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جامـــــعة المـــــستـقـبـل
AL MUSTAQBAL UNIVERSITY

كلية العلـوم
قســـم الادلة الجنائية

المحاضرة الثامنة

Partial Fractions

المادة : حساب التفاضل والتكامل
المرحلة : الاولى
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Partial fractions:

Success in separating $\frac{f(x)}{g(x)}$ into a sum of partial fractions hinges on two things:-

1- The degree of $f(x)$ must be less than the degree of $g(x)$. (If this is not case, we first perform a long division, and then work with the remainder term).

2- The factors of $g(x)$ must be known. If these two conditions are met we can carry out the following steps:

Step I - let $x-r$ be a linear factor of $g(x)$. Suppose $(x-r)^m$ is the highest power of $(x-r)$ that divides $g(x)$. Then assign the sum of m partial factors to this factor, as follows:

$$\frac{A_1}{x-r}+\frac{A_2}{(x-r)^2}+\dots\dots\dots+\frac{A_m}{(x-r)^m}$$

Do this for each distinct linear factor of $f(x)$.

Step II - let x^2+px+q be an irreducible quadratic factor of $g(x)$. Suppose $(x^2+px+q)^n$ is the highest power of this factor that divides $g(x)$. Then, to this factor, assign the sum of the n partial fractions:

$$\frac{B_1x+C_1}{x^2+px+q}+\frac{B_2x+C_2}{(x^2+px+q)^2}+\dots\dots\dots+\frac{B_nx+C_n}{(x^2+px+q)^n}$$

Do this for each distinct linear factor of $g(x)$.

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Step III - set the original fraction $\frac{f(x)}{g(x)}$ equal to the sum of all these partial fractions. Clear the resulting equation of fractions and arrange the sums in decreasing powers of x.

Step IV - equate the coefficients of corresponding powers of x and solve the resulting equations for the undetermined coefficients.

EX-8 **– Evaluate the following integrals:**

1) $\int \frac{2x+5}{x^2-9}dx$

2) $\int \frac{x\,dx}{x^2+4x+3}$

3) $\int \frac{x^3-x}{(x^2+1)\cdot(x-1)^2}dx$

4) $\int \frac{\sin x\ dx}{\cos^2 x-5\cos x+4}$

5) $\int \frac{2x^2-3x+2}{(x-1)^2(x-2)}dx$

6) $\int \frac{x^3+4x^2}{x^2+4x+3}dx$

Sol.-

1) $\int \frac{2x+5}{x^2-9}dx = \int \frac{2x+5}{(x-3)\cdot(x+3)}dx$

$$\frac{2x+5}{(x-3)\cdot(x+3)} = \frac{A}{x-3}+\frac{B}{x+3} \quad \Rightarrow \quad 2x+5 = A(x+3)+B(x-3)$$

$$at \quad x=3 \quad \Rightarrow \quad 6A = 6+5 \quad \Rightarrow \quad A = \frac{11}{6}$$

$$at \quad x=-3 \quad \Rightarrow \quad -6B = -6+5 \quad \Rightarrow \quad B = \frac{1}{6}$$

$$\int \frac{2x+5}{x^2-9}dx = \int\left(\frac{11/6}{x-3}+\frac{1/6}{x+3}\right)dx = \frac{11}{6}ln(x-3)+\frac{1}{6}ln(x+3)+c$$

2) $\int \frac{x\,dx}{x^2+4x+3} = \int \frac{x\,dx}{(x+3)(x+1)}$

$$\frac{x}{(x+3)(x+1)} = \frac{A}{x+3}+\frac{B}{x+1} \quad \Rightarrow \quad x = A(x+1)+B(x+3)$$

$$at \quad x=-3 \quad \Rightarrow \quad A=\frac{3}{2} \quad and \quad at \quad x=-1 \quad \Rightarrow \quad B=-\frac{1}{2}$$

$$\int \frac{x\,dx}{x^2+4x+3} = \int\left(\frac{3/2}{x+3}+\frac{-1/2}{x+1}\right)dx = \frac{3}{2}ln(x+3)-\frac{1}{2}ln(x+1)+c$$

