Al-Mustaqbal University

College of Engineering & Technology

Biomedical Engineering Department

Subject Name: Dynamics

1st Class, Second Semester

Subject Code: [UOMU011024]

Academic Year: 2024-2025

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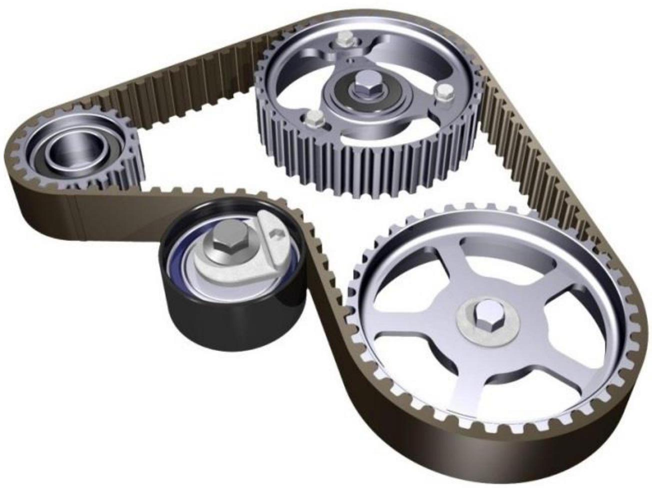
Lecture No.:- 6

Lecture Title: [Plane Kinematics of Rigid Bodies]



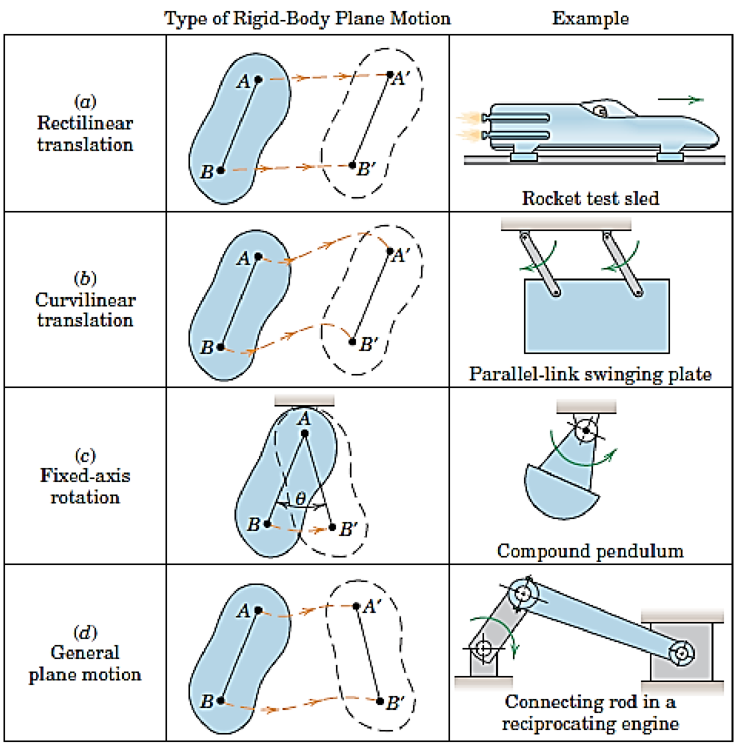
Dynamics of Rigid Bodies

Plane Kinematics of Rigid Bodies

- It can be defined as a *rigid body* as a system of particles for which the distances between the particles remain unchanged.

Plane Motion:

- ***Translation***
- ***Rotation***
- ***General plane motion***



Angular-Motion Relations

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} \qquad \text{or} \qquad \alpha = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\omega\, d\omega = \alpha\, d\theta \qquad \text{or} \qquad \dot{\theta}\, d\dot{\theta} = \ddot{\theta}\, d\theta$$

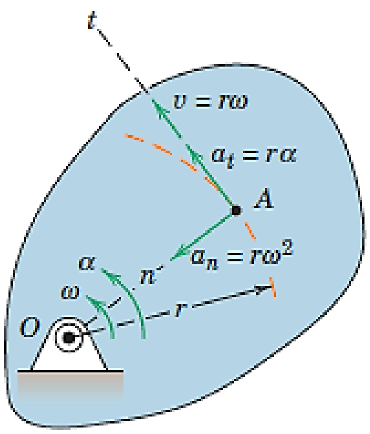
For rotation with *constant* angular acceleration

