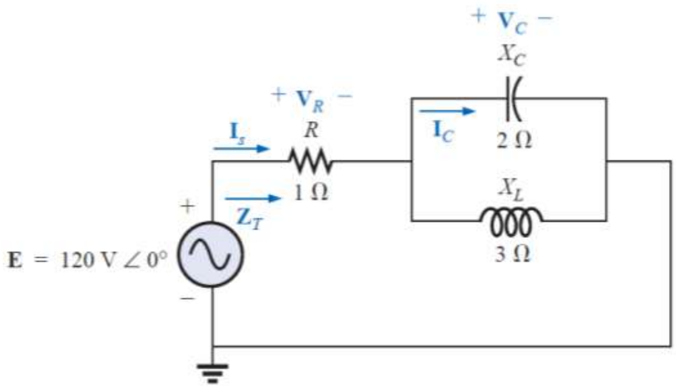
Series-Parallel AC Networks examples:

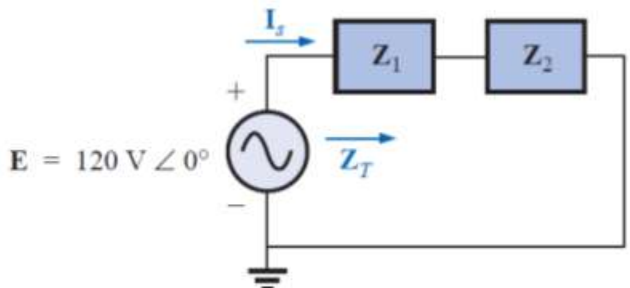
EXAMPLE 1: For the network shown below.

a. Calculate $\mathbf{Z}_T$.
b. Determine $\mathbf{I}_s$.
c. Calculate $\mathbf{V}_R$ and $\mathbf{V}_C$.
d. Find $\mathbf{I}_C$.



Solutions:

a)



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$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2$$
$$\mathbf{Z}_1 = R\angle 0° = 1\ \Omega\angle 0°$$
$$\mathbf{Z}_2 = \mathbf{Z}_C \parallel \mathbf{Z}_L = \frac{(X_C\angle -90°)(X_L\angle 90°)}{-jX_C + jX_L} = \frac{(2\ \Omega\angle -90°)(3\ \Omega\angle 90°)}{-j\,2\ \Omega + j\,3\ \Omega}$$
$$= \frac{6\ \Omega\angle 0°}{j\,1} = \frac{6\ \Omega\angle 0°}{1\angle 90°} = 6\ \Omega\angle -90°$$
$$\mathbf{Z}_T = \mathbf{Z}_1 + \mathbf{Z}_2 = 1\ \Omega - j\,6\ \Omega = \mathbf{6.08\ \Omega\angle -80.54°}$$

b)

$$\mathbf{I}_s = \frac{\mathbf{E}}{\mathbf{Z}_T} = \frac{120\text{ V}\angle 0°}{6.08\ \Omega\angle -80.54°} = \mathbf{19.74\ A\angle 80.54°}$$

c)

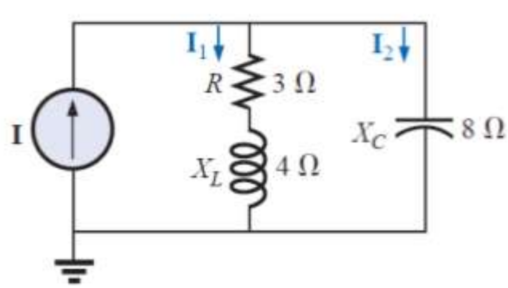
$$\mathbf{V}_R = \mathbf{I}_s\mathbf{Z}_1 = (19.74\text{ A}\angle 80.54°)(1\ \Omega\angle 0°) = \mathbf{19.74\ V\angle 80.54°}$$
$$\mathbf{V}_C = \mathbf{I}_s\mathbf{Z}_2 = (19.74\text{ A}\angle 80.54°)(6\ \Omega\angle -90°)$$
$$= \mathbf{118.44\ V\angle -9.46°}$$

d)

$$\mathbf{I}_C = \frac{\mathbf{V}_C}{\mathbf{Z}_C} = \frac{118.44\text{ V}\angle -9.46°}{2\ \Omega\angle -90°} = \mathbf{59.22\ A\angle 80.54°}$$

EXAMPLE 2: For the network shown below.

a. If $\mathbf{I}$ is 50 A $\angle 30°$, calculate $\mathbf{I}_1$ using the current divider rule.
b. Repeat part (a) for $\mathbf{I}_2$.
c. Verify Kirchhoff's current law at one node.



