Al-Mustaqbal University

College of Engineering & Technology

Building and Construction Techniques Engineering Dep.

Class (2nd)

STRENGTH OF MATERIALS

Lecture 3 & 4

Beams &

Shear Force and Bending Moment diagram

Lecturer:

Dr. Ammar Adil Albakri

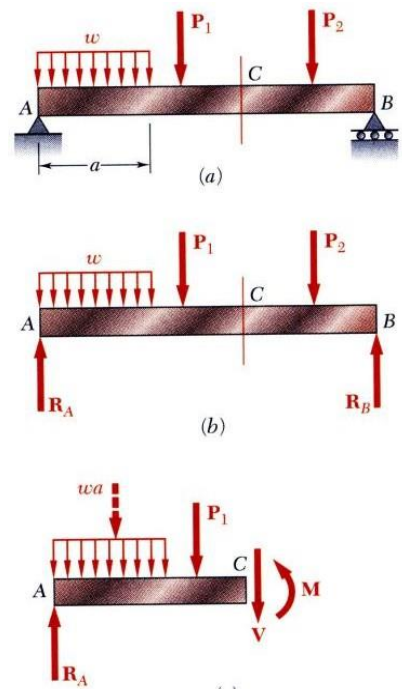
MECHANICS OF MATERIALS

STRESSES IN BEAMS

BEAMS

Introduction:

- *Beams* - structural members supporting loads at various points along the member.
- Transverse loadings of beams are classified as *concentrated* loads or *distributed* loads.
- Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution).

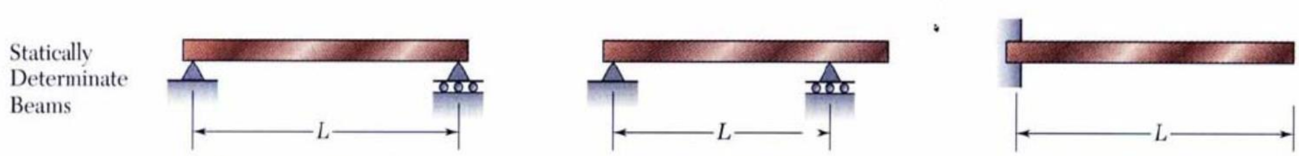


Classification of Beams:

1- Statically Determinate Beams:

Statically determinate beams are those beams in which the reactions of the supports may be determined by the use of the equations of static equilibrium. The beams shown below are examples of statically of statically determinate beams.