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = {\omega_0}^2 + 2\alpha(\theta - \theta_0)$$

$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$

Rotation about a Fixed Axis



$$v = r\omega$$

$$a_n = r\omega^2 = v^2/r = v\omega$$

$$a_t = r\alpha$$

SAMPLE PROBLEM 5/1

A flywheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time $t = 0$. The torque produces a counterclockwise angular acceleration $\alpha = 4t$ rad/s^2, where t is the time in seconds during which the torque is applied. Determine (*a*) the time required for the flywheel to reduce its clockwise angular speed to 900 rev/min, (*b*) the time required for the flywheel to reverse its direction of rotation, and (*c*) the total number of revolutions, clockwise plus counterclockwise, turned by the flywheel during the first 14 seconds of torque application.

Solution. The counterclockwise direction will be taken arbitrarily as positive.

(a) Since α is a known function of the time, we may integrate it to obtain angular
1 velocity. With the initial angular velocity of $-1800(2\pi)/60 = -60\pi$ rad/s, we have

$$[d\omega = \alpha\, dt] \qquad \int_{-60\pi}^{\omega} d\omega = \int_0^t 4t\, dt \qquad \omega = -60\pi + 2t^2$$

Substituting the clockwise angular speed of 900 rev/min or $\omega = -900(2\pi)/60 = -30\pi$ rad/s gives

$$-30\pi = -60\pi + 2t^2 \qquad t^2 = 15\pi \qquad t = 6.86 \text{ s} \qquad \textit{Ans.}$$

(b) The flywheel changes direction when its angular velocity is momentarily zero. Thus,

$$0 = -60\pi + 2t^2 \qquad t^2 = 30\pi \qquad t = 9.71 \text{ s} \qquad \textit{Ans.}$$

(c) The total number of revolutions through which the flywheel turns during 14 seconds is the number of clockwise turns N_1 during the first 9.71 seconds, plus the number of counterclockwise turns N_2 during the remainder of the interval. Integrating the expression for ω in terms of t gives us the angular displacement in radians. Thus, for the first interval

$$[d\theta = \omega\, dt] \qquad \int_0^{\theta_1} d\theta = \int_0^{9.71} (-60\pi + 2t^2)\, dt$$

2
$$\theta_1 = [-60\pi t + \tfrac{2}{3}t^3]_0^{9.71} = -1220 \text{ rad}$$

or $N_1 = 1220/2\pi = 194.2$ revolutions clockwise.

For the second interval

$$\int_0^{\theta_2} d\theta = \int_{9.71}^{14} (-60\pi + 2t^2)\, dt$$

3
$$\theta_2 = [-60\pi t + \tfrac{2}{3}t^3]_{9.71}^{14} = 410 \text{ rad}$$

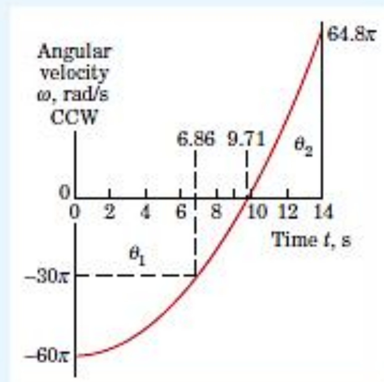
or $N_2 = 410/2\pi = 65.3$ revolutions counterclockwise. Thus, the total number of revolutions turned during the 14 seconds is

$$N = N_1 + N_2 = 194.2 + 65.3 = 259 \text{ rev} \qquad \textit{Ans.}$$

We have plotted ω versus t and we see that θ_1 is represented by the negative area and θ_2 by the positive area. If we had integrated over the entire interval in one step, we would have obtained $|\theta_2| - |\theta_1|$.

Helpful Hints

1 We must be very careful to be consistent with our algebraic signs. The lower limit is the negative (clockwise) value of the initial angular velocity. Also we must convert revolutions to radians since α is in radian units.