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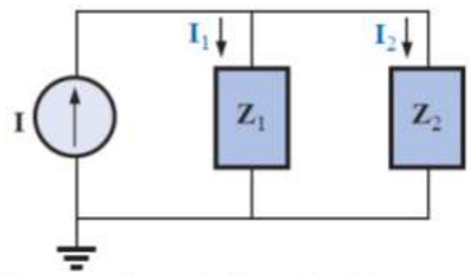
Solutions:

a)



$$\mathbf{Z}_1 = R + j\,X_L = 3\ \Omega + j\,4\ \Omega = 5\ \Omega \angle 53.13^\circ$$
$$\mathbf{Z}_2 = -j\,X_C = -j\,8\ \Omega = 8\ \Omega \angle -90^\circ$$
$$\mathbf{I}_1 = \frac{\mathbf{Z}_2\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(8\ \Omega \angle -90^\circ)(50\ \text{A} \angle 30^\circ)}{(-j\,8\ \Omega) + (3\ \Omega + j\,4\ \Omega)} = \frac{400 \angle -60^\circ}{3 - j\,4}$$
$$= \frac{400 \angle -60^\circ}{5 \angle -53.13^\circ} = \mathbf{80\ A \angle -6.87^\circ}$$

b)

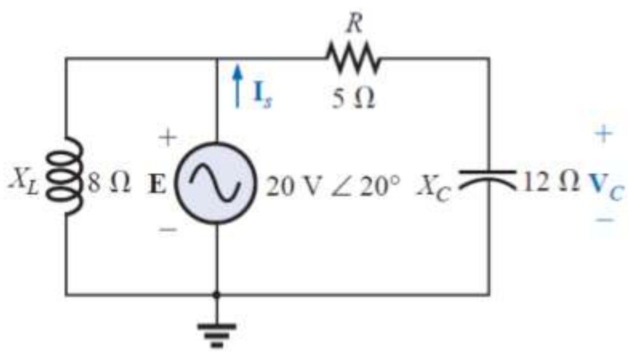
$$\mathbf{I}_2 = \frac{\mathbf{Z}_1\mathbf{I}}{\mathbf{Z}_2 + \mathbf{Z}_1} = \frac{(5\ \Omega \angle 53.13^\circ)(50\ \text{A} \angle 30^\circ)}{5\ \Omega \angle -53.13^\circ} = \frac{250 \angle 83.13^\circ}{5 \angle -53.13^\circ}$$
$$= \mathbf{50\ A \angle 136.26^\circ}$$

c)

$$\mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2$$
$$50\ \text{A} \angle 30^\circ = 80\ \text{A} \angle -6.87^\circ + 50\ \text{A} \angle 136.26^\circ$$
$$= (79.43 - j\,9.57) + (-36.12 + j\,34.57)$$
$$= 43.31 + j\,25.0$$
$$50\ \text{A} \angle 30^\circ = 50\ \text{A} \angle 30^\circ \quad \text{(checks)}$$

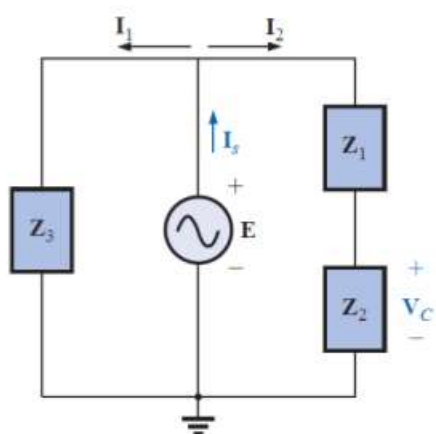
EXAMPLE 3: For the network shown below .

a. Calculate the voltage $\mathbf{V}_C$ using the voltage divider rule.
b. Calculate the current $\mathbf{I}_s$.



Solutions:

a)



$$\mathbf{V}_C = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(12\ \Omega\ \angle -90^\circ)(20\ \text{V}\ \angle 20^\circ)}{5\ \Omega - j\ 12\ \Omega} = \frac{240\ \text{V}\ \angle -70^\circ}{13\ \angle -67.38^\circ}$$
$$= \mathbf{18.46\ V\ \angle -2.62^\circ}$$

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b)

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_3} = \frac{20\text{ V}\angle 20^\circ}{8\ \Omega\angle 90^\circ} = 2.5\text{ A}\angle -70^\circ$$

