Al-Mustaqbal University

College of Engineering & Technology

Biomedical Engineering Department

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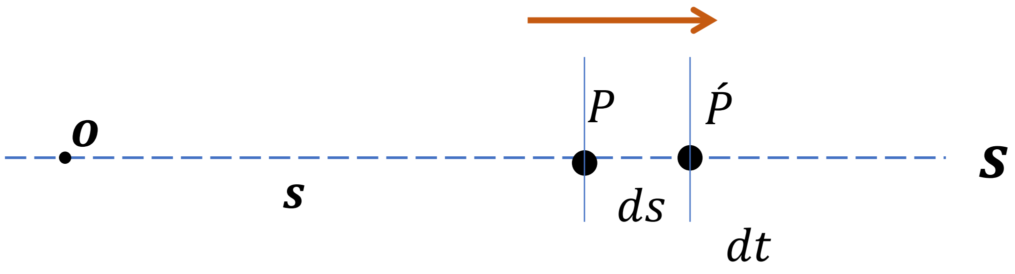
Lecture No.:- 3

Lecture Title: [Curvilinear Motion]

Curvilinear Motion

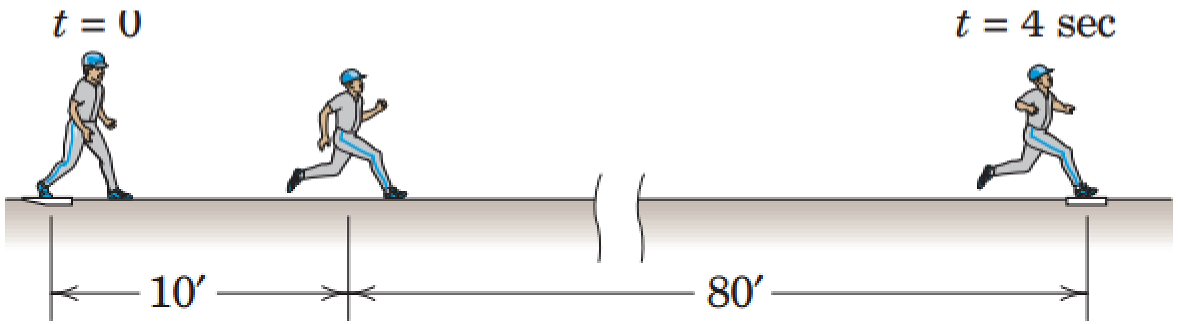
Rectilinear Motion

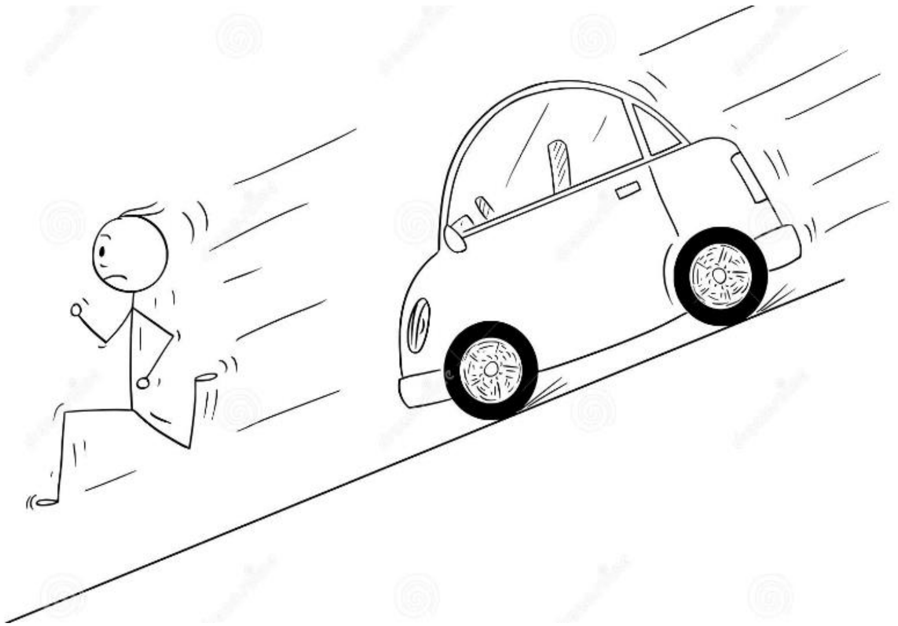


