Al-Mustaqbal University

College of Engineering & Technology

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Lecture No.:- 5

Lecture Title: [Constrained Motion]

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2/9 Constrained Motion of Connected Particles

Sometimes the motions of particles are interrelated because of the constraints imposed by interconnecting members. In such cases it is necessary to account for these constraints in order to determine the respective motions of the particles.

One Degree of Freedom

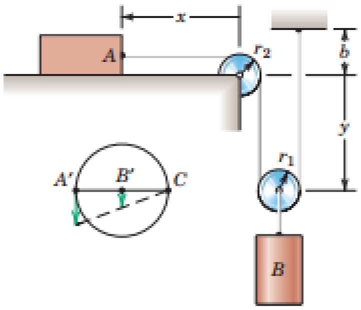


Figure 2/19

Consider first the very simple system of two interconnected particles A and B shown in Fig. 2/19. It should be quite evident by inspection that the horizontal motion of A is twice the vertical motion of B. Nevertheless we will use this example to illustrate the method of analysis which applies to more complex situations where the results cannot be easily obtained by inspection. The motion of B is clearly the same as that of the center of its pulley, so we establish position coordinates y and x measured from a convenient fixed datum. The total length of the cable is

$$L = x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b$$

With L, r_2, r_1, and b all constant, the first and second time derivatives of the equation give

$$0 = \dot{x} + 2\dot{y} \qquad \text{or} \qquad 0 = v_A + 2v_B$$

$$0 = \ddot{x} + 2\ddot{y} \qquad \text{or} \qquad 0 = a_A + 2a_B$$

The velocity and acceleration constraint equations indicate that, for the coordinates selected, the velocity of A must have a sign which is opposite to that of the velocity of B, and similarly for the accelerations. The constraint equations are valid for the motion of the system in either direction. We emphasize that $v_A = \dot{x}$ is positive to the left and that $v_B = \dot{y}$ is positive down.

Because the results do not depend on the lengths or pulley radii, we should be able to analyze the motion without considering them. In the lower-left portion of Fig. 2/19 is shown an enlarged view of the horizontal diameter $A'B'C'$ of the lower pulley at an instant of time. Clearly, A' and A have the same motion magnitudes, as do B and B'. During an infinitesimal motion of A', it is easy to see from the triangle that B' moves half as far as A' because point C as a point on the fixed portion of the cable momentarily has no motion. Thus, with differentiation by time in mind, we can obtain the velocity and acceleration magnitude relationships by inspection. The pulley, in effect, is a wheel which rolls on the fixed vertical cable. (The kinematics of a rolling wheel will be treated more extensively in Chapter 5 on rigid-body motion.) The system of Fig. 2/19 is said to have *one degree of freedom* since only one variable, either x or y, is needed to specify the positions of all parts of the system.

Two Degrees of Freedom

The system with *two degrees of freedom* is shown in Fig. 2/20. Here the positions of the lower cylinder and pulley C depend on the separate specifications of the two coordinates y_A and y_B. The lengths of the cables attached to cylinders A and B can be written, respectively, as

$$L_A = y_A + 2y_D + \text{constant}$$

$$L_B = y_B + y_C + (y_C - y_D) + \text{constant}$$

and their time derivatives are

$$0 = \dot{y}_A + 2\dot{y}_D \quad \text{and} \quad 0 = \dot{y}_B + 2\dot{y}_C - \dot{y}_D$$

$$0 = \ddot{y}_A + 2\ddot{y}_D \quad \text{and} \quad 0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$$

Eliminating the terms in $\dot{y}_D$ and $\ddot{y}_D$ gives

$$\dot{y}_A + 2\dot{y}_B + 4\dot{y}_C = 0 \quad \text{or} \quad v_A + 2v_B + 4v_C = 0$$

$$\ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_C = 0 \quad \text{or} \quad a_A + 2a_B + 4a_C = 0$$

It is clearly impossible for the signs of all three terms to be positive simultaneously. So, for example, if both A and B have downward (positive) velocities, then C will have an upward (negative) velocity.