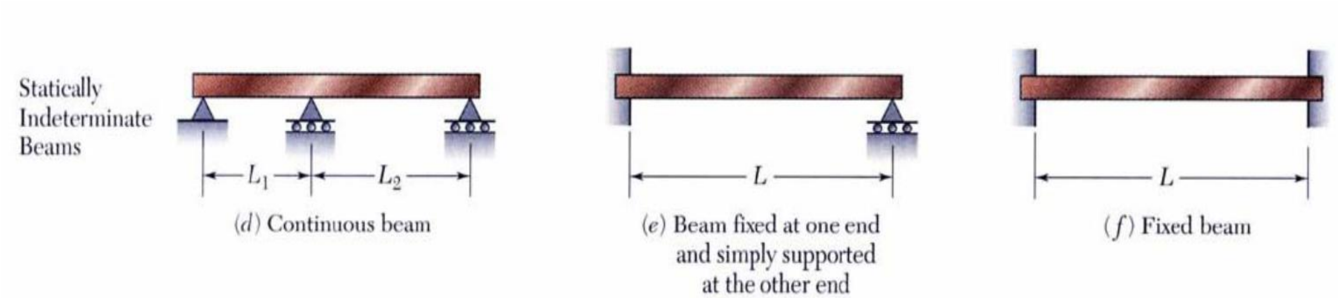
Classification of Beam Supports



(a) Simply supported beam (b) Overhanging beam (c) Cantilever beam

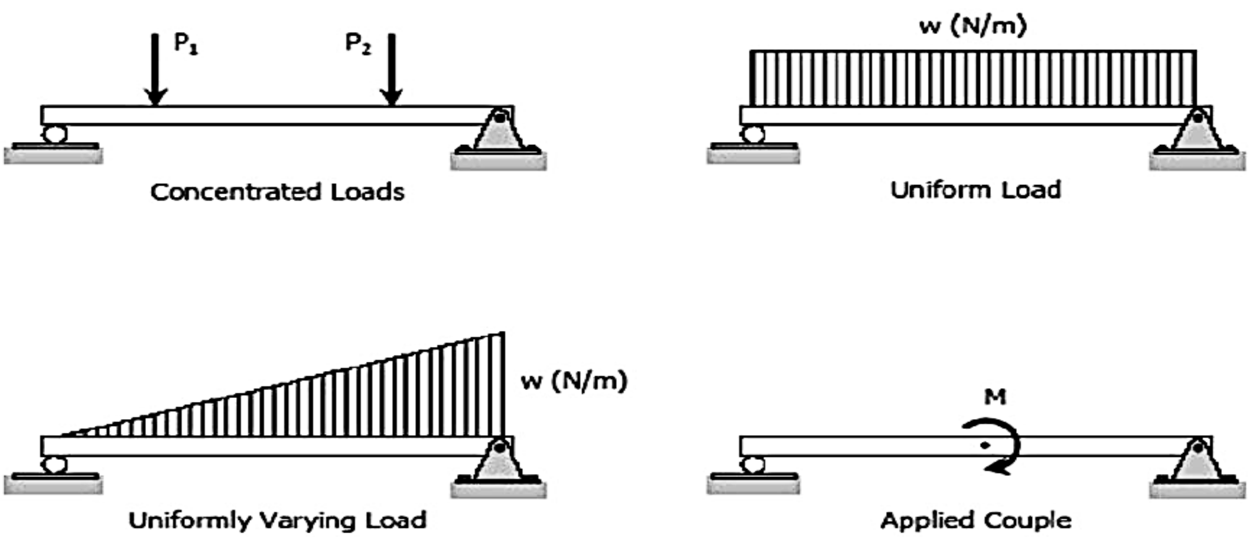
2- Statically Indeterminate Beams:

If the number of reactions exerted upon a beam exceeds the number of equations in static equilibrium, the beam is said to be statically indeterminate. In order to solve the reactions of the beam, the static equations must be supplemented by equations based upon the elastic deformations of the beam.



<u>TYPES OF LOADING</u>

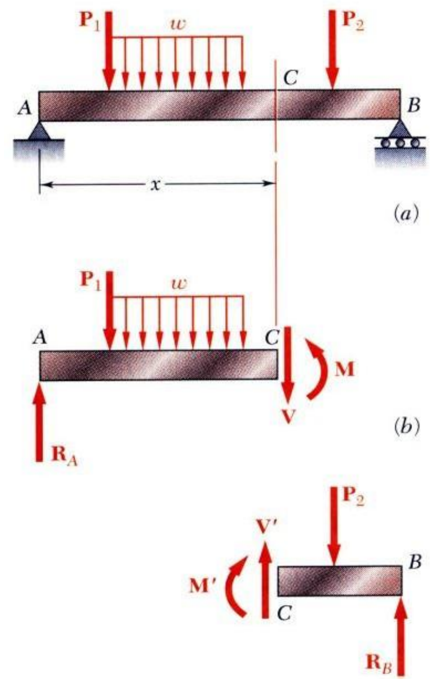
Loads applied to the beam may consist of a concentrated load (load applied at a point), uniform load, uniformly varying load, or an applied couple or moment. These loads are shown in the following figures.



Shear Force and Bending Moment diagram

Shear Force and Bending Moment Diagrams are plots of the shear forces and bending moments, respectively, along the length of a beam. **The purpose of these plots is to clearly show maximum of the shear force and bending moment, which are important in the design of beams.**

The most common sign convention for the shear force and bending moment in beams is shown in Fig. 1.