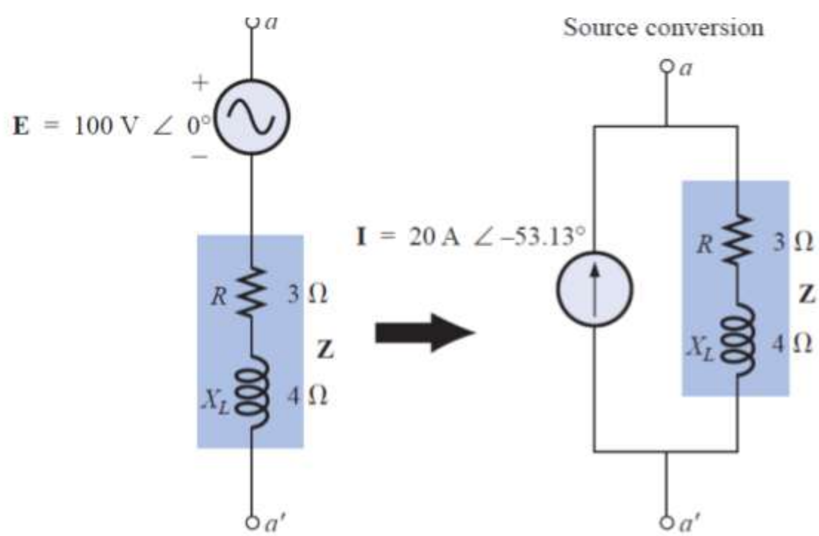
$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_1+\mathbf{Z}_2} = \frac{20\text{ V}\angle 20^\circ}{13\ \Omega\angle -67.38^\circ} = 1.54\text{ A}\angle 87.38^\circ$$

$$\begin{aligned}\mathbf{I}_s &= \mathbf{I}_1 + \mathbf{I}_2\\ &= 2.5\text{ A}\angle -70^\circ + 1.54\text{ A}\angle 87.38^\circ\\ &= (0.86 - j\,2.35) + (0.07 + j\,1.54)\end{aligned}$$

$$\mathbf{I}_s = 0.93 - j\,0.81 = \mathbf{1.23\text{ A}\angle -41.05^\circ}$$

Methods of Analysis (AC):

EXAMPLE 1 : Convert the *<u>voltage</u>* source to a *<u>current</u>* source.



Solutions:

$$\mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}} = \frac{100\text{ V}\angle 0^\circ}{5\ \Omega\angle 53.13^\circ} = \mathbf{20\text{ A}\angle -53.13^\circ}$$

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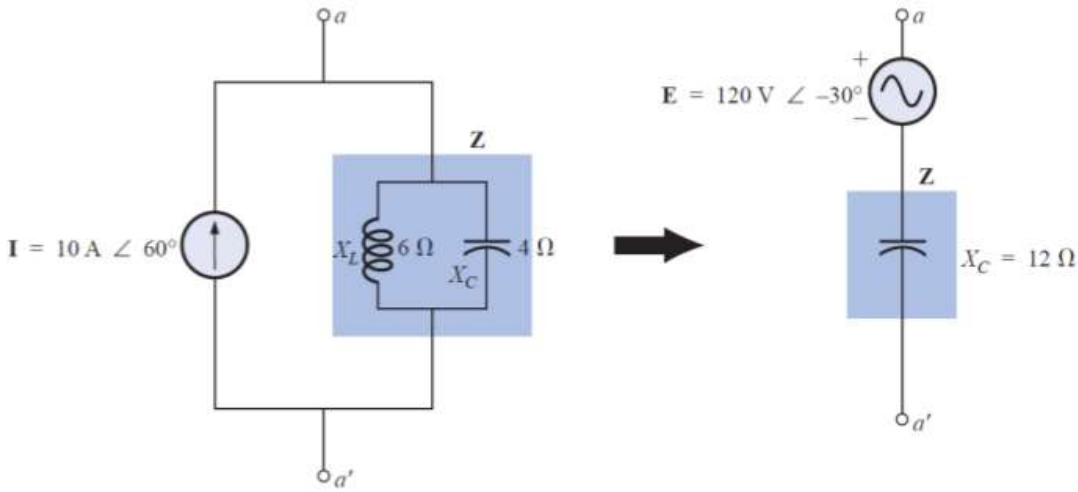
EXAMPLE 2: Convert the current source to a voltage source.



