Active Filters

There are three major limitations to the passive filters considered in the previous section. First, they cannot generate gain greater than 1; passive elements cannot add energy to the network. Second, they may require bulky and expensive inductors. Third, they perform poorly at frequencies below the audio frequency range (300 Hz < f < 3,000 Hz). Nevertheless, passive filters are useful at high frequencies.

Active filters consist of combinations of resistors, capacitors, and op amps. They offer some advantages over passive RLC filters. First, they are often smaller and less expensive, because they do not require inductors. This makes feasible the integrated circuit realizations of filters. Second, they can provide amplifier gain in addition to providing the same frequency response as RLC filters. Third, active filters can be combined with buffer amplifiers (voltage followers) to isolate each stage of the filter from source and load impedance effects. This isolation allows designing the stages independently and then cascading them to realize the desired transfer function. (Bode plots, being logarithmic, may be added when transfer functions are cascaded.) However, active filters are less reliable and less stable. The practical limit of most active filters is about 100 kHz—most active filters operate well below that frequency. Filters are often classified according to their order (or number of poles) or their specific design type.

1. First-Order Lowpass Filter

One type of first-order filter is shown in Fig. 1.

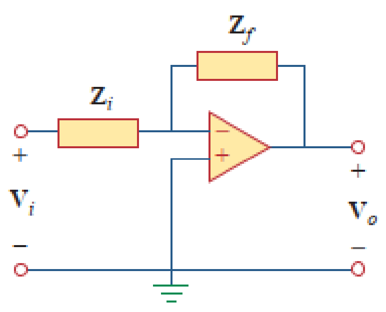


Fig .1 A general first-order active filter.

The components selected for and determine whether the filter is lowpass or high-pass, but one of the components must be reactive. Figure 2 shows a typical active lowpass filter.

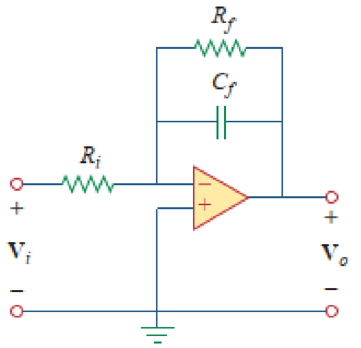


Fig.2 Active first-order lowpass filter.

For this filter, the transfer function is:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} \qquad (1)$$

where $\mathbf{Z}_i = R_i$ and

$$\mathbf{Z}_f = R_f \left\| \frac{1}{j\omega C_f} = \frac{R_f/j\omega C_f}{R_f + 1/j\omega C_f} = \frac{R_f}{1 + j\omega C_f R_f} \right. \qquad (2)$$

Therefore,

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_f R_f} \qquad (3)$$

the corner frequency is:

$$\omega_c = \frac{1}{R_f C_f} \qquad (4)$$

2. First-Order Highpass Filter

Figure 3 shows a typical highpass filter.

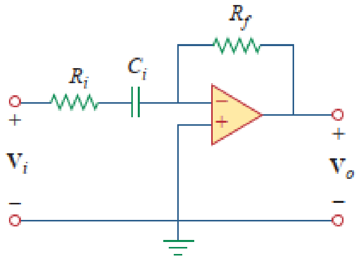


Fig.3 Active first-order highpass filter.

As before,

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{\mathbf{Z}_f}{\mathbf{Z}_i} \quad (5)$$

where $\mathbf{Z}_i = R_i + 1/j\omega C_i$ and $\mathbf{Z}_f = R_f$ so that

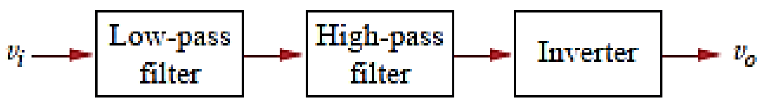
$$\mathbf{H}(\omega) = -\frac{R_f}{R_i + 1/j\omega C_i} = -\frac{j\omega C_i R_f}{1 + j\omega C_i R_i} \quad (6)$$

the corner frequency is:

$$\omega_c = \frac{1}{R_i C_i} \quad (7)$$

3. Bandpass Filter

The circuit in Fig. 2 may be combined with that in Fig. 3 to form a bandpass filter that will have a gain K over the required range of frequencies. By cascading a unity-gain lowpass filter, a unity-gain highpass filter, and an inverter with gain $-R_f / R_i$ as shown in the block diagram of Fig. 4(a), we can construct a bandpass filter whose frequency response is that in Fig. 4(b).



(a)

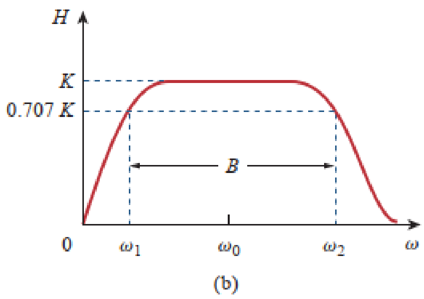


Fig.4 Active bandpass filter: (a) block diagram, (b) frequency response.