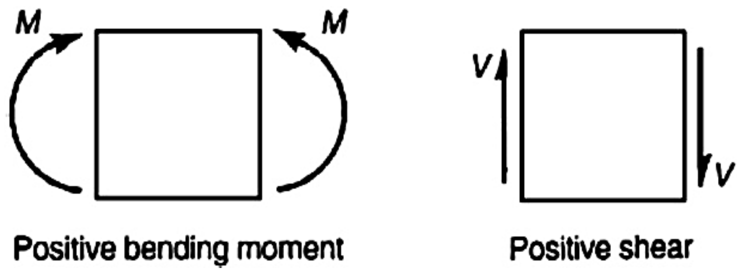


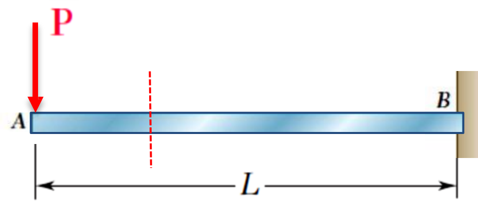
Fig. 1. Sign convention for the shear force and bending moment in beams.

Determining shear forces and bending moments along the length of a beam typically involves three steps:

1. Draw the free body diagram of our beam.
2. Determine the reactions forces and moments from equilibrium of the entire beam by using the equilibrium equations.
3. Cut the beam at a single location and use the equilibrium equations to determine the shear force and bending moment at that location.
4. Repeat this process for each location along the beam.
5. Draw the result on our shear force and bending moment diagrams.

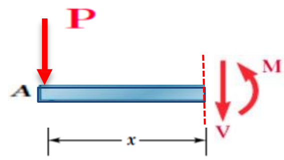
Example (1)

For the beam shown, derive equations for shear force and bending moment at any point along the beam.



Solution:

We cut the beam at a point between A and B at distance x from A and draw thefree-body diagram of the left part of the beam, directing **V** and **M** as indicated in the figure.



$\Sigma F_y = 0$:

$$P + V = 0$$

$$V = -P \quad (\downarrow)$$

$\Sigma M_x = 0$:

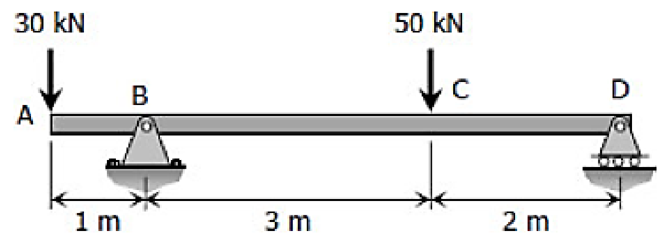
$$P.x + M = 0$$

$$M = -Px\ (\circlearrowleft)$$

- ***Note** that shear force is constant (equal P) along the beam, and bending moment is a linear function of (**x**).*

Example 2:

Beam loaded as shown in figure. Write the shear and moment equations then draw the shear and moment diagrams.



Solution:

From the load diagram:

$\Sigma F_y = 0$

$R_B + R_D = 30 + 50$ …… (1)

$\Sigma M_B = 0$

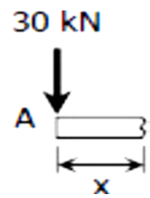
$5R_D + 1(30) = 3(50)$ ➡ $R_D = 24$ kN

$R_B = 56$ kN

Segment AB:

$V_{AB} = -30\,\text{kN}$

$M_{AB} = -30x\,\text{kN}\cdot\text{m}$



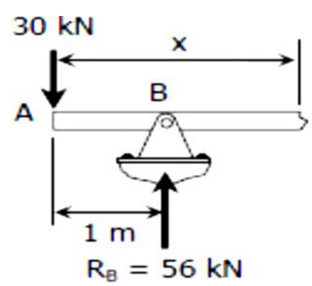
Segment BC:

$V_{BC} = -30 + 56$

$V_{BC} = 26\,\text{kN}$

$M_{BC} = -30x + 56(x-1)$

$M_{BC} = 26x - 56\,\text{kN}\cdot\text{m}$



Segment CD:

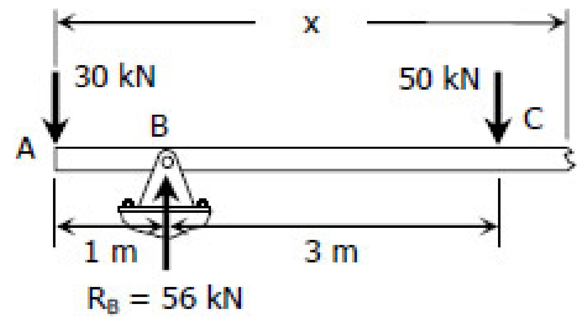
$V_{CD} = -30 + 56 - 50$

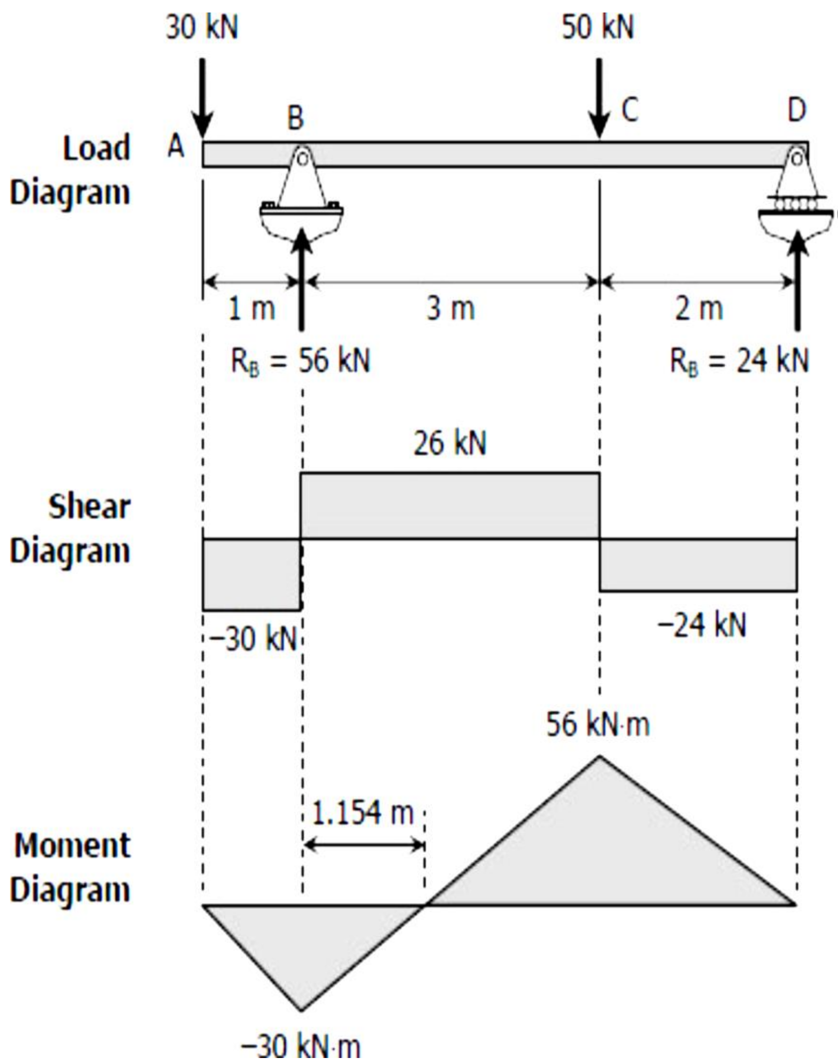
$= -24 \text{ kN}$

$M_{CD} = -30x + 56(x - 1) - 50(x - 4)$

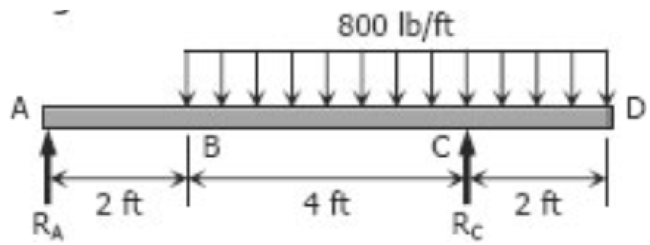
$= -30x + 56x - 56 - 50x + 200$

$= -24x + 144$





Example 3:

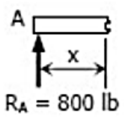


