Al-Mustaqbal University

College of Engineering & Technology

Biomedical Engineering Department

Subject Name: Dynamics

1st Class, Second Semester

Subject Code: [UOMU011024]

Academic Year: 2024-2025

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Lecture No.:- 4

Lecture Title: [Curvilinear Motion P2]

2/62 A particle which moves with curvilinear motion has coordinates in millimeters which vary with the time t in seconds according to $x = 3t^2 - 4t$ and $y = 4t^2 - \frac{1}{3}t^3$. Determine the magnitudes of the velocity **v** and acceleration **a** and the angles which these vectors make with the x-axis when $t = 2$ s.

$\boxed{2/62}$ $x = 3t^2 - 4t$, $\dot{x} = 6t - 4$, $\ddot{x} = 6 \text{ mm/s}^2$

$y = 4t^2 - \frac{1}{3}t^3$, $\dot{y} = 8t - t^2$, $\ddot{y} = 8 - 2t \text{ mm/s}^2$

When $t = 2$ s, $\left.\begin{array}{l}\dot{x} = 12 - 4 = 8 \text{ mm/s} \\ \dot{y} = 16 - 4 = 12 \text{ mm/s}\end{array}\right\}$ $v = \sqrt{\dot{x}^2 + \dot{y}^2}$

$= \sqrt{8^2 + 12^2} = \underline{14.42 \frac{\text{mm}}{\text{s}}}$

$\theta_x = \tan^{-1}\frac{\dot{y}}{\dot{x}} = \tan^{-1}\frac{12}{8}$

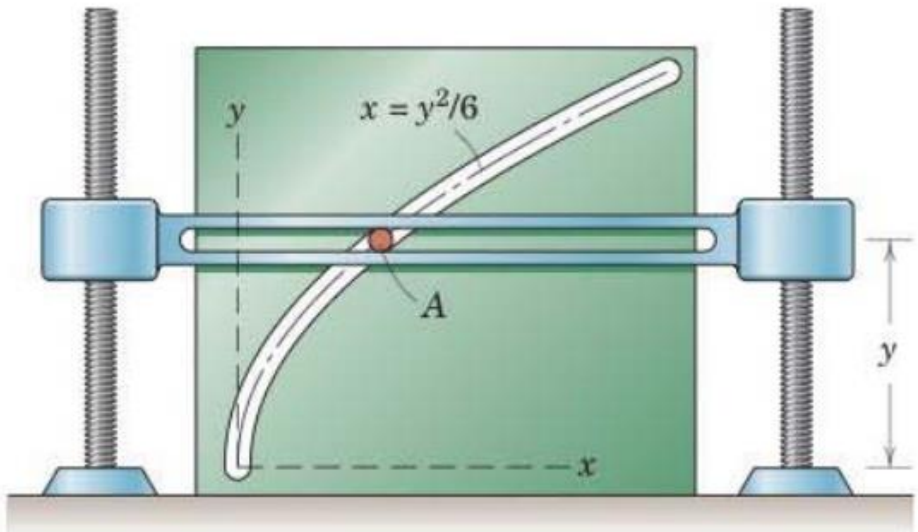
$= \underline{56.3°}$

$\ddot{x} = 6 \text{ mm/s}^2$, $\ddot{y} = 8 - 4 = 4 \text{ mm/s}^2$

$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{6^2 + 4^2} = \underline{7.21 \text{ mm/s}^2}$

$\theta_x = \tan^{-1}\frac{\ddot{y}}{\ddot{x}} = \tan^{-1}\frac{4}{6} = \underline{33.7°}$

2/64 For a certain interval of motion the pin A is forced to move in the fixed parabolic slot by the horizontal slotted arm which is elevated in the y-direction at the constant rate of 3 in./sec. All measurements are in inches and seconds. Calculate the velocity v and acceleration a of pin A when $x = 6$ in.