Solutions:

$$\mathbf{Z} = \frac{\mathbf{Z}_C\mathbf{Z}_L}{\mathbf{Z}_C + \mathbf{Z}_L} = \frac{(X_C \angle -90^\circ)(X_L \angle 90^\circ)}{-j\,X_C + j\,X_L}$$

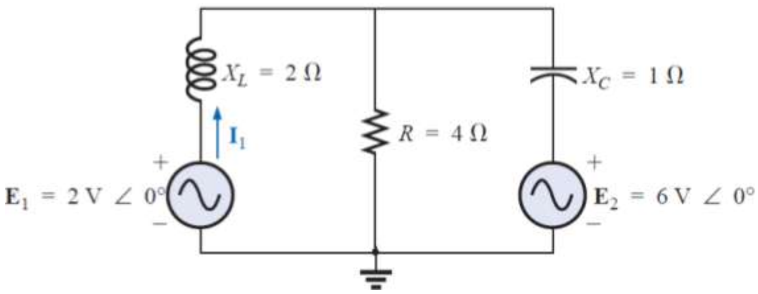
$$= \frac{(4\ \Omega \angle -90^\circ)(6\ \Omega \angle 90^\circ)}{-j\,4\ \Omega + j\,6\ \Omega} = \frac{24\ \Omega \angle 0^\circ}{2 \angle 90^\circ}$$

$$= \mathbf{12\ \Omega \angle -90^\circ}$$

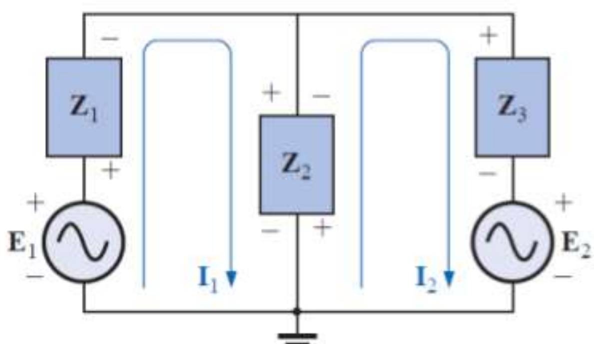
$$\mathbf{E} = \mathbf{IZ} = (10\ \text{A} \angle 60^\circ)(12\ \Omega \angle -90^\circ)$$

$$= \mathbf{120\ V \angle -30^\circ}$$

EXAMPLE 3: Using the general approach to mesh analysis, find the current I_1 in the network shown below .



Solutions:



$$Z_1 = +j X_L = +j\,2\ \Omega \qquad E_1 = 2\ V \angle 0°$$
$$Z_2 = R = 4\ \Omega \qquad E_2 = 6\ V \angle 0°$$
$$Z_3 = -j X_C = -j\,1\ \Omega$$

$$\begin{aligned} I_1(Z_1 + Z_2) - I_2 Z_2 &= E_1 \\ I_2(Z_2 + Z_3) - I_1 Z_2 &= -E_2 \end{aligned}$$

$$\begin{aligned} I_1(Z_1 + Z_2) - I_2 Z_2 \qquad\qquad &= E_1 \\ -I_1 Z_2 \qquad + I_2(Z_2 + Z_3) &= -E_2 \end{aligned}$$

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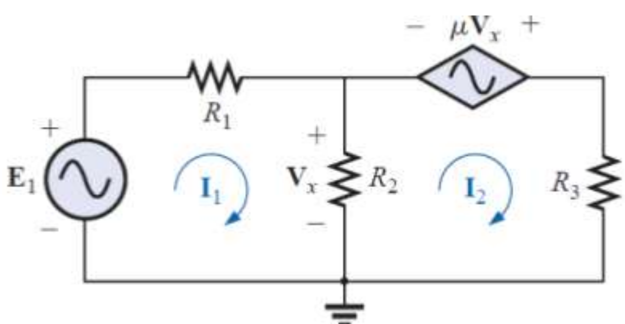
$$\mathbf{I}_1 = \frac{\begin{vmatrix} \mathbf{E}_1 & -\mathbf{Z}_2 \\ -\mathbf{E}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{Z}_1 + \mathbf{Z}_2 & -\mathbf{Z}_2 \\ -\mathbf{Z}_2 & \mathbf{Z}_2 + \mathbf{Z}_3 \end{vmatrix}} = \frac{\mathbf{E}_1(\mathbf{Z}_2 + \mathbf{Z}_3) - \mathbf{E}_2(\mathbf{Z}_2)}{(\mathbf{Z}_1 + \mathbf{Z}_2)(\mathbf{Z}_2 + \mathbf{Z}_3) - (\mathbf{Z}_2)^2} = \frac{(\mathbf{E}_1 - \mathbf{E}_2)\mathbf{Z}_2 + \mathbf{E}_1\mathbf{Z}_3}{\mathbf{Z}_1\mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_3 + \mathbf{Z}_2\mathbf{Z}_3}$$

$$\mathbf{I}_1 = \frac{(2\text{ V} - 6\text{ V})(4\ \Omega) + (2\text{ V})(-j\,1\ \Omega)}{(+j\,2\ \Omega)(4\ \Omega) + (+j\,2\ \Omega)(-j\,2\ \Omega) + (4\ \Omega)(-j\,2\ \Omega)}$$

$$= \frac{-16 - j\,2}{j\,8 - j^2 2 - j\,4} = \frac{-16 - j\,2}{2 + j\,4} = \frac{16.12\text{ A}\angle -172.87^\circ}{4.47\angle 63.43^\circ}$$

$$= \mathbf{3.61\text{ A}\angle -236.30^\circ} \quad \text{or} \quad \mathbf{3.61\text{ A}\angle 123.70^\circ}$$

EXAMPLE 4: Write the mesh currents for the network having a dependent voltage source.



