جامــــعـة المـــســتـقـبـل
AL MUSTAQBAL UNIVERSITY

قســـم الامــــــن الـــــــــســـيبرانـــــــــي

Department of Cyber Security

Subject:

Functions

Class:

First

Lecturer:

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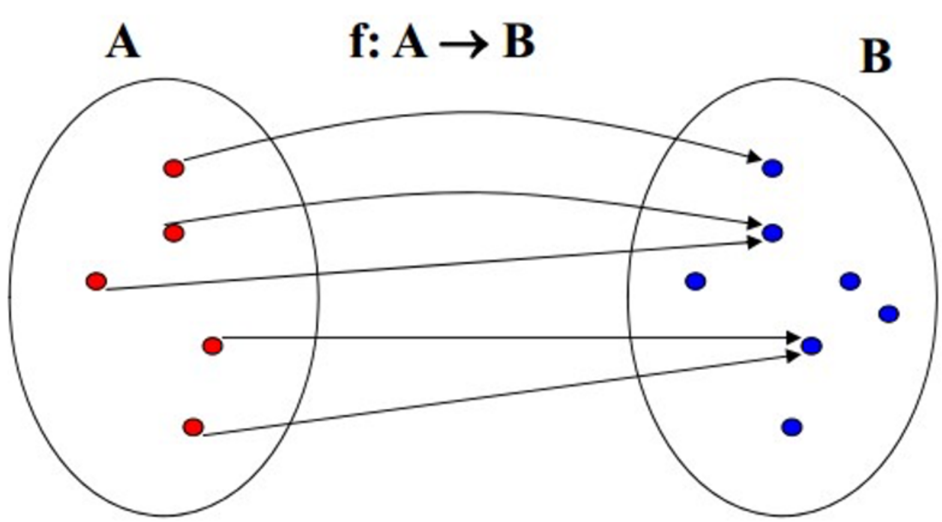
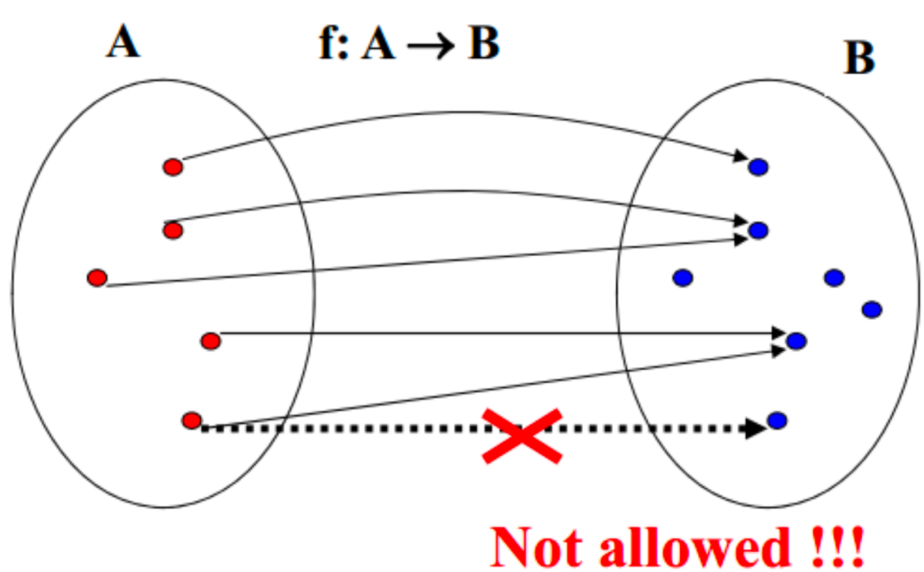
Lecture: (7)

Functions

Definition: Let A and B be two sets. A function from A to B, denoted

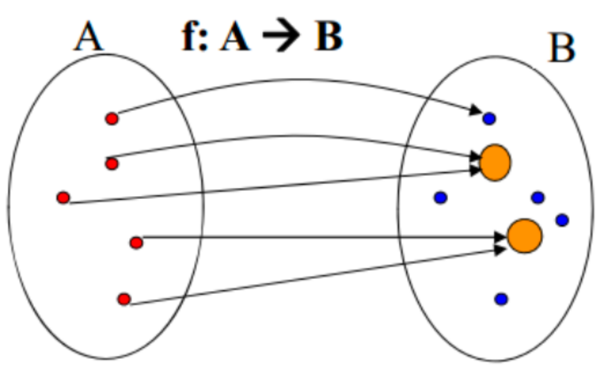
f: A → B, is an assignment of exactly one element of B to each element of A. We write f(a) = b to denote the assignment of b to an element a of A by the function f.