$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}$$

$$v\,dv = a\,ds$$



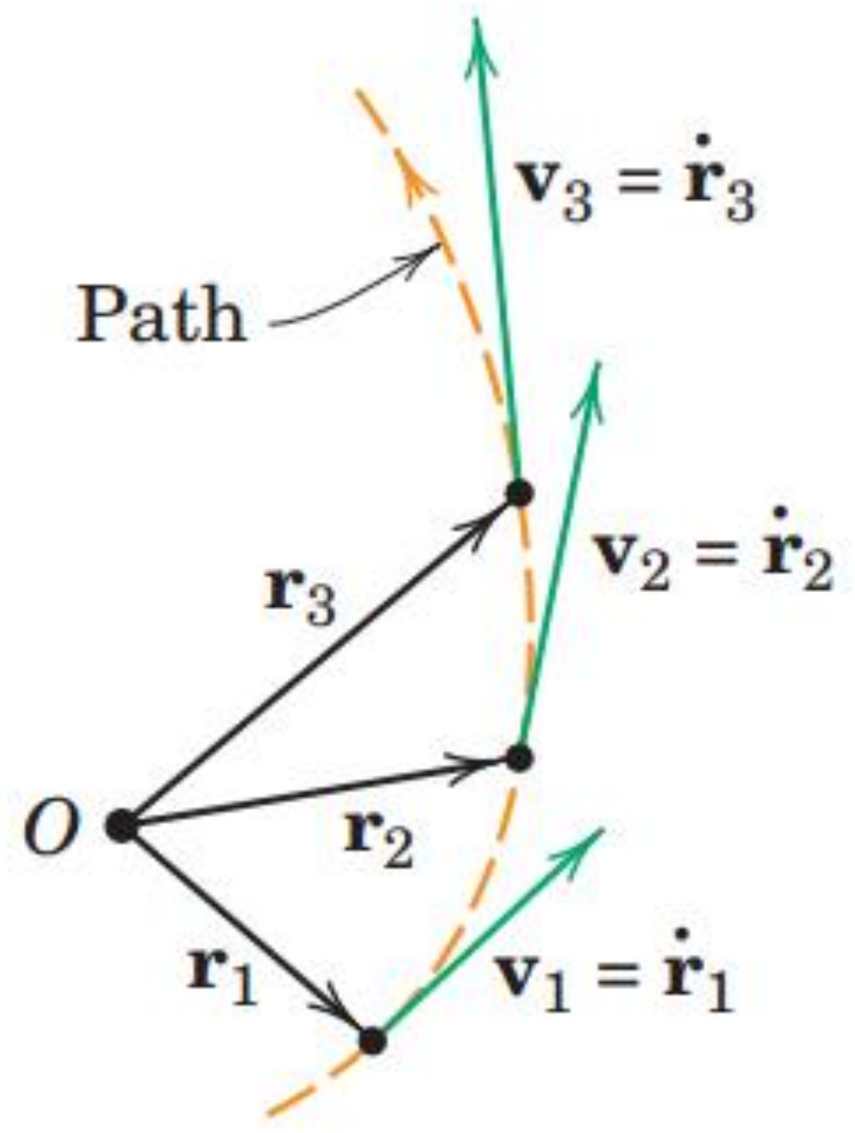


Curvilinear Motion

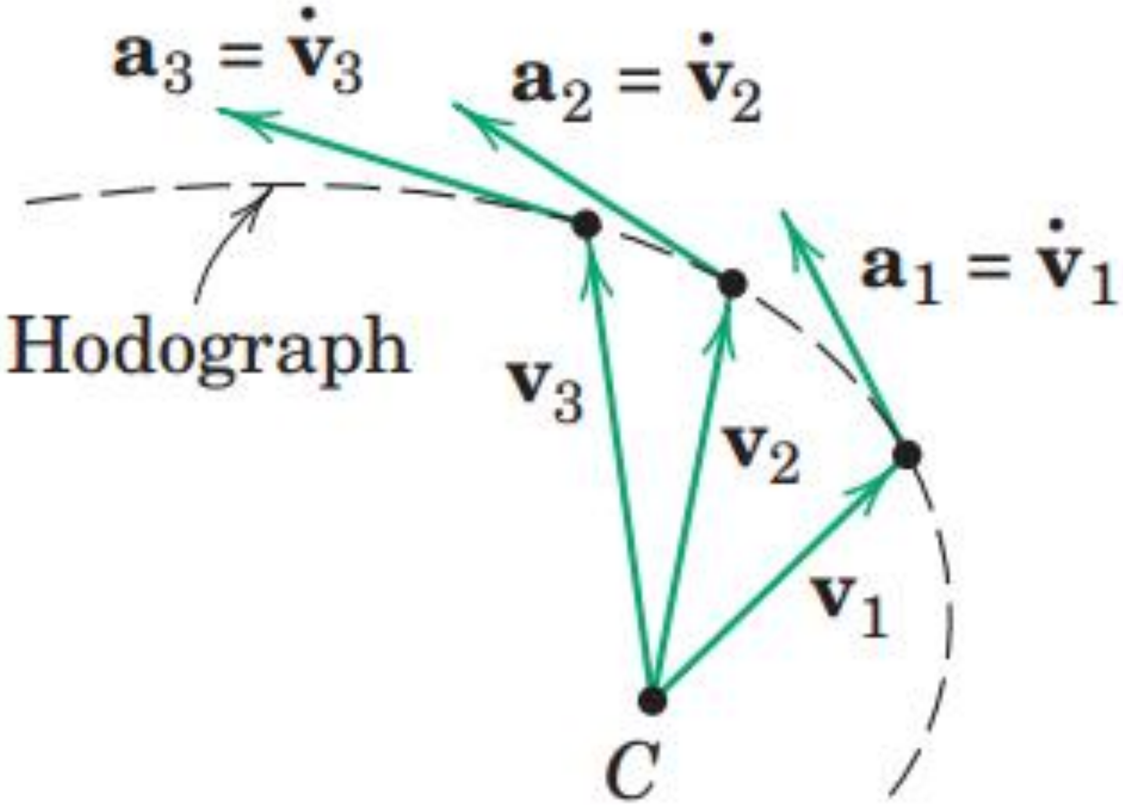
Examples of Curvilinear Motion

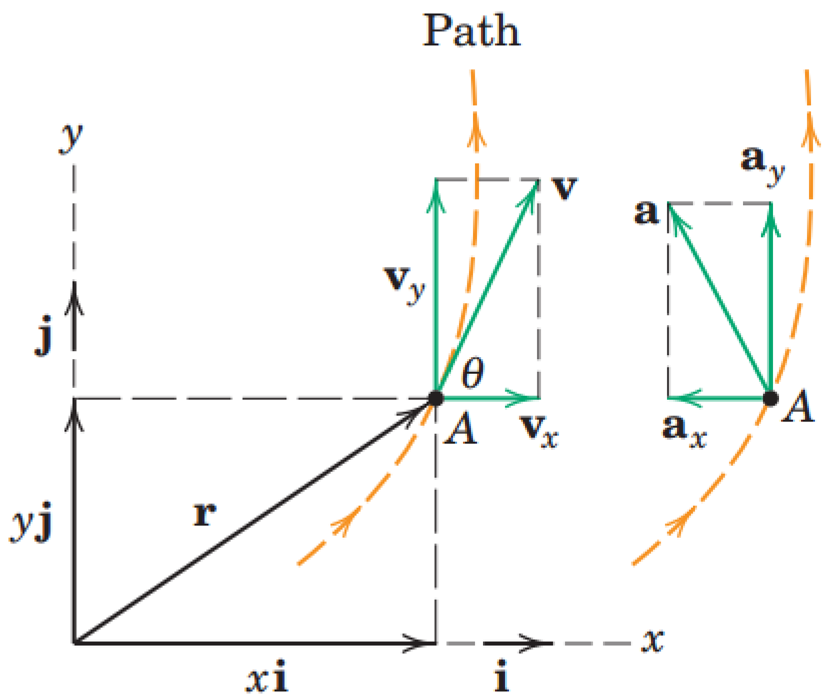
Curvilinear Motion

Displacement and Velocity



Acceleration





$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