Solutions:

$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I}_1 - \mathbf{I}_2) = 0$$

$$R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu\mathbf{V}_x - \mathbf{I}_2 R_3 = 0 \qquad \mathbf{V}_x = (\mathbf{I}_1 - \mathbf{I}_2)R_2$$

$$\mathbf{E}_1 - \mathbf{I}_1 R_1 - R_2(\mathbf{I} - \mathbf{I}_2) = 0$$

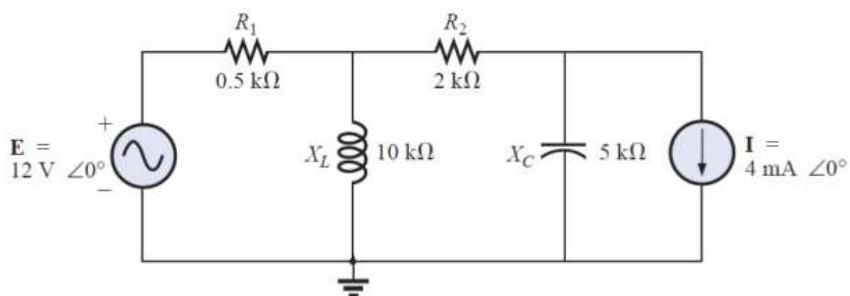
$$\underline{R_2(\mathbf{I}_2 - \mathbf{I}_1) + \mu R_2(\mathbf{I}_1 - \mathbf{I}_2) - \mathbf{I}_2 R_3 = 0}$$

Week 8:

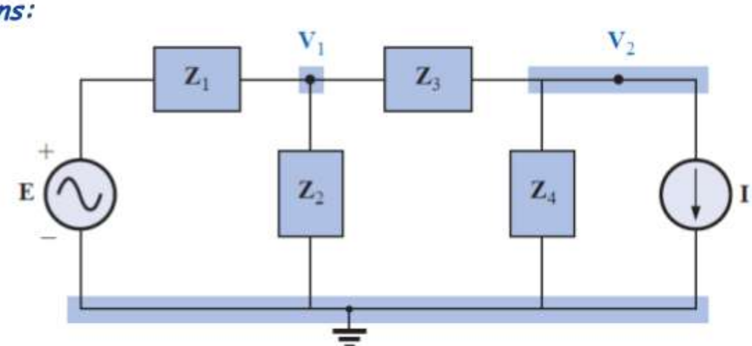
Methods of Analysis (AC) Network :

Different network "Nodal Equations" for the network having a dependent current source.

EXAMPLE 5: Write the nodal equations for the network having a dependent current source shown below.



Solutions:



$$\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}+\frac{1}{\mathbf{Z}_3}\right]-\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}\right]=\frac{\mathbf{E}_1}{\mathbf{Z}_1}$$

$$\mathbf{V}_2\left[\frac{1}{\mathbf{Z}_3}+\frac{1}{\mathbf{Z}_4}\right]-\mathbf{V}_1\left[\frac{1}{\mathbf{Z}_3}\right]=-\mathbf{I}$$