$\boxed{2/64}$ $x = y^2/6$ & $\dot{y} = 3$ in./sec

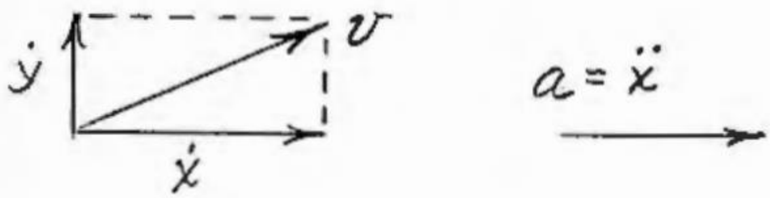
$\dot{x} = \frac{y}{3}\dot{y}$, $\ddot{x} = \frac{\dot{y}^2}{3} + \frac{y}{3}\ddot{y}$ but $\ddot{y} = 0$ & $\dot{y} = 3$ in/sec

Also when $x = 6$ in., $y = \sqrt{36} = 6$ in.

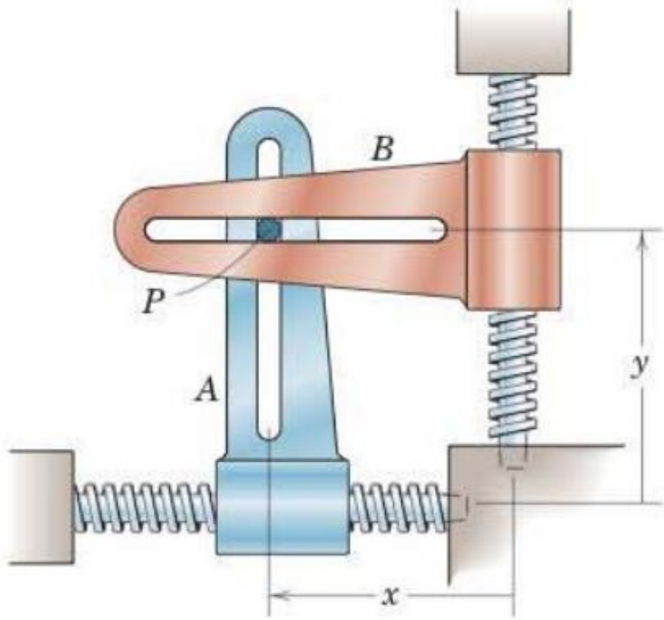
So $\dot{x} = \frac{6}{3}(3) = 6$ in/sec,

Hence $v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{6^2 + 3^2} = \underline{3\sqrt{5}\text{ in/sec}}$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(3^2/3)^2 + 0} = \underline{3\text{ in./sec}^2}$



2/66 The x- and y-motions of guides A and B with right-angle slots control the curvilinear motion of the connecting pin P, which slides in both slots. For a short interval, the motions are governed by $x = 20 + \frac{1}{4}t^2$ and $y = 15 - \frac{1}{6}t^3$, where x and y are in millimeters and t is in seconds. Calculate the magnitudes of the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the pin for $t = 2$ s. Sketch the direction of the path and indicate its curvature for this instant.



2/66 $x = 20 + \frac{1}{4}t^2, \quad \dot{x} = \frac{1}{2}t, \quad \ddot{x} = \frac{1}{2} \text{ mm/s}^2$

$y = 15 - \frac{1}{6}t^3, \quad \dot{y} = -\frac{1}{2}t^2, \quad \ddot{y} = -t \text{ mm/s}^2$