The actual construction of the bandpass filter is shown in Fig. 5.

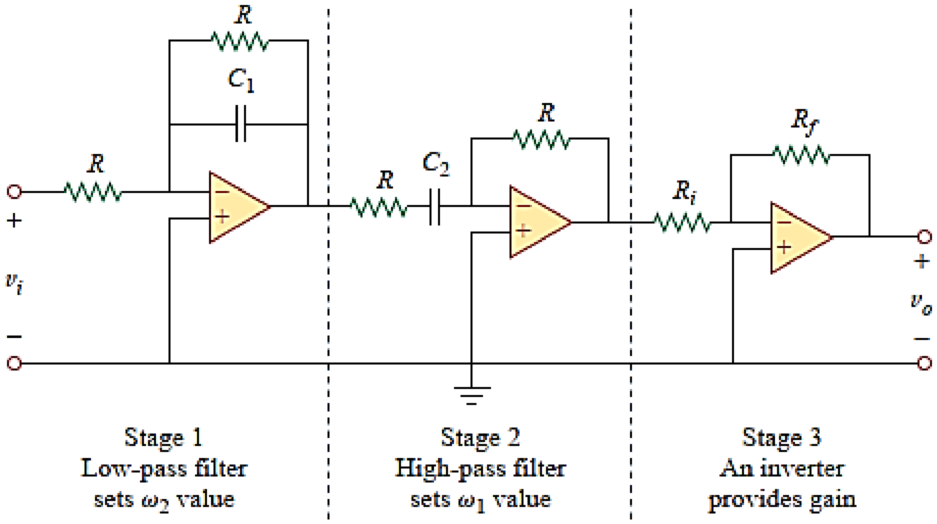


Fig. 5 Active bandpass filter.

The analysis of the bandpass filter is relatively simple. Its transfer function is:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \left(-\frac{1}{1 + j\omega C_1 R}\right)\left(-\frac{j\omega C_2 R}{1 + j\omega C_2 R}\right)\left(-\frac{R_f}{R_i}\right) \qquad (8)$$

$$= -\frac{R_f}{R_i}\frac{1}{1 + j\omega C_1 R}\frac{j\omega C_2 R}{1 + j\omega C_2 R}$$

The lowpass section sets the upper corner frequency as

$$\omega_2 = \frac{1}{RC_1} \tag{9}$$

while the highpass section sets the lower corner frequency as

$$\omega_1 = \frac{1}{RC_2} \tag{10}$$

With these values of ω_1 and ω_2 the center frequency, bandwidth, and quality factor are found as follows:

$$\omega_0 = \sqrt{\omega_1 \omega_2} \tag{11}$$

$$B = \omega_2 - \omega_1 \tag{12}$$

$$Q = \frac{\omega_0}{B} \tag{13}$$

To find the passband gain K,

$$\mathbf{H}(\omega) = -\frac{R_f}{R_i}\frac{j\omega/\omega_1}{(1 + j\omega/\omega_1)(1 + j\omega/\omega_2)} = -\frac{R_f}{R_i}\frac{j\omega\omega_2}{(\omega_1 + j\omega)(\omega_2 + j\omega)} \tag{14}$$

At the center frequency $\omega_0 = \sqrt{\omega_1 \omega_2}$, the magnitude of the transfer function is

$$|\mathbf{H}(\omega_0)| = \left|\frac{R_f}{R_i}\frac{j\omega_0\omega_2}{(\omega_1 + j\omega_0)(\omega_2 + j\omega_0)}\right| = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1 + \omega_2} \tag{15}$$

Thus, the passband gain is

$$K = \frac{R_f}{R_i}\frac{\omega_2}{\omega_1 + \omega_2} \tag{16}$$

4. Bandreject (or Notch) Filter

A bandreject filter may be constructed by parallel combination of a lowpass filter and a highpass filter and a summing amplifier, as shown in the block diagram of Fig. 6(a).