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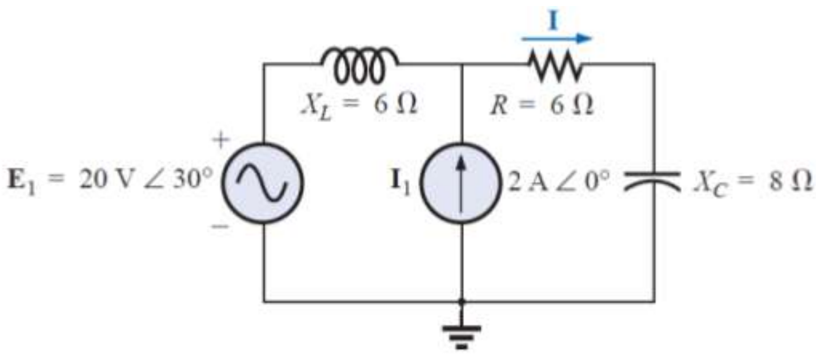
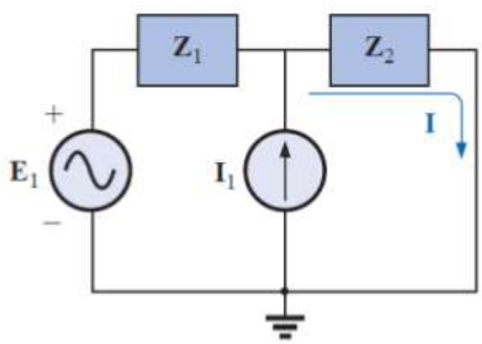
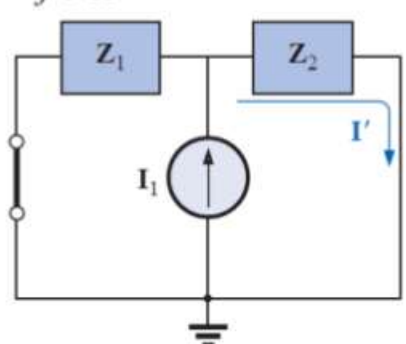
EXAMPLE 6 : Using superposition, find the current I through the 6 Ω resistor shown in the network below.



Solutions:



$Z_1 = j\,6\ \Omega \qquad Z_2 = 6 - j\,8\ \Omega$



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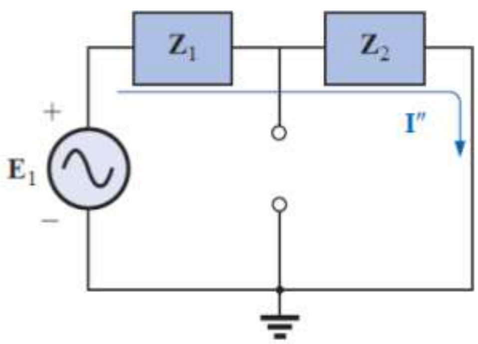
$$\mathbf{I}' = \frac{\mathbf{Z}_1\mathbf{I}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,6\,\Omega)(2\,\text{A})}{j\,6\,\Omega + 6\,\Omega - j\,8\,\Omega} = \frac{j\,12\,\text{A}}{6 - j\,2}$$

$$= \frac{12\,\text{A}\,\angle 90^\circ}{6.32\,\angle -18.43^\circ}$$

$$\mathbf{I}' = 1.9\,\text{A}\,\angle 108.43^\circ$$



$$\mathbf{I}'' = \frac{\mathbf{E}_1}{\mathbf{Z}_T} = \frac{\mathbf{E}_1}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{20\,\text{V}\,\angle 30^\circ}{6.32\,\Omega\,\angle -18.43^\circ}$$

$$= 3.16\,\text{A}\,\angle 48.43^\circ$$

$$\mathbf{I} = \mathbf{I}' + \mathbf{I}''$$

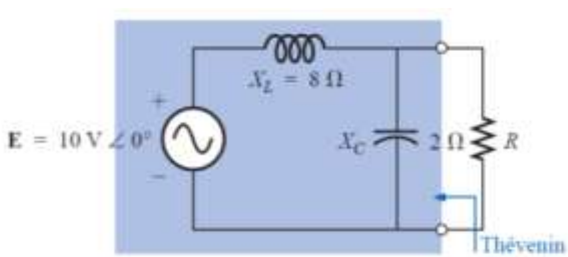
$$= 1.9\,\text{A}\,\angle 108.43^\circ + 3.16\,\text{A}\,\angle 48.43^\circ$$

$$= (-0.60\,\text{A} + j\,1.80\,\text{A}) + (2.10\,\text{A} + j\,2.36\,\text{A})$$

$$= 1.50\,\text{A} + j\,4.16\,\text{A}$$

$$\mathbf{I} = \mathbf{4.42\,A\,\angle 70.2^\circ}$$

EXAMPLE 7 : Find the Thévenin equivalent circuit for the network external to resistor R for the network shown below.