For $t = 2$ s, $\quad \dot{x} = 1$ mm/s

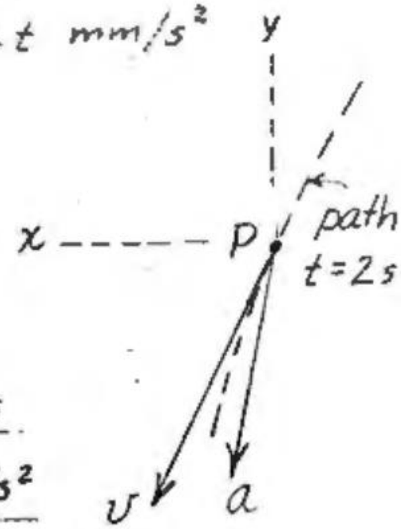
$\dot{y} = -2$ mm/s

$\ddot{x} = \frac{1}{2}$ mm/s^2

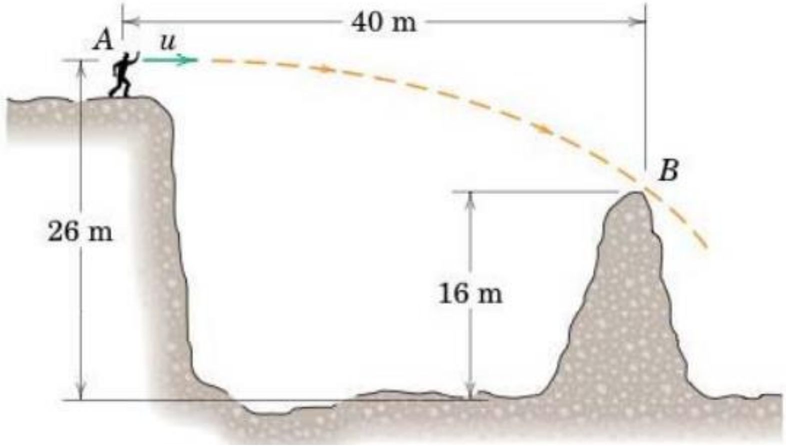
$\ddot{y} = -2$ mm/s^2

$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{1^2 + (-2)^2} = \underline{2.24 \text{ mm/s}}$

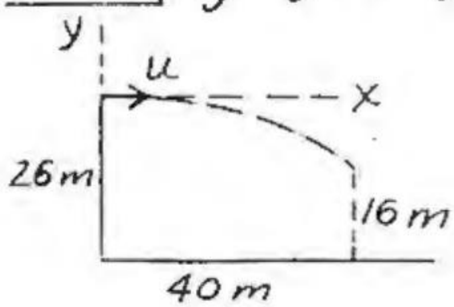
$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(1/2)^2 + (-2)^2} = \underline{2.06 \text{ mm/s}^2}$



2/72 With what minimum horizontal velocity u can a boy throw a rock at A and have it just clear the obstruction at B?



2/72 $a_y = -g$ so $y = 0 - \frac{1}{2}gt^2$, $t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(26-16)}{9.81}}$



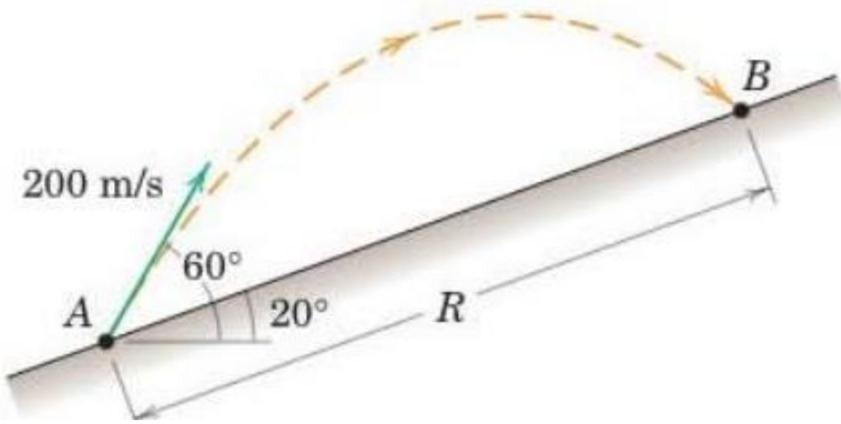
$= 1.428$ s

$x = ut$; $u = 40/1.428$

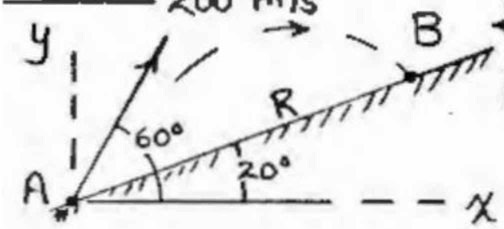
$= \underline{28.0 \text{ m/s}}$

2/85 A projectile is launched with an initial speed of 200 m/s at an angle of 60° with respect to the horizontal. Compute the range R as measured up the incline.