Beam loaded as shown in figure. Write the shear and moment equations then draw the shear and moment diagrams.

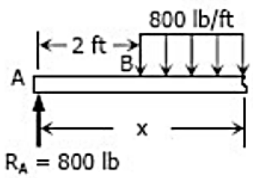
<u>Solution</u>

$\sum M_A = 0$
$6R_C = 5[6(800)]$
$R_C = 4000$ lb

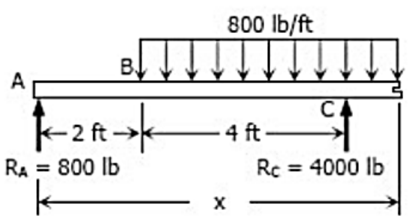
$\sum M_C = 0$
$6R_A = 1[6(800)]$
$R_A = 800$ lb



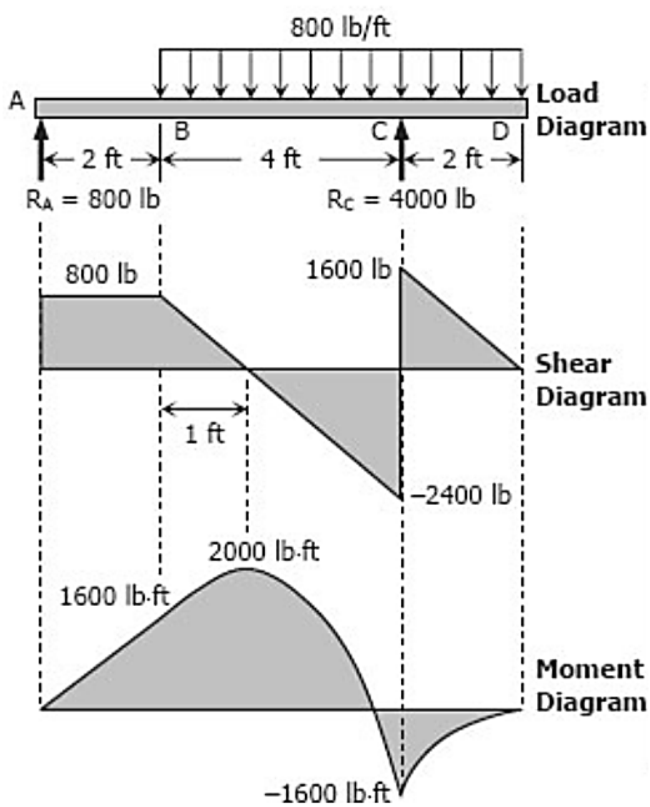
Segment AB:
$V_{AB} = 800$ lb
$M_{AB} = 800x$



Segment BC:
$V_{BC} = 800 - 800(x - 2)$
$= 2400 - 800x$
$M_{BC} = 800x - 800(x - 2)(x - 2)/2$
$= 800x - 400(x - 2)^2$