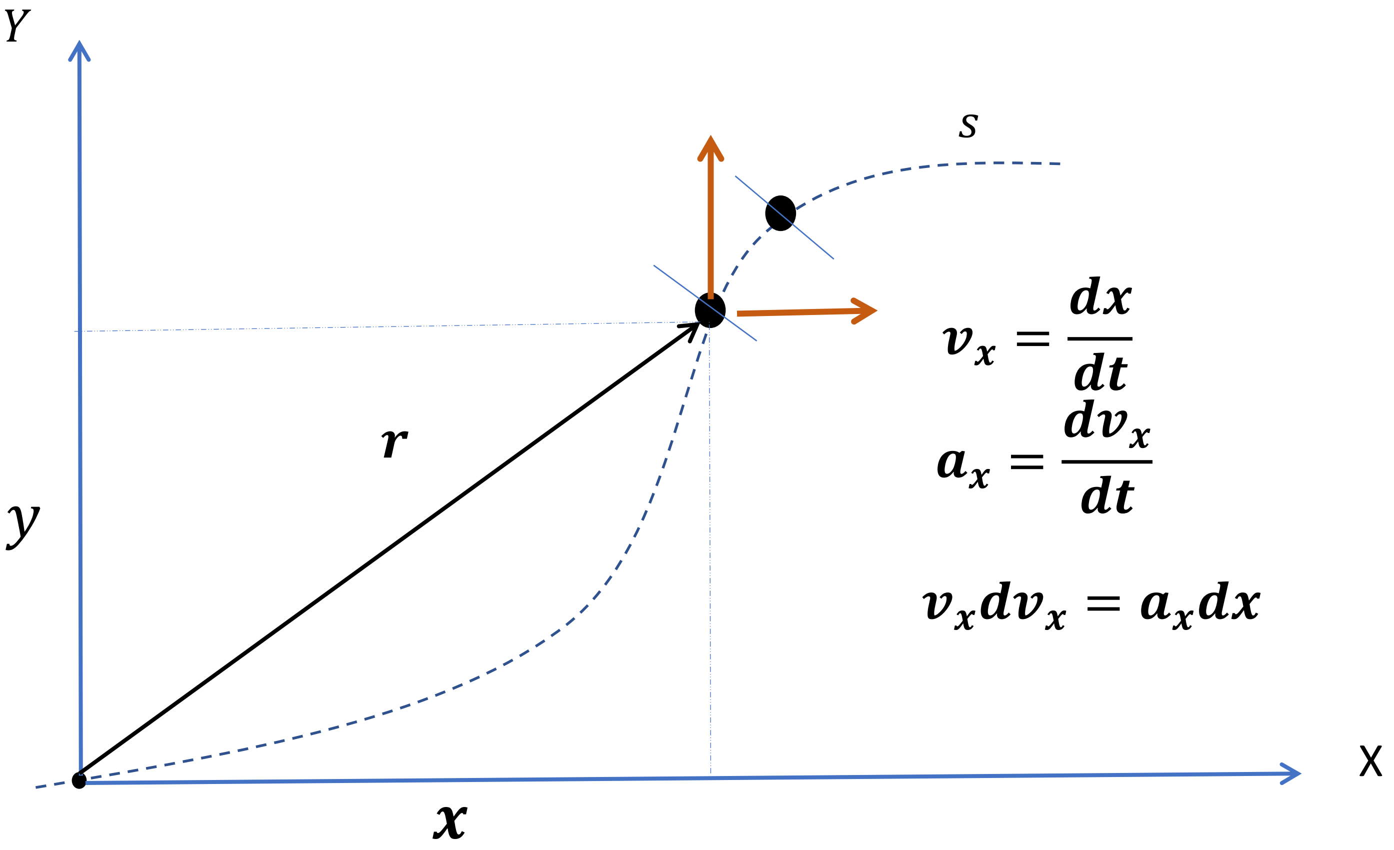
$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

$$v^2 = {v_x}^2 + {v_y}^2 \qquad v = \sqrt{{v_x}^2 + {v_y}^2} \qquad \tan\theta = \frac{v_y}{v_x}$$

$$a^2 = {a_x}^2 + {a_y}^2 \qquad a = \sqrt{{a_x}^2 + {a_y}^2}$$

Rectangular Coordinates (x-y)

$$v_y = \frac{dy}{dt} \qquad a_y = \frac{dv_y}{dt} \qquad v_y dv_y = a_y dy$$



$$v_x = \frac{dx}{dt}$$

$$a_x = \frac{dv_x}{dt}$$

$$v_x dv_x = a_x dx$$

SAMPLE PROBLEM 2/5

The curvilinear motion of a particle is defined by $v_x = 50 - 16t$ and $y = 100 - 4t^2$, where v_x is in meters per second, y is in meters, and t is in seconds. It is also known that $x = 0$ when $t = 0$. Plot the path of the particle and determine its velocity and acceleration when the position $y = 0$ is reached.

