Al-Mustaqbal University

College of Engineering & Technology

Biomedical Engineering Department

Subject Name: Dynamics

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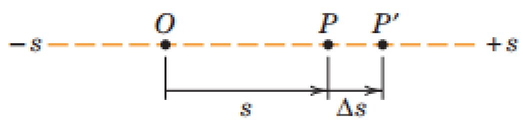
Lecture No.:- 2

Lecture Title: [Kinematics of Particles]

Kinematics of Particles

1) Rectilinear Motion:

As mentioned in our first lectures, only three equations will be used to solve problems related to particle kinematics. As below:



$$v = \frac{ds}{dt} = \dot{s}$$

$$a = \frac{dv}{dt} = \dot{v} \quad \textbf{or} \quad a = \frac{d^2s}{dt^2} = \ddot{s}$$

$$v\,dv = a\,ds \quad \text{or} \quad \dot{s}\,d\dot{s} = \ddot{s}\,ds$$

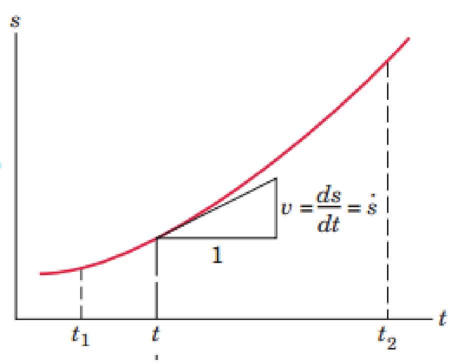
Solution Methods: Two main methods can be used

1. **Graphical Method:**

In this methods, five different curves are used to understand and solve the dynamic behaviour of particle motion. These curves are (s-t), (v-t), (a-t), A-s), and (v-s).

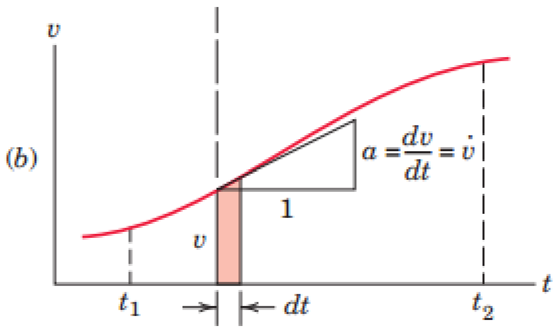
a) (s-t) curve is used to determine the velocity of any point on the curve by making a tangent at the point and find its slope. The slope of the tangent of any point gives the velocity of this point.

(a)



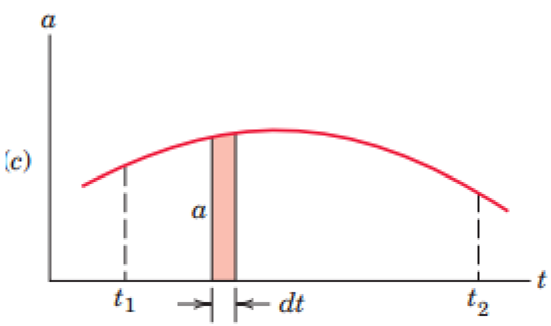
$v = ds/dt.$

b) (v-t) Curve is used to determine the velocity of any point on the curve by making a tangent at that point and from the slope of the tangent the velocity can be obtained. Additionally, the curve can be used to find the displacement of any point on the curve. The area between any two points under the curve represents the displacement difference between those two points.



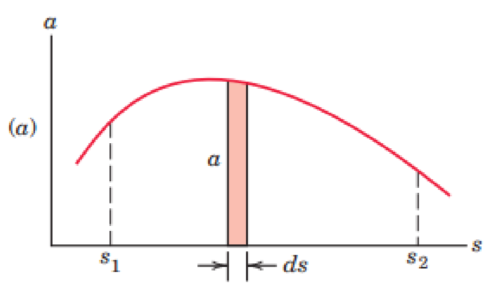
$$\int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} v\,dt \qquad \text{or} \qquad s_2 - s_1 = (\text{area under } v\text{-}t \text{ curve})$$

c) (a-t) curve is only used to determine the difference in velocities between two points on the curve by calculating the area under the curve between these two points