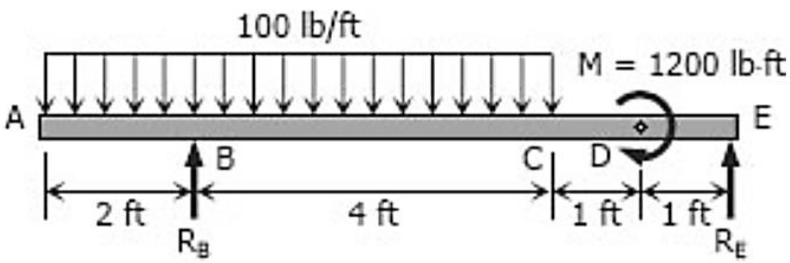


Segment CD:
$V_{CD} = 800 + 4000 - 800(x - 2)$
$= 4800 - 800x + 1600$
$= 6400 - 800x$
$M_{CD} = 800x + 4000(x - 6) - 800(x - 2)(x - 2)/2$
$= 800x + 4000(x - 6) - 400(x - 2)^2$



Example 4:

Beam loaded as shown in figure. Write the shear and moment equations then draw the shear and moment diagrams.



Solution

$\sum M_B = 0$

$6R_E = 1200 + 1[6(100)]$

$R_E = 300$ lb

$\sum M_E = 0$

$6R_B + 1200 = 5[6(100)]$

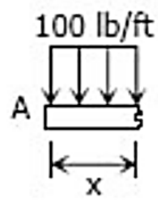
$R_B = 300$ lb

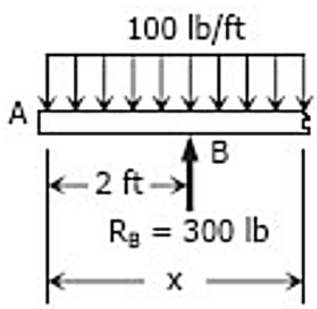
Segment *AB*:

$V_{AB} = -100x$ lb

$M_{AB} = -100x(x/2)$

$= -50x^2$ lb·ft



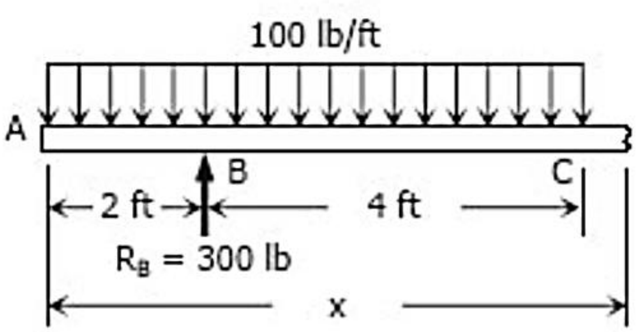


Segment *BC*:

$V_{BC} = -100x + 300$ lb

$M_{BC} = -100x(x/2) + 300(x - 2)$

$= -50x^2 + 300x - 600$ lb·ft



Segment *CD*:

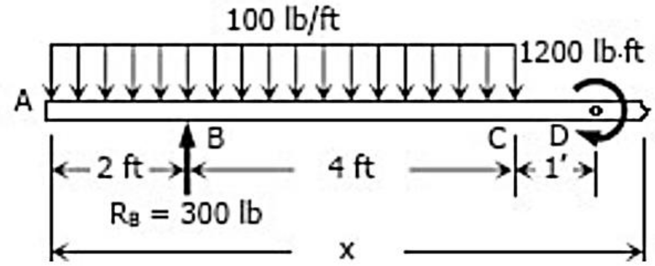
$V_{CD} = -100(6) + 300$

$= -300$ lb

$M_{CD} = -100(6)(x - 3) + 300(x - 2)$

$= -600x + 1800 + 300x - 600$

$= -300x + 1200$ lb·ft



Segment DE:

$$V_{DE} = -100(6) + 300$$
$$= -300 \text{ lb}$$

$$M_{DE} = -100(6)(x-3) + 1200 + 300(x-2)$$
$$= -600x + 1800 + 1200 + 300x - 600$$
$$= -300x + 2400$$