These results can also be found by inspection of the motions of the two pulleys at C and D. For an increment dy_A (with y_B held fixed), the center of D moves up an amount $dy_A/2$, which causes an upward movement $dy_A/4$ of the center of C. For an increment dy_B (with y_A held fixed), the center of C moves up a distance $dy_B/2$. A combination of the two movements gives an upward movement

$$-dy_C = \frac{dy_A}{4} + \frac{dy_B}{2}$$

so that $-v_C = v_A/4 + v_B/2$ as before. Visualization of the actual geometry of the motion is an important ability.

A second type of constraint where the direction of the connecting member changes with the motion is illustrated in the second of the two sample problems which follow.

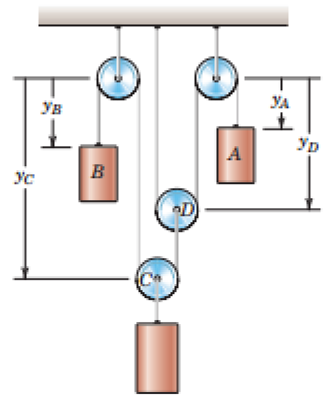
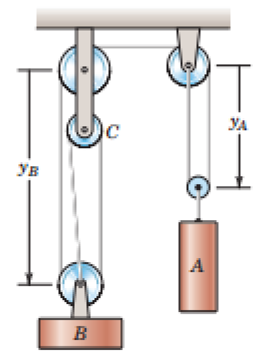


Figure 2/20

SAMPLE PROBLEM 2/15

In the pulley configuration shown, cylinder A has a downward velocity of 0.3 m/s. Determine the velocity of B. Solve in two ways.



Solution (I). The centers of the pulleys at A and B are located by the coordinates y_A and y_B measured from fixed positions. The total constant length of cable in the pulley system is

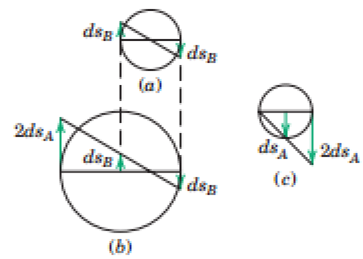
$$L = 3y_B + 2y_A + \text{constants}$$

1 where the constants account for the fixed lengths of cable in contact with the circumferences of the pulleys and the constant vertical separation between the two upper left-hand pulleys. Differentiation with time gives

$$0 = 3\dot{y}_B + 2\dot{y}_A$$

Substitution of $v_A = \dot{y}_A = 0.3$ m/s and $v_B = \dot{y}_B$ gives

2 $$0 = 3(v_B) + 2(0.3) \qquad \text{or} \qquad v_B = -0.2 \text{ m/s} \qquad \textit{Ans.}$$



Solution (II). An enlarged diagram of the pulleys at A, B, and C is shown. During a differential movement ds_A of the center of pulley A, the left end of its horizontal diameter has no motion since it is attached to the fixed part of the cable. Therefore, the right-hand end has a movement of $2ds_A$ as shown. This movement is transmitted to the left-hand end of the horizontal diameter of the pulley at B. Further, from pulley C with its fixed center, we see that the displacements on each side are equal and opposite. Thus, for pulley B, the right-hand end of the diameter has a downward displacement equal to the upward displacement ds_B of its center. By inspection of the geometry, we conclude that

$$2ds_A = 3ds_B \qquad \text{or} \qquad ds_B = \tfrac{2}{3}ds_A$$

Dividing by dt gives

$$|v_B| = \tfrac{2}{3}v_A = \tfrac{2}{3}(0.3) = 0.2 \text{ m/s (upward)} \qquad \textit{Ans.}$$