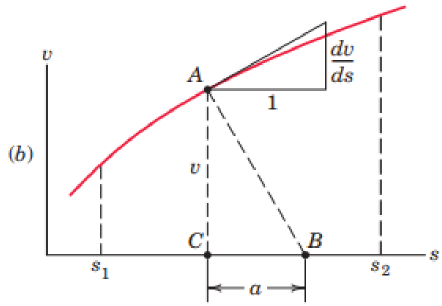
$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt \qquad \text{or} \qquad v_2 - v_1 = (\text{area under } a\text{-}t \text{ curve})$$

d) (a-s) Curve is used to find the velocity of any point by calculating the area under the curve.



$$\int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds \qquad \text{or} \qquad \tfrac{1}{2}(v_2^2 - v_1^2) = (\text{area under } a\text{-}s \text{ curve})$$

e) (S-v) curve is used to calculate the acceleration of any point on the curve.



$\overline{CB} = v(dv/ds) = a$,

2. **Analytical Integration Method:**

(a) Constant Acceleration. When a is constant, the first of Eqs. 2/2 and 2/3 can be integrated directly. For simplicity with $s = s_0$, $v = v_0$, and $t = 0$ designated at the beginning of the interval, then for a time interval t the integrated equations become

$$\int_{v_0}^{v} dv = a \int_0^t dt \qquad \text{or} \qquad v = v_0 + at$$

$$\int_{v_0}^{v} v\, dv = a \int_{s_0}^{s} ds \qquad \text{or} \qquad v^2 = {v_0}^2 + 2a(s - s_0)$$

Substitution of the integrated expression for v into Eq. 2/1 and integration with respect to t give

$$\int_{s_0}^{s} ds = \int_0^t (v_0 + at)\, dt \qquad \text{or} \qquad s = s_0 + v_0 t + \tfrac{1}{2}at^2$$

(b) Acceleration Given as a Function of Time, $a = f(t)$. Substitution of the function into the first of Eqs. 2/2 gives $f(t) = dv/dt$. Multiplying by dt separates the variables and permits integration. Thus,

$$\int_{v_0}^{v} dv = \int_0^t f(t)\, dt \qquad \text{or} \qquad v = v_0 + \int_0^t f(t)\, dt$$

From this integrated expression for v as a function of t, the position coordinate s is obtained by integrating Eq. 2/1, which, in form, would be

$$\int_{s_0}^{s} ds = \int_0^t v\, dt \qquad \text{or} \qquad s = s_0 + \int_0^t v\, dt$$

(c) Acceleration Given as a Function of Velocity, a = f(v). Substitution of the function into the first of Eqs. 2/2 gives $f(v) = dv/dt$, which permits separating the variables and integrating. Thus,

$$t = \int_0^t dt = \int_{v_0}^{v} \frac{dv}{f(v)}$$

This result gives t as a function of v. Then it would be necessary to solve for v as a function of t so that Eq. 2/1 can be integrated to obtain the position coordinate s as a function of t.

Another approach is to substitute the function $a = f(v)$ into the first of Eqs. 2/3, giving $v\,dv = f(v)\,ds$. The variables can now be separated and the equation integrated in the form

$$\int_{v_0}^{v} \frac{v\,dv}{f(v)} = \int_{s_0}^{s} ds \qquad \text{or} \qquad s = s_0 + \int_{v_0}^{v} \frac{v\,dv}{f(v)}$$

(d) Acceleration Given as a Function of Displacement, a = f(s). Substituting the function into Eq. 2/3 and integrating give the form

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} f(s)\,ds \qquad \text{or} \qquad v^2 = {v_0}^2 + 2\int_{s_0}^{s} f(s)\,ds$$

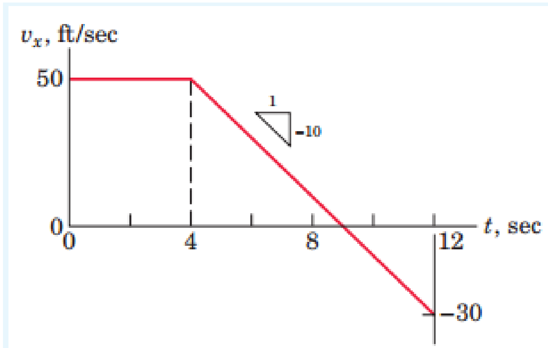
Next we solve for v to give $v = g(s)$, a function of s. Now we can substitute ds/dt for v, separate variables, and integrate in the form

$$\int_{s_0}^{s} \frac{ds}{g(s)} = \int_0^t dt \qquad \text{or} \qquad t = \int_{s_0}^{s} \frac{ds}{g(s)}$$

which gives t as a function of s. Finally, we can rearrange to obtain s as a function of t.