Ans. $R = 2970$ m



2/85



$$\begin{cases} v_{x_0} = 200\cos 60^\circ = 100 \text{ m/s} \\ v_{y_0} = 200\sin 60^\circ = 173.2 \text{ m/s} \end{cases}$$

t_f = flight time

$x = x_0 + v_{x_0} t$ @ B: $R\cos 20^\circ = 100\, t_f$ (1)

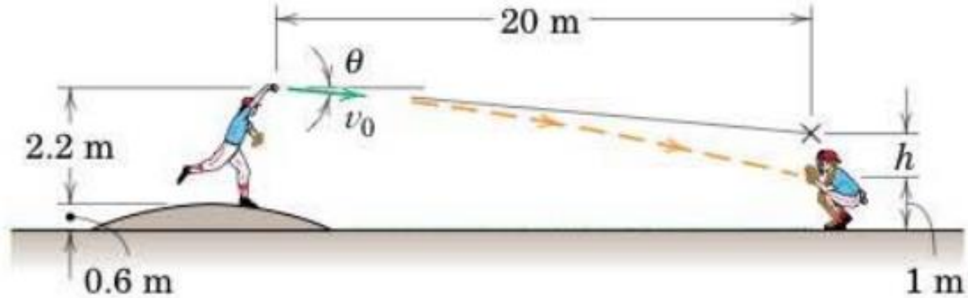
$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ @ B: $R\sin 20^\circ = 173.2\, t_f - \frac{9.81}{2} t_f^2$ (2)

(1): $t_f = 0.00940\, R$

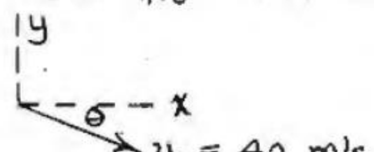
(2): $R\sin 20^\circ = 173.2(0.00940\, R) - \frac{9.81}{2}(0.00940 R)^2$

$\underline{R = 2970 \text{ m}}$

2/92 Determine the location h of the spot toward which the pitcher must throw if the ball is to hit the catcher's mitt. The ball is released with a speed of 40 m/s.



2/92 Set up x-y coordinates with origin at release point :



$x = x_0 + v_{x_0} t$ at mitt : $20 = (40 \cos\theta) t_f$ (1)

$y = y_0 + v_{y_0} t - \frac{1}{2} g t^2$ at mitt :

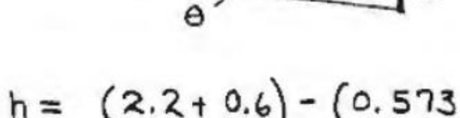
$-1.8 = 0 + (-40 \sin\theta) t_f - \frac{9.81}{2} t_f^2$ (2)

(1) : $t_f = \frac{1}{2\cos\theta}$

(2) : $-1.8 = -40 \sin\theta \left(\frac{1}{2\cos\theta}\right) - \frac{9.81}{2}\left(\frac{1}{2\cos\theta}\right)^2$

Use $\frac{1}{\cos^2\theta} = \tan^2\theta + 1$: $1.226 \tan^2\theta + 20 \tan\theta - 0.574 = 0$

$\Rightarrow \theta = 1.640°$



$d = 20 \tan 1.640$
$= 0.573'$

$h = (2.2 + 0.6) - (0.573 + 1) = \underline{1.227 \text{ m}}$