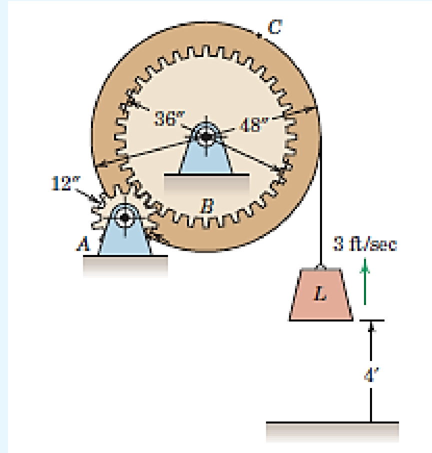


2 Again note that the minus sign signifies clockwise in this problem.

3 We could have converted the original expression for α into the units of rev/s^2, in which case our integrals would have come out directly in revolutions.

SAMPLE PROBLEM 5/2

The pinion A of the hoist motor drives gear B, which is attached to the hoisting drum. The load L is lifted from its rest position and acquires an upward velocity of 3 ft/sec in a vertical rise of 4 ft with constant acceleration. As the load passes this position, compute (*a*) the acceleration of point C on the cable in contact with the drum and (*b*) the angular velocity and angular acceleration of the pinion A.



Solution. ***(a)*** If the cable does not slip on the drum, the vertical velocity and acceleration of the load L are, of necessity, the same as the tangential velocity v and tangential acceleration a_t of point C. For the rectilinear motion of L with constant acceleration, the n- and t-components of the acceleration of C become

$[v^2 = 2as]$ $\quad a = a_t = v^2/2s = 3^2/[2(4)] = 1.125 \text{ ft/sec}^2$

① $[a_n = v^2/r]$ $\quad a_n = 3^2/(24/12) = 4.5 \text{ ft/sec}^2$

$[a = \sqrt{a_n^2 + a_t^2}]$ $\quad a_C = \sqrt{(4.5)^2 + (1.125)^2} = 4.64 \text{ ft/sec}^2$ *Ans.*

(b) The angular motion of gear A is determined from the angular motion of gear B by the velocity v_1 and tangential acceleration a_1 of their common point of contact. First, the angular motion of gear B is determined from the motion of point C on the attached drum. Thus,

$[v = r\omega]$ $\quad \omega_B = v/r = 3/(24/12) = 1.5 \text{ rad/sec}$

$[a_t = r\alpha]$ $\quad \alpha_B = a_t/r = 1.125/(24/12) = 0.562 \text{ rad/sec}^2$

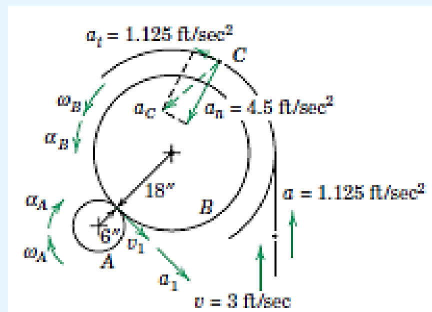
Then from $v_1 = r_A\omega_A = r_B\omega_B$ and $a_1 = r_A\alpha_A = r_B\alpha_B$, we have

$$\omega_A = \frac{r_B}{r_A}\omega_B = \frac{18/12}{6/12}1.5 = 4.5 \text{ rad/sec CW} \qquad \textit{Ans.}$$

$$\alpha_A = \frac{r_B}{r_A}\alpha_B = \frac{18/12}{6/12}0.562 = 1.688 \text{ rad/sec}^2 \text{ CW} \qquad \textit{Ans.}$$

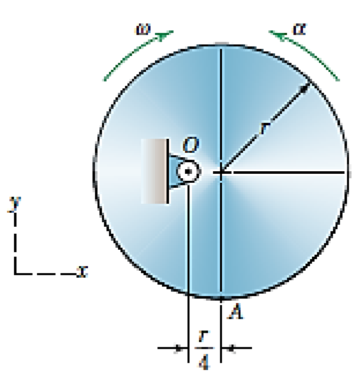
Helpful Hint

① Recognize that a point on the cable changes the direction of its velocity after it contacts the drum and acquires a normal component of acceleration.