$v_x = 50 - 16t$ $\qquad$ $x = 0$ when $t = 0$. $\qquad$ $y = 100 - 4t^2$

$$\left[\int dx = \int v_x\, dt\right] \qquad \int_0^x dx = \int_0^t (50 - 16t)\, dt \qquad x = 50t - 8t^2 \text{ m}$$

$$[a_x = \dot{v}_x] \qquad a_x = \frac{d}{dt}(50 - 16t) \qquad a_x = -16 \text{ m/s}^2$$

The y-components of velocity and acceleration are

$y = 100 - 4t^2$

$[v_y = \dot{y}]$ $\qquad v_y = \frac{d}{dt}(100 - 4t^2) \qquad v_y = -8t \text{ m/s}$

$[a_y = \dot{v}_y]$ $\qquad a_y = \frac{d}{dt}(-8t) \qquad a_y = -8 \text{ m/s}^2$

We now calculate corresponding values of x and y for various values of t and plot x against y to obtain the path as shown

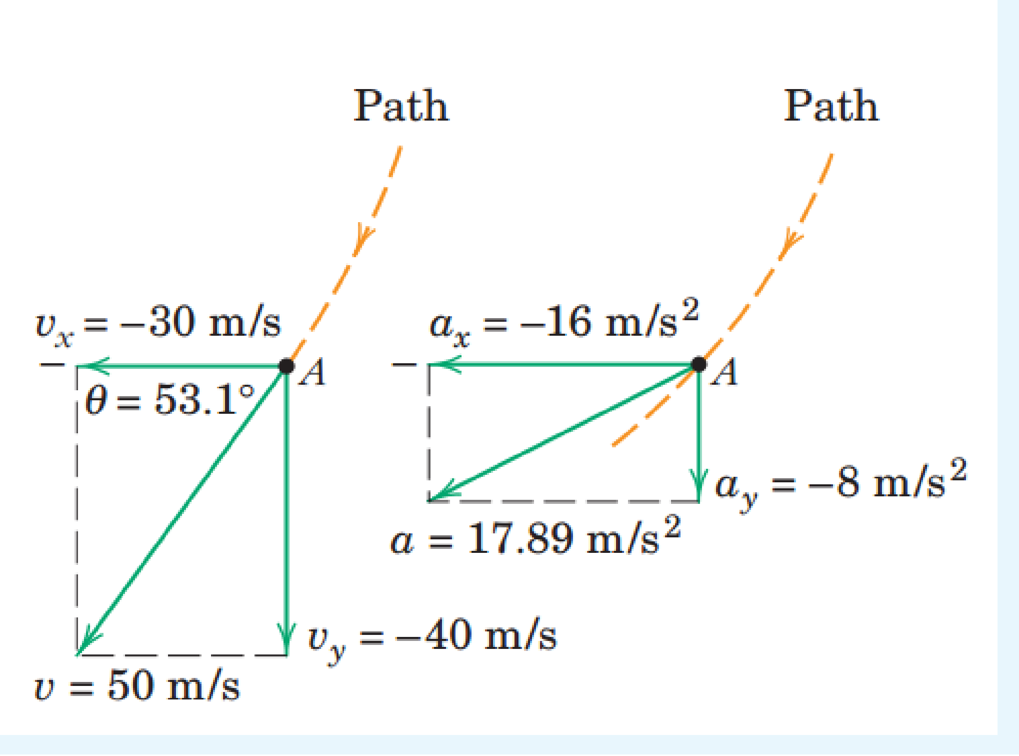
When $y = 0$, $0 = 100 - 4t^2$, so $t = 5$ s. For this value of the time, we have