Example 1: Graphical Method

A particle moves along the x-axis with an initial velocity $v_x = 50$ ft/sec at the origin when $t = 0$. For the first 4 seconds it has no acceleration, and thereafter it is acted on by a retarding force which gives it a constant acceleration $a_x = -10$ ft/sec^2. Calculate the velocity and the x-coordinate of the particle for the conditions of $t = 8$ sec and $t = 12$ sec and find the maximum positive x-coordinate reached by the particle.



Solution. The velocity of the particle after $t = 4$ sec is computed from

$$\left[\int dv = \int a\,dt\right] \qquad \int_{50}^{v_x} dv_x = -10\int_4^t dt \qquad v_x = 90 - 10t \text{ ft/sec}$$

and is plotted as shown. At the specified times, the velocities are

$$t = 8 \text{ sec}, \qquad v_x = 90 - 10(8) = 10 \text{ ft/sec}$$

$$t = 12 \text{ sec}, \qquad v_x = 90 - 10(12) = -30 \text{ ft/sec} \qquad \textit{Ans.}$$

The x-coordinate of the particle at any time greater than 4 seconds is the distance traveled during the first 4 seconds plus the distance traveled after the discontinuity in acceleration occurred. Thus,

$$\left[\int ds = \int v\,dt\right] \qquad x = 50(4) + \int_4^t (90 - 10t)\,dt = -5t^2 + 90t - 80 \text{ ft}$$

For the two specified times,

$$t = 8 \text{ sec}, \qquad x = -5(8^2) + 90(8) - 80 = 320 \text{ ft}$$

$$t = 12 \text{ sec}, \qquad x = -5(12^2) + 90(12) - 80 = 280 \text{ ft} \qquad \textit{Ans.}$$

The x-coordinate for $t = 12$ sec is less than that for $t = 8$ sec since the motion is in the negative x-direction after $t = 9$ sec. The maximum positive x-coordinate is, then, the value of x for $t = 9$ sec which is

$$x_{max} = -5(9^2) + 90(9) - 80 = 325 \text{ ft} \qquad \textit{Ans.}$$

Example 2: integration method (acceleration as a function of time)

The position coordinate of a particle which is confined to move along a straight line is given by $s = 2t^3 - 24t + 6$, where s is measured in meters from a convenient origin and t is in seconds. Determine (*a*) the time required for the particle to reach a velocity of 72 m/s from its initial condition at $t = 0$, (*b*) the acceleration of the particle when $v = 30$ m/s, and (*c*) the net displacement of the particle during the interval from $t = 1$ s to $t = 4$ s.

Solution. The velocity and acceleration are obtained by successive differentiation of s with respect to the time. Thus,

$[v = \dot{s}]$ $$v = 6t^2 - 24 \text{ m/s}$$

$[a = \dot{v}]$ $$a = 12t \text{ m/s}^2$$

(a) Substituting $v = 72$ m/s into the expression for v gives us $72 = 6t^2 - 24$, from which $t = \pm 4$ s. The negative root describes a mathematical solution for t before the initiation of motion, so this root is of no physical interest. Thus, the desired result is

$$t = 4 \text{ s}$$ *Ans.*

(b) Substituting $v = 30$ m/s into the expression for v gives $30 = 6t^2 - 24$, from which the positive root is $t = 3$ s, and the corresponding acceleration is

$$a = 12(3) = 36 \text{ m/s}^2$$ *Ans.*

(c) The net displacement during the specified interval is

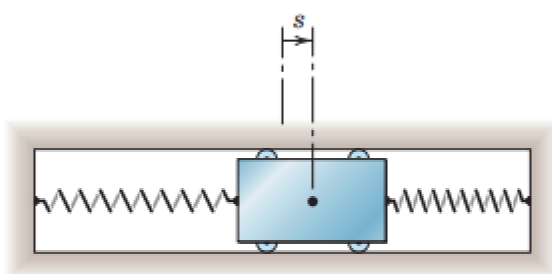
$$\Delta s = s_4 - s_1 \qquad \text{or}$$

$$\Delta s = [2(4^3) - 24(4) + 6] - [2(1^3) - 24(1) + 6]$$

$$= 54 \text{ m}$$ *Ans.*

Example 3. Integration Method (acceleration as a function of s)