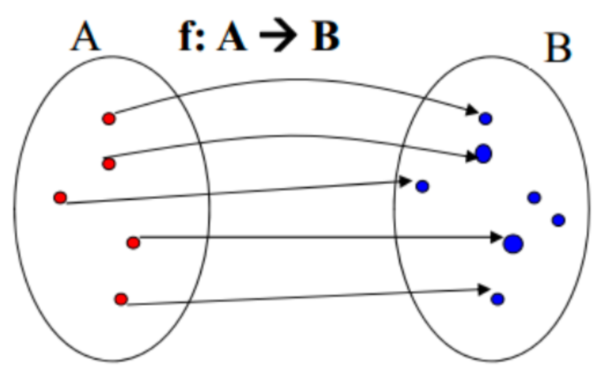


Injective function

Definition: A function f is said to be one-to-one, or injective, if and only if f(x) = f(y) implies x = y for all x, y in the domain of f. A function is said to be an injection if it is one-to-one. Alternative: A function is one-to-one if and only if f(x) # f(y), whenever x # y. This is contrapositive of the definition.



Not injective function

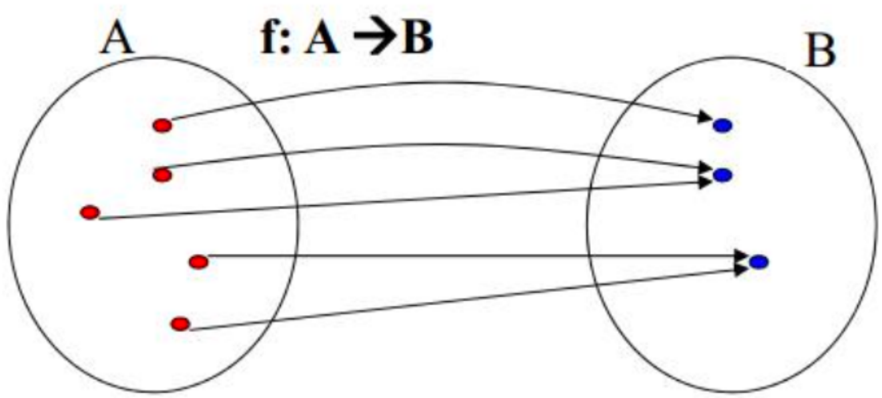


Injective function

Surjective function

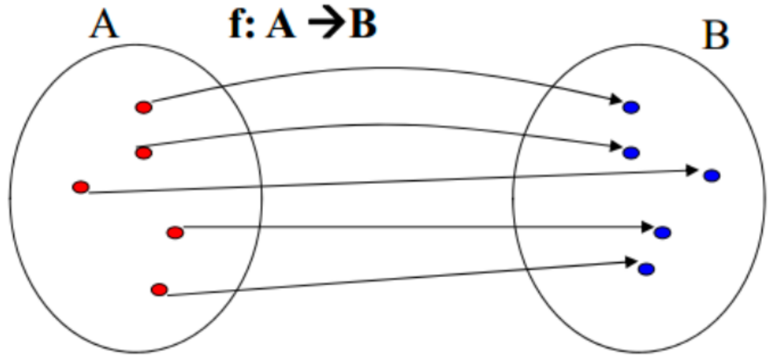
Definition: A function f from A to B is called onto, or surjective, if and only if for every b € B there is an element a e A such that f(a) = b.

Alternative: all co-domain elements are covered



Bijective functions

Definition: A function f is called a bijection if it is both one-to-one (injection) and onto (surjection).



Example 1:

- Let A = {1,2,3} and B = {a,b,c}
- Define fa
- 1 → c
- 2 → a
- 3 → b
- Is fa bijection?

Yes. It is both one-to-one and onto.

Example 2:

- Define g: W → W (whole numbers), where g(n) = [n/2] (floor function).
- 0 → [0/2] = [0] = 0
- 1 → [1/2] = [1/2] = 0
- 2 → [2/2] = [1] = 1
- 3→ [3/2] = [3/2] = 1

• Is g a bijection?

No. g is onto but not 1-1 (g(0) = g(1) = 0 however 0 # 1.

Theorem: Let f be a function f: A → A from a set A to itself, where A is finite. Then f is one-to-one if and only if f is onto.

Assume → A is finite, and f is one-to-one (injective)

• Is f an onto function (surjection)?

Theorem: Let f be a function f: A → A from a set A to itself, where A is finite. Then f is one-to-one if and only if f is onto.

Proof:

→ A is **finite, and f is one-to-one (injective)**

Is f an onto function (surjection)?

Yes. Every element point to exactly one element. Injection assures they are different. So, we have A different elements A points to. Since f: A → A the co-domain is covered thus the function is also a surjection (and a bijection)

←A is finite, and f is an onto function

• Is the function one-to-one?

Theorem: Let f be a function f: A →A from a set A to itself, where A is finite. Then f is one-to-one if and only if f is onto.

Proof:

→ A is finite, and f is one-to-one (injective)

Is f an onto function (surjection)?

Yes. Every element points to exactly one element. Injection assures they are different. So, we have A different elements A points to. Since f: A → A the co-domain is covered thus the function is also a surjection (and a bijection)

A is finite, and f is an onto function

- Is the function one-to-one?

Yes. Every element map to exactly one element and all elements in A are covered. Thus, the mapping must be one-to-