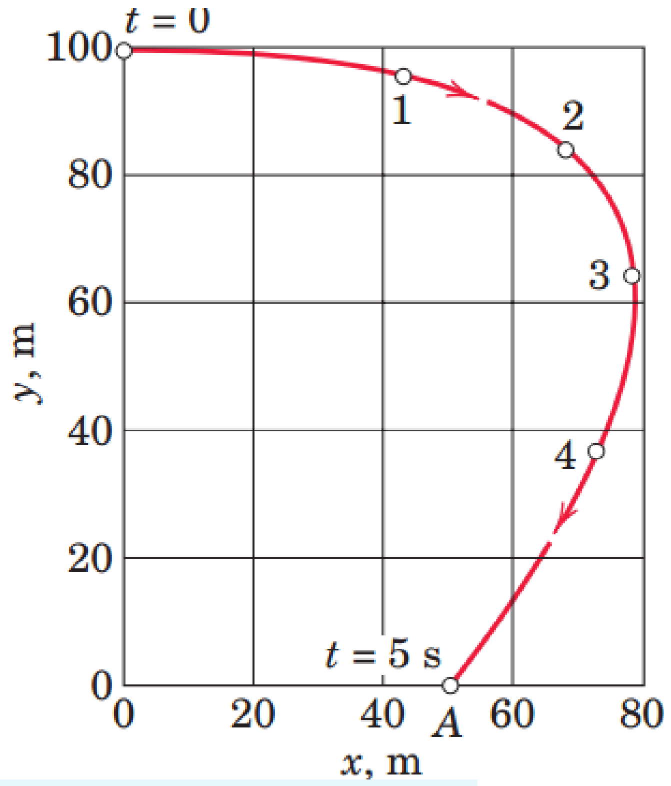
$$v_x = 50 - 16(5) = -30 \text{ m/s}$$

$$v_y = -8(5) = -40 \text{ m/s}$$

$$v = \sqrt{(-30)^2 + (-40)^2} = 50 \text{ m/s}$$

$$a = \sqrt{(-16)^2 + (-8)^2} = 17.89 \text{ m/s}^2$$





The velocity and acceleration components and their resultants are shown on the separate diagrams for point A, where $y = 0$. Thus, for this condition we may write

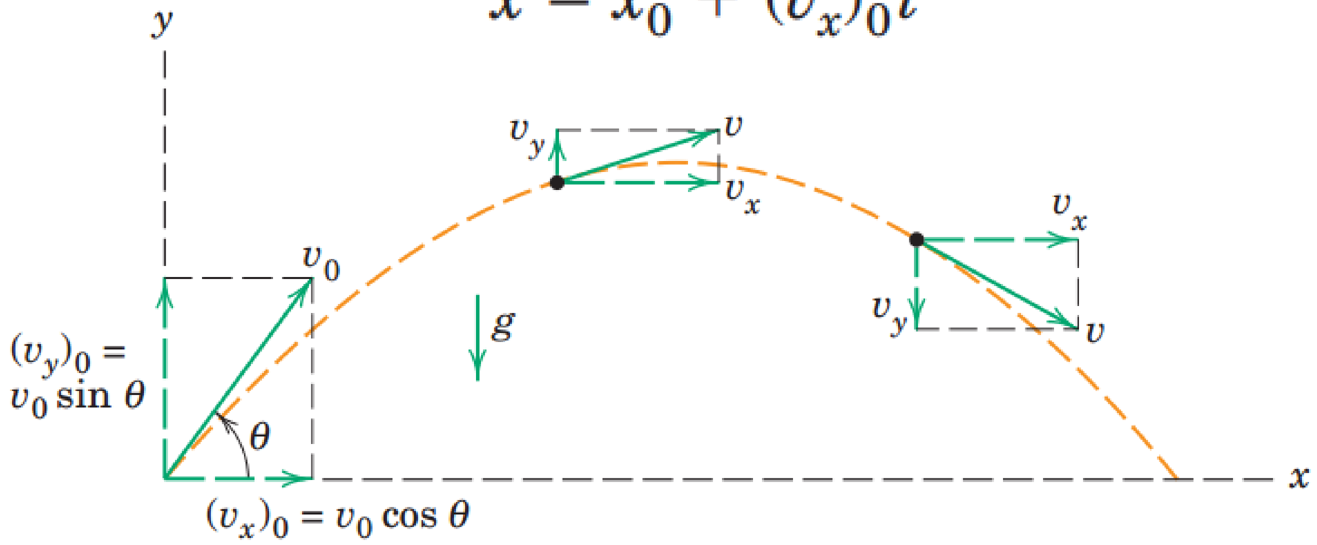
$$\mathbf{v} = -30\mathbf{i} - 40\mathbf{j} \text{ m/s} \qquad \textit{Ans.}$$