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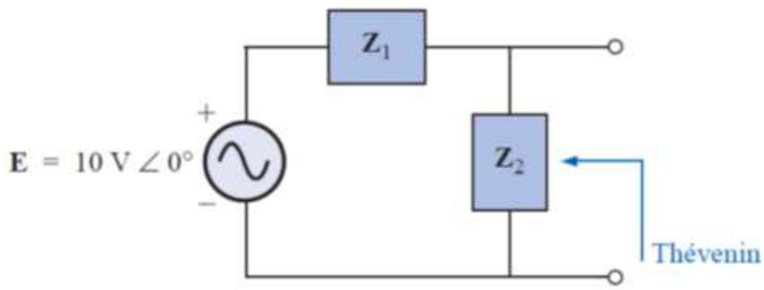
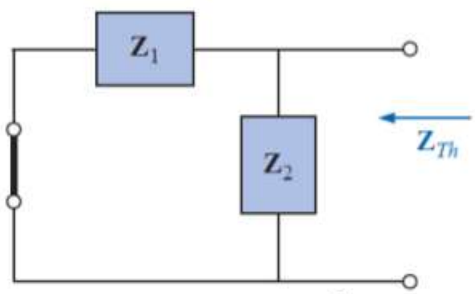
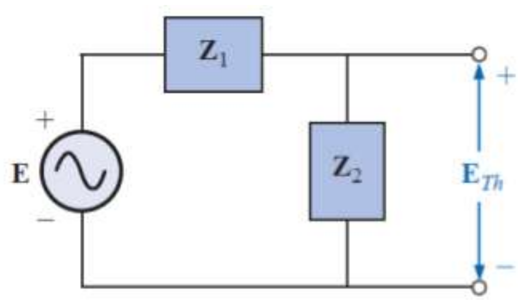
Solutions:



$$\mathbf{Z}_1 = jX_L = j\,8\,\Omega \qquad \mathbf{Z}_2 = -jX_C = -j\,2\,\Omega$$



$$\mathbf{Z}_{Th} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(j\,8\,\Omega)(-j\,2\,\Omega)}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j^2 16\,\Omega}{j\,6} = \frac{16\,\Omega}{6\,\angle 90^\circ}$$
$$= \mathbf{2.67\,\Omega\,\angle -90^\circ}$$



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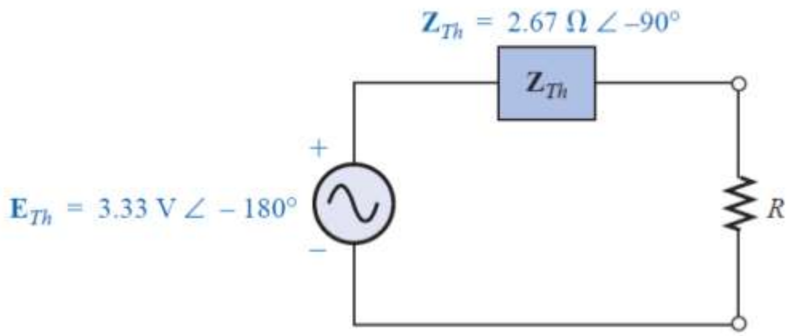
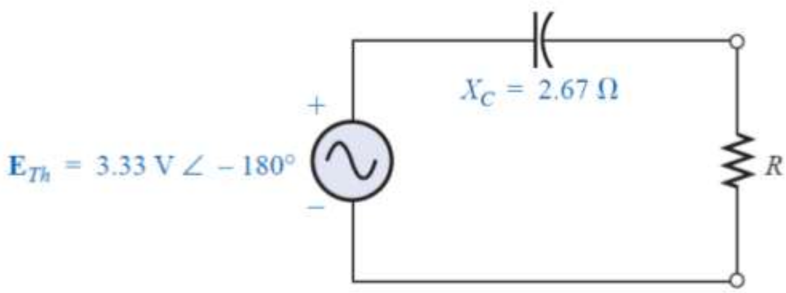
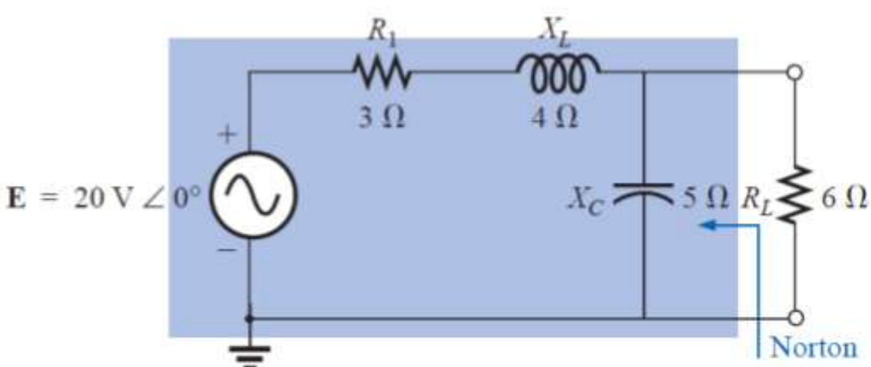
$$\mathbf{E}_{Th} = \frac{\mathbf{Z}_2\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} \qquad \text{(voltage divider rule)}$$

$$= \frac{(-j\,2\,\Omega)(10\text{ V})}{j\,8\,\Omega - j\,2\,\Omega} = \frac{-j\,20\text{ V}}{j\,6} = \mathbf{3.33\text{ V}\angle -180^\circ}$$





EXAMPLE 8: Determine the Norton equivalent circuit for the network external to the 6 Ω resistor for the network shown below.