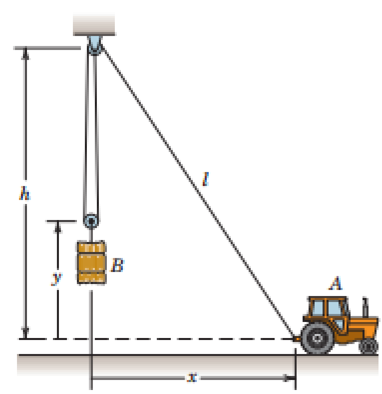
Helpful Hints

1 We neglect the small angularity of the cables between B and C.

2 The negative sign indicates that the velocity of B is *upward*.

SAMPLE PROBLEM 2/16

The tractor A is used to hoist the bale B with the pulley arrangement shown. If A has a forward velocity v_A, determine an expression for the upward velocity v_B of the bale in terms of x.



Solution. We designate the position of the tractor by the coordinate x and the position of the bale by the coordinate y, both measured from a fixed reference. The total constant length of the cable is

$$L = 2(h - y) + l = 2(h - y) + \sqrt{h^2 + x^2}$$

1 Differentiation with time yields

$$0 = -2\dot{y} + \frac{x\dot{x}}{\sqrt{h^2 + x^2}}$$

Substituting $v_A = \dot{x}$ and $v_B = \dot{y}$ gives

$$v_B = \frac{1}{2}\frac{xv_A}{\sqrt{h^2 + x^2}} \qquad \textit{Ans.}$$

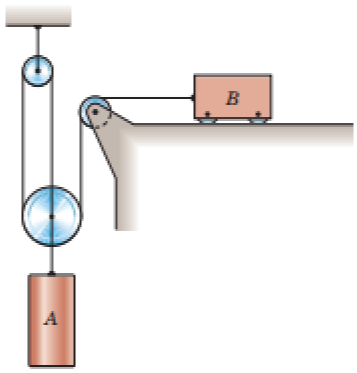
Helpful Hint

1 Differentiation of the relation for a right triangle occurs frequently in mechanics.

PROBLEMS

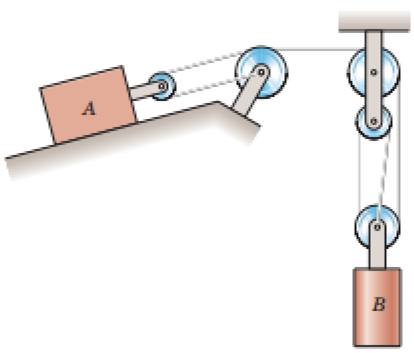
Introductory Problems

2/207 If block B has a leftward velocity of 1.2 m/s, determine the velocity of cylinder A.



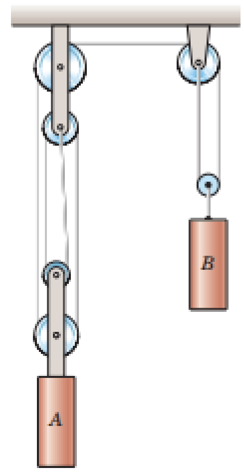
Problem 2/207

2/208 At a certain instant, the velocity of cylinder B is 1.2 m/s down and its acceleration is 2 m/s² up. Determine the corresponding velocity and acceleration of block A.



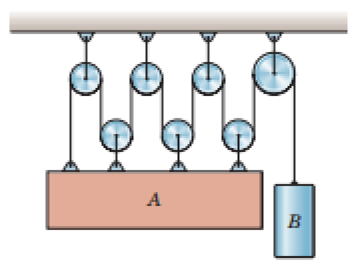
Problem 2/208

2/209 Cylinder B has a downward velocity in feet per second given by $v_B = t^2/2 + t^3/6$, where t is in seconds. Calculate the acceleration of A when $t = 2$ sec.



Problem 2/209

2/210 Determine the constraint equation which relates the accelerations of bodies A and B. Assume that the upper surface of A remains horizontal.



Problem 2/210