$$\mathbf{a} = -16\mathbf{i} - 8\mathbf{j} \text{ m/s}^2 \qquad \textit{Ans.}$$

Projectile Motion

$$a_x = 0 \qquad v_x = (v_x)_0$$

$$x = x_0 + (v_x)_0 t$$



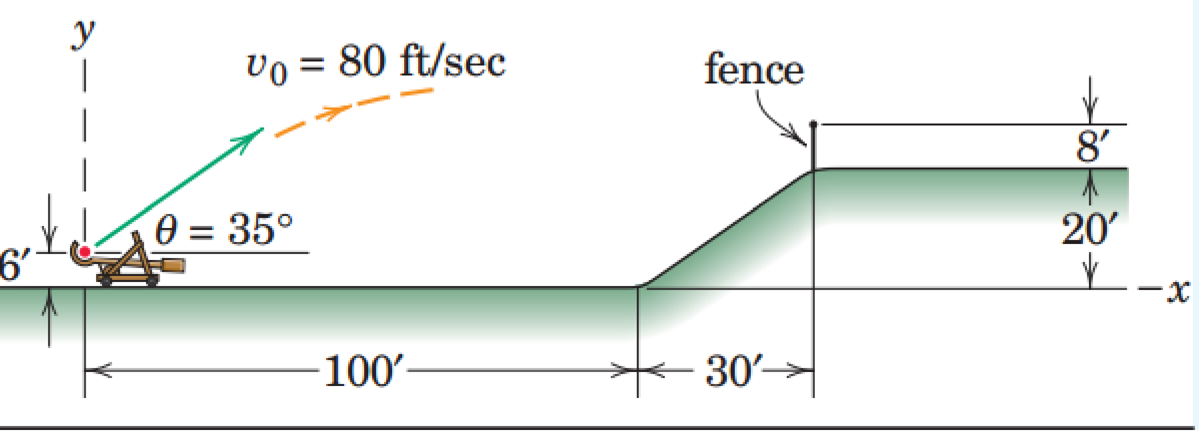
$$a_y = -g$$

$$v_y = (v_y)_0 - gt$$

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

$$v_y{}^2 = (v_y)_0{}^2 - 2g(y - y_0)$$

A team of engineering students designs a medium-size catapult which launches 8-lb steel spheres. The launch speed is $v_0 = 80$ ft/sec, the launch angle is $\theta = 35°$ above the horizontal, and the launch position is 6 ft above ground level. The students use an athletic field with an adjoining slope topped by an 8-ft fence as shown. Determine:



(a) the x-y coordinates of the point of first impact

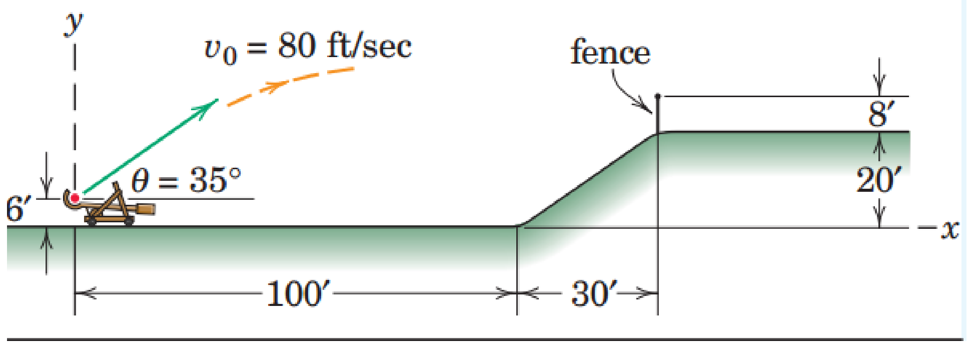
(b) the time duration t_f of the flight

(c) the maximum height h above the horizontal field attained by the ball

(d) the velocity (expressed as a vector) with which the projectile strikes the ground

Repeat part (a) for a launch speed of $v_0 = 75$ ft/sec.

Solution. We make the assumptions of constant gravitational acceleration and no aerodynamic drag. With the latter assumption, the 8-lb weight of the projectile is irrelevant. Using the given x-y coordinate system, we begin by checking the y-displacement at the horizontal position of the fence.



$[x = x_0 + (v_x)_0 t] \qquad 100 + 30 = 0 + (80 \cos 35°)t \qquad t = 1.984 \text{ sec}$

$[y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2] \qquad y = 6 + 80 \sin 35°(1.984) - \frac{1}{2}(32.2)(1.984)^2 = 33.7 \text{ ft}$

(a) Because the y-coordinate of the top of the fence is $20 + 8 = 28$ feet, the projectile clears the fence. We now find the flight time by setting $y = 20$ ft:

$[y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2] \qquad 20 = 6 + 80 \sin 35°(t_f) - \frac{1}{2}(32.2)t_f^{\,2} \qquad t_f = 2.50 \text{ s} \qquad$ *Ans*

$[x = x_0 + (v_x)_0 t] \qquad x = 0 + 80 \cos 35°(2.50) = 164.0 \text{ ft}$

(b) Thus the point of first impact is $(x, y) = (164.0, 20)$ ft.