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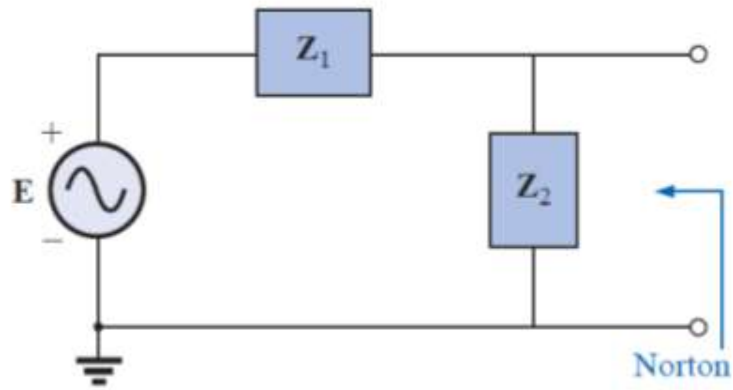
Solutions:



$$\mathbf{Z}_1 = R_1 + j X_L = 3\ \Omega + j\,4\ \Omega = 5\ \Omega\ \angle 53.13^\circ$$

$$\mathbf{Z}_2 = -j X_C = -j\,5\ \Omega$$

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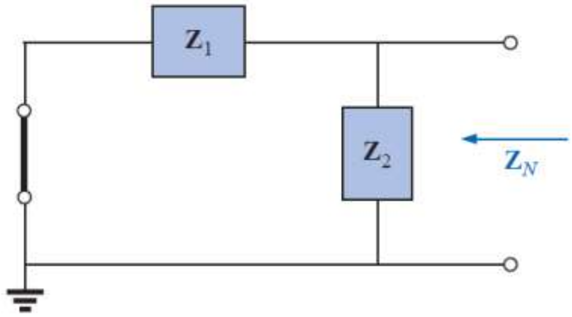
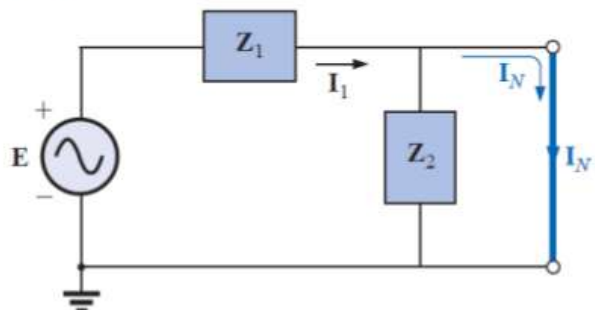
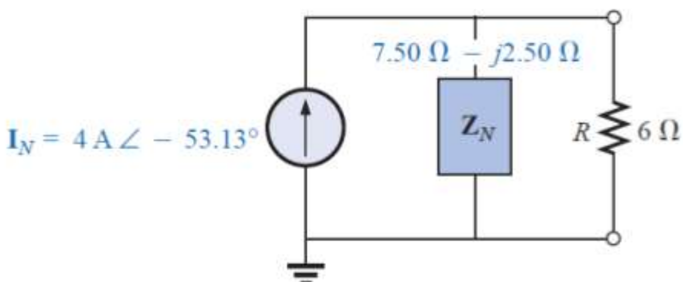
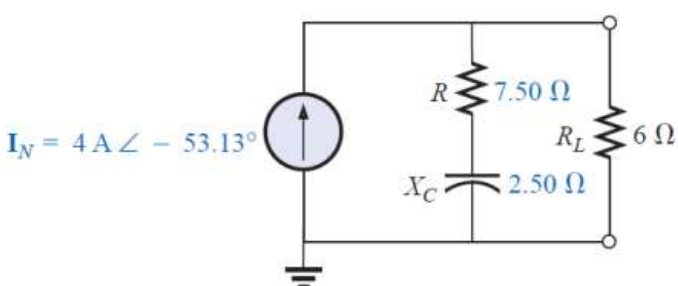


$$\mathbf{Z}_N = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(5\ \Omega \angle 53.13^\circ)(5\ \Omega \angle -90^\circ)}{3\ \Omega + j\,4\ \Omega - j\,5\ \Omega} = \frac{25\ \Omega \angle -36.87^\circ}{3 - j\,1}$$

$$= \frac{25\ \Omega \angle -36.87^\circ}{3.16 \angle -18.43^\circ} = 7.91\ \Omega \angle -18.44^\circ = \mathbf{7.50\ \Omega - j\,2.50\ \Omega}$$







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