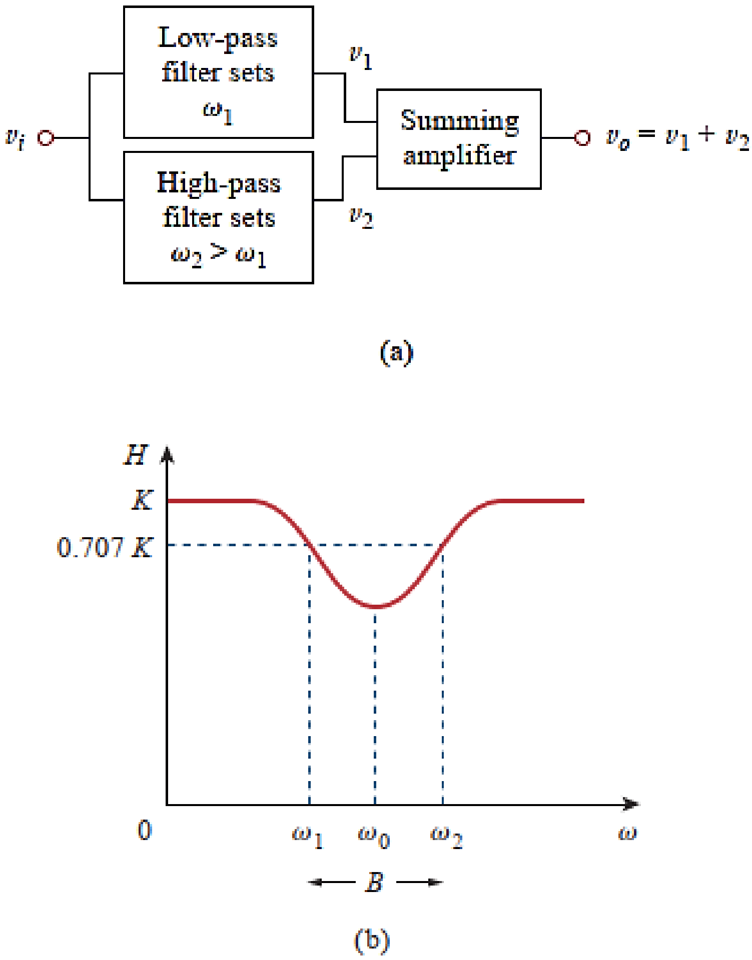


Fig. 6 Active bandreject filter: (a) block diagram, (b) frequency response.

The circuit is designed such that the lower cutoff frequency ω_1 is set by the lowpass filter while the upper cutoff frequency $\omega_2$is set by the highpass filter. The gap between $\omega_1$and ω_2 is the bandwidth of the filter. As shown in Fig. 6(b), the filter passes frequencies below $\omega_1$and above ω_2.

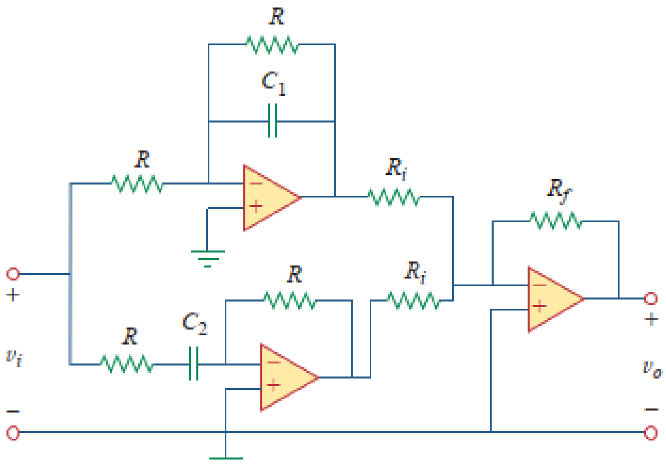


Fig.7 Active bandreject filter.

The block diagram in Fig. 6(a) is actually constructed as shown in Fig. 7. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = -\frac{R_f}{R_i}\left(-\frac{1}{1 + j\omega C_1 R} - \frac{j\omega C_2 R}{1 + j\omega C_2 R}\right) \qquad (17)$$

To determine the passband gain K

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{R_f}{R_i}\left(\frac{1}{1 + j\omega/\omega_2} + \frac{j\omega/\omega_1}{1 + j\omega/\omega_1}\right) \\ &= \frac{R_f}{R_i}\frac{(1 + j2\omega/\omega_1 + (j\omega)^2/\omega_1\omega_1)}{(1 + j\omega/\omega_2)(1 + j\omega/\omega_1)}\end{aligned} \qquad (18)$$

The gain is

$$K = \frac{R_f}{R_i} \qquad (19)$$

We can also find the gain at the center frequency by finding the magnitude of the transfer function at $\omega_0 = \sqrt{\omega_1\omega_2}$, writing

$$H(\omega_0) = \left| \frac{R_f}{R_i} \frac{(1 + j2\omega_0/\omega_1 + (j\omega_0)^2/\omega_1\omega_1)}{(1 + j\omega_0/\omega_2)(1 + j\omega_0/\omega_1)} \right|$$
$$= \frac{R_f}{R_i} \frac{2\omega_1}{\omega_1 + \omega_2}$$

Example 1:

Design a lowpass active filter with a dc gain of 4 and a corner frequency of 500 Hz.

Solution:

$$\omega_c = 2\pi f_c = 2\pi(500) = \frac{1}{R_f C_f}$$

The dc gain is

$$H(0) = -\frac{R_f}{R_i} = -4$$

We have two equations and three unknowns. If we select $C_f = 0.2\ \mu F$, then

$$R_f = \frac{1}{2\pi(500)0.2 \times 10^{-6}} = 1.59\ k\Omega$$

and

$$R_i = \frac{R_f}{4} = 397.5\ \Omega$$

Figure 2 show the filter.