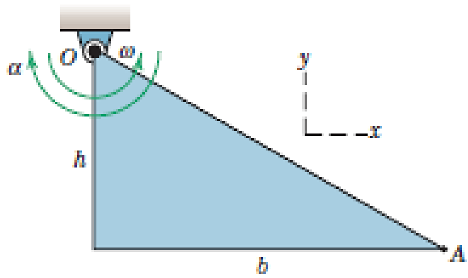


Problems for solving.

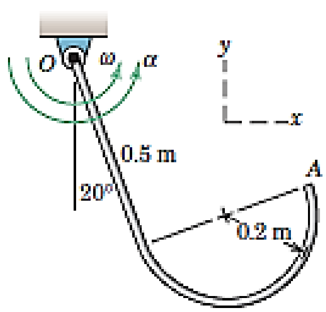
5/1 The circular disk of radius $r = 0.16$ m rotates about a fixed axis through point O with the angular properties $\omega = 2$ rad/s and $\alpha = 3$ rad/s² with directions as shown in the figure. Determine the instantaneous values of the velocity and acceleration of point A.



5/2 The triangular plate rotates about a fixed axis through point O with the angular properties indicated. Determine the instantaneous velocity and acceleration of point A. Take all given variables to be positive.

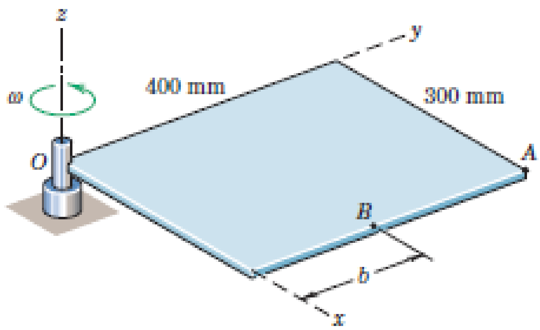


5/3 The body is formed of slender rod and rotates about a fixed axis through point O with the indicated angular properties. If $\omega = 4$ rad/s and $\alpha = 7$ rad/s^2, determine the instantaneous velocity and acceleration of point A.



5/4 A torque applied to a flywheel causes it to accelerate uniformly from a speed of 200 rev/min to a speed of 800 rev/min in 4 seconds. Determine the number of revolutions N through which the wheel turns during this interval. (*Suggestion:* Use revolutions and minutes for units in your calculations.)

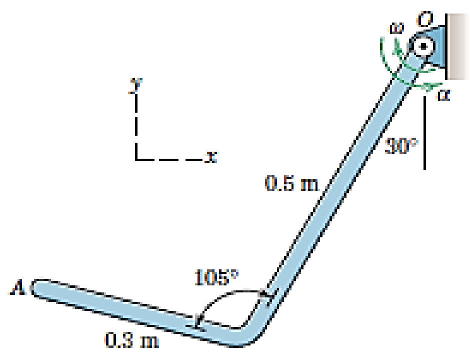
5/7 The rectangular plate is rotating about its corner axis through O with a constant angular velocity $\omega = 10$ rad/s. Determine the magnitudes of the velocity $\mathbf{v}$ and acceleration $\mathbf{a}$ of the corner A by (*a*) using the scalar relations and (*b*) using the vector relations.



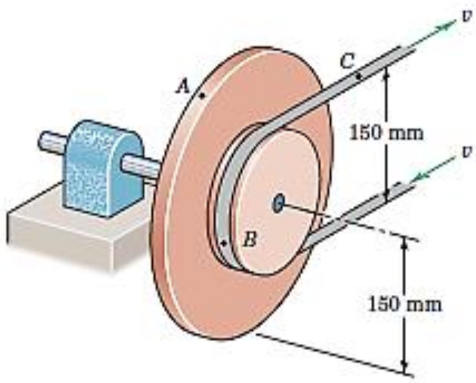
5/8 If the rectangular plate of Prob. 5/7 starts from rest and point B has an initial acceleration of 5.5 m/s^2, determine the distance b if the plate reaches an angular speed of 300 rev/min in 2 seconds with a constant angular acceleration.

5/9 A shaft is accelerated from rest at a constant rate to a speed of 3600 rev/min and then is immediately decelerated to rest at a constant rate within a total time of 10 seconds. How many revolutions N has the shaft turned during this interval?