3) $\int \frac{x^3-x}{(x^2+1)(x-1)^2}dx = \int \frac{x(x-1)(x+1)}{(x^2+1)(x-1)^2}dx = \int \frac{x^2+x}{(x^2+1)(x-1)}dx$

$$\frac{x^2+x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} \quad \Rightarrow \quad x^2+x = (Ax+B)(x-1)+C(x^2+1)$$

$$x^2+x = (A+C)x^2 + (-A+B)x + (-B+C)$$

$$\left.\begin{aligned} A+C=1 \; ......(1) \\ -A+B=1 \; ......(2) \\ -B+C=0 \; ......(3) \end{aligned}\right\} \Rightarrow \quad A=0 \;,\; B=1 \;,\; C=1$$

$$\int \frac{x^3-x}{(x^2+1)(x-1)^2}dx = \int \left(\frac{1}{x^2+1} + \frac{1}{x-1}\right)dx = \tan^{-1}x + \ln(x-1) + c$$

4) let $y=\cos x \quad \Rightarrow \quad dy = -\sin x\, dx$

$$\int \frac{\sin x\; dx}{\cos^2 x - 5\cos x + 4} = -\int \frac{dy}{y^2-5y+4} = -\int \frac{dy}{(y-4)(y-1)}$$

$$\frac{dy}{(y-4)(y-1)} = \frac{A}{y-4} + \frac{B}{y-1} \quad \Rightarrow \quad 1 = A(y-1) + B(y-4)$$

$$\text{at} \quad y=4 \;\Rightarrow\; A=\frac{1}{3} \quad \text{and} \quad \text{at} \quad y=1 \;\Rightarrow\; B=-\frac{1}{3}$$

$$\int \frac{\sin x\; dx}{\cos^2 x - 5\cos x + 4} = -\int \left(\frac{1/3}{y-4} + \frac{-1/3}{y-1}\right)dy$$

$$= -\frac{1}{3}\ln(y-4) + \frac{1}{3}\ln(y-1) + c = -\frac{1}{3}\ln(\cos x - 4) + \frac{1}{3}\ln(\cos x - 1) + c$$

5) $\frac{2x^2-3x+2}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2}$

$$2x^2-3x+2 = A(x-1)(x-2) + B(x-2) + C(x-1)^2$$

$$\left.\begin{aligned} A+C=2 \; ....(1) \\ -3A+B-2C=-3 \; ......(2) \\ 2A-2B+C=2 \; ....(3) \end{aligned}\right\} \Rightarrow \quad A=-2 \;,\; B=-1 \;,\; C=4$$

$$\int \frac{2x^2-3x+2}{(x-1)^2(x-2)}dx = \int \left(\frac{-2}{x-1} + \frac{1}{(x-1)^2} + \frac{4}{x-2}\right)dx$$

$$= -2\ln(x-1) + \frac{1}{x-1} + 4\ln(x-2) + C$$

6) $\frac{x^3+4x^2}{x^2+4x+3} = x - \frac{3x}{(x+3)(x+1)}$

$$\begin{array}{r} x \\ x^2+4x+3 \overline{\smash{)}\; x^3+4x^2 \qquad} \\ \mp x^3 \mp 4x^2 \mp 3x \\ \hline -3x \end{array}$$

$$\frac{3x}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1} \quad \Rightarrow \quad 3x = A(x+1) + B(x+3)$$

$$at \;\; x = -3 \Rightarrow A = \frac{9}{2} \;\; and \;\; at \;\; x = -1 \Rightarrow B = -\frac{3}{2}$$

$$\int \frac{x^3+4x^2}{x^2+4x+3}\,dx = \int \left(x - \frac{9/2}{x+3} + \frac{3/2}{x+1} \right) dx$$

$$= \frac{x^2}{2} - \frac{9}{2}\ln(x+3) - \frac{3}{2}\ln(x+1) + c$$