The spring-mounted slider moves in the horizontal guide with negligible friction and has a velocity v_0 in the s-direction as it crosses the mid-position where $s = 0$ and $t = 0$. The two springs together exert a retarding force to the motion of the slider, which gives it an acceleration proportional to the displacement but oppositely directed and equal to $a = -k^2 s$, where k is constant. (The constant is arbitrarily squared for later convenience in the form of the expressions.) Determine the expressions for the displacement s and velocity v as functions of the time t.



Solution I. Since the acceleration is specified in terms of the displacement, the differential relation $v\,dv = a\,ds$ may be integrated. Thus,

$$\int v\,dv = \int -k^2 s\,ds + C_1 \text{ a constant,} \quad \text{or} \quad \frac{v^2}{2} = -\frac{k^2 s^2}{2} + C_1$$

When $s = 0$, $v = v_0$, so that $C_1 = v_0^2/2$, and the velocity becomes

$$v = +\sqrt{v_0^2 - k^2 s^2}$$

The plus sign of the radical is taken when v is positive (in the plus s-direction). This last expression may be integrated by substituting $v = ds/dt$. Thus,

$$\int \frac{ds}{\sqrt{v_0^2 - k^2 s^2}} = \int dt + C_2 \text{ a constant,} \quad \text{or} \quad \frac{1}{k}\sin^{-1}\frac{ks}{v_0} = t + C_2$$

With the requirement of $t = 0$ when $s = 0$, the constant of integration becomes $C_2 = 0$, and we may solve the equation for s so that

$$s = \frac{v_0}{k}\sin kt \qquad \textit{Ans.}$$

The velocity is $v = \dot{s}$, which gives

$$v = v_0 \cos kt \qquad \textit{Ans.}$$

Example 4: integration method (acceleration as a function of v)

A freighter is moving at a speed of 8 knots when its engines are suddenly stopped. If it takes 10 minutes for the freighter to reduce its speed to 4 knots, determine and plot the distance s in nautical miles moved by the ship and its speed v in knots as functions of the time t during this interval. The deceleration of the ship is proportional to the square of its speed, so that $a = -kv^2$.

Solution. The speeds and the time are given, so we may substitute the expression for acceleration directly into the basic definition $a = dv/dt$ and integrate. Thus,

$$-kv^2 = \frac{dv}{dt} \qquad \frac{dv}{v^2} = -k\,dt \qquad \int_8^v \frac{dv}{v^2} = -k\int_0^t dt$$

$$-\frac{1}{v} + \frac{1}{8} = -kt \qquad v = \frac{8}{1 + 8kt}$$

Now we substitute the end limits of $v = 4$ knots and $t = \frac{10}{60} = \frac{1}{6}$ hour and get

$$4 = \frac{8}{1 + 8k(1/6)} \qquad k = \frac{3}{4}\,\text{mi}^{-1} \qquad v = \frac{8}{1 + 6t} \qquad \textit{Ans.}$$

The speed is plotted against the time as shown.

The distance is obtained by substituting the expression for v into the definition $v = ds/dt$ and integrating. Thus,

$$\frac{8}{1 + 6t} = \frac{ds}{dt} \qquad \int_0^t \frac{8\,dt}{1 + 6t} = \int_0^s ds \qquad s = \frac{4}{3}\ln(1 + 6t) \qquad \textit{Ans.}$$

The distance s is also plotted against the time as shown, and we see that the ship has moved through a distance $s = \frac{4}{3}\ln(1 + \frac{6}{6}) = \frac{4}{3}\ln 2 = 0.924$ mi (nautical) during the 10 minutes.

Assignment 1(Homework)

Problems from Page 33- page 39 in the Textbook