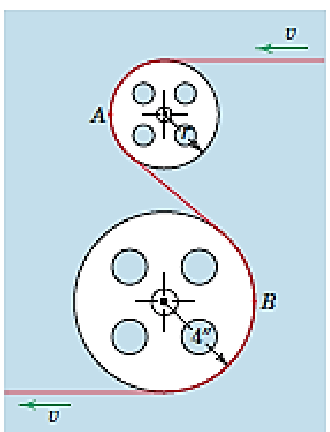
5/10 The bent flat bar rotates about a fixed axis through point O. At the instant depicted, its angular properties are $\omega = 5$ rad/s and $\alpha = 8$ rad/s^2 with directions as indicated in the figure. Determine the instantaneous velocity and acceleration of point A.



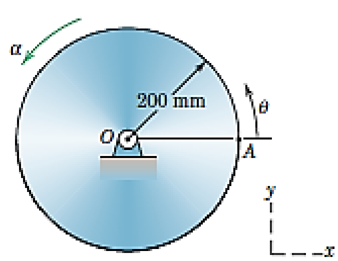
5/17 The belt-driven pulley and attached disk are rotating with increasing angular velocity. At a certain instant the speed v of the belt is 1.5 m/s, and the total acceleration of point A is 75 m/s^2. For this instant determine (*a*) the angular acceleration α of the pulley and disk, (*b*) the total acceleration of point B, and (*c*) the acceleration of point C on the belt.



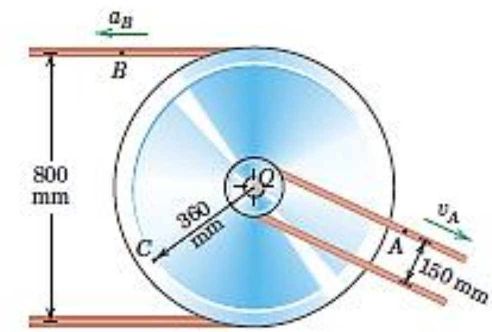
5/18 Magnetic tape is being fed over and around the light pulleys mounted in a computer. If the speed v of the tape is constant and if the magnitude of the acceleration of point A on the tape is 4/3 times that of point B, calculate the radius r of the smaller pulley.



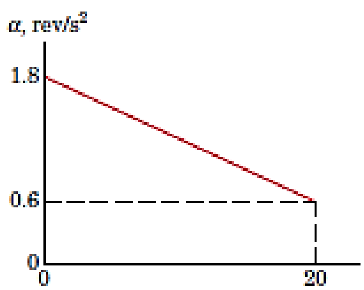
5/20 Point A of the circular disk is at the angular position $\theta = 0$ at time $t = 0$. The disk has angular velocity $\omega_0 = 0.1$ rad/s at $t = 0$ and subsequently experiences a constant angular acceleration $\alpha = 2$ rad/s^2. Determine the velocity and acceleration of point A in terms of fixed **i** and **j** unit vectors at time $t = 1$ s.



5/26 The two V-belt pulleys form an integral unit and rotate about the fixed axis at O. At a certain instant, point A on the belt of the smaller pulley has a velocity $v_A = 1.5$ m/s, and point B on the belt of the larger pulley has an acceleration $a_B = 45$ m/s^2 as shown. For this instant determine the magnitude of the acceleration $\mathbf{a}_C$ of point C and sketch the vector in your solution.



5/27 A clockwise variable torque is applied to a flywheel at time $t = 0$ causing its clockwise angular acceleration to decrease linearly with angular displacement θ during 20 revolutions of the wheel as shown. If the clockwise speed of the flywheel was 300 rev/min at $t = 0$, determine its speed N after turning the 20 revolutions. (*Suggestion:* Use units of revolutions instead of radians.)



5/28 The design characteristics of a gear-reduction unit are under review. Gear B is rotating clockwise with a speed of 300 rev/min when a torque is applied to gear A at time $t = 2$ s to give gear A a counterclockwise acceleration α which varies with time for a duration of 4 seconds as shown. Determine the speed N_B of gear B when $t = 6$ s.