(c) For the maximum height:

$[v_y{}^2 = (v_y)_0{}^2 - 2g(y - y_0)]$ $\quad 0^2 = (80 \sin 35°)^2 - 2(32.2)(h - 6) \quad h = 38.7 \text{ ft}$ *Ans.*

(d) For the impact velocity:

$[v_x = (v_x)_0] \qquad v_x = 80 \cos 35° = 65.5 \text{ ft/sec}$

$[v_y = (v_y)_0 - gt] \qquad v_y = 80 \sin 35° - 32.2(2.50) = -34.7 \text{ ft/sec}$

So the impact velocity is $\mathbf{v} = 65.5\mathbf{i} - 34.7\mathbf{j}$ ft/sec. *Ans.*

If $v_0 = 75$ ft/sec, the time from launch to the fence is found by

$[x = x_0 + (v_x)_0 t] \qquad 100 + 30 = (75 \cos 35°)t \qquad t = 2.12 \text{ sec}$

and the corresponding value of y is

$[y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2] \qquad y = 6 + 80 \sin 35°(2.12) - \frac{1}{2}(32.2)(2.12)^2 = 24.9 \text{ ft}$

For this launch speed, we see that the projectile hits the fence, and the point of impact is

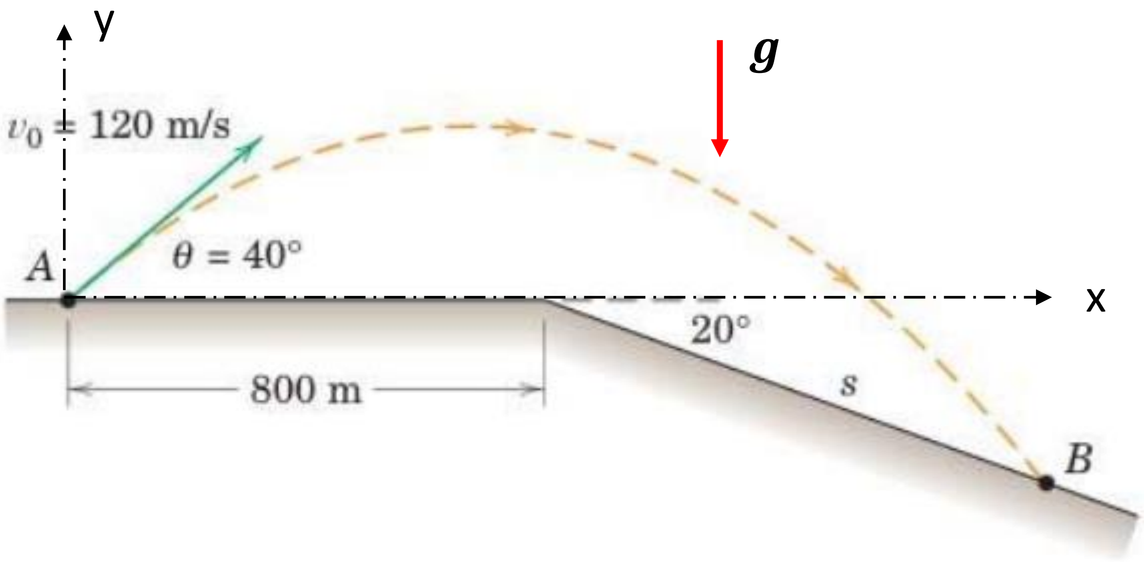
$$(x, y) = (130, 24.9) \text{ ft} \qquad \textit{Ans.}$$

For lower launch speeds, the projectile could land on the slope or even on the level portion of the athletic field.

Thank you for your listening to our presentation

Example:

A projectile is launched from point A with the initial conditions shown in the figure. Determine the slant distance s which locates the point B of impact. Calculate the time of flight t.



Set up x-y coordinates with origin at point A

$x = x_o + v_{x_o}t$ at point B

$800 + s\cos 20 = 0 + (120\cos 40)\,t$ …… (1)

$y = y_o + v_{y_o}t - \frac{1}{2}gt^2$ at point B

$-s\sin 20 = 120\sin 40 - \frac{1}{2}(9.81)t^2$ ………………(2)

simultaneously

$s = 1057m, t = 19.50\ s$