Three-Phase Circuits

Unbalanced Three-Phase Systems

An unbalanced system is caused by two possible situations:

(1) the source voltages are not equal in magnitude and/or differ in phase by angles that are unequal.

(2) load impedances are unequal. Thus,

> An **unbalanced** system is due to unbalanced voltage sources or an unbalanced load

To simplify analysis, we will assume balanced source voltages, but an unbalanced load.

Unbalanced three-phase systems are solved by direct application of mesh and nodal analysis. Fig.1 shows an example of an unbalanced three-phase system that consists of balanced source voltages (not shown in the figure) and an unbalanced Y-connected load (shown in the figure). Since the load is unbalanced, Z_A, $Z_B$and Z_C are not equal.

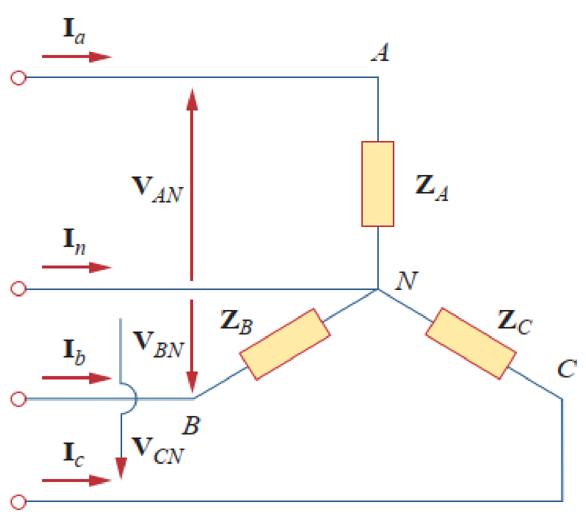

Fig.1 Unbalanced three-phase Y-connected load.

The line currents are determined by Ohm's law as

$$\mathbf{I}_a = \frac{\mathbf{V}_{AN}}{\mathbf{Z}_A}, \quad \mathbf{I}_b = \frac{\mathbf{V}_{BN}}{\mathbf{Z}_B}, \quad \mathbf{I}_c = \frac{\mathbf{V}_{CN}}{\mathbf{Z}_C} \qquad (1)$$

This set of unbalanced line currents produces current in the neutral line, which is not zero as in a balanced system. Applying KCL at node N gives the neutral line current as

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) \qquad (2)$$

In a three-wire system where the neutral line is absent, we can still find the line currents $\mathbf{I_a}$, $\mathbf{I_b}$ and $\mathbf{I_c}$ using mesh analysis. At node N, KCL must be satisfied so that $\mathbf{I_a} + \mathbf{I_b} + \mathbf{I_c} = 0$in this case. The same could be done for an unbalanced Δ-Y, Y- Δ or Δ - Δ three-wire system.

Example 1:

The unbalanced Y-load of Fig.2 has balanced voltages of 100 V and the *acb* sequence. Calculate the line currents and the neutral current. Take $Z_A = 15\ \Omega$, $Z_B = 10 + j5\ \Omega$ and $Z_C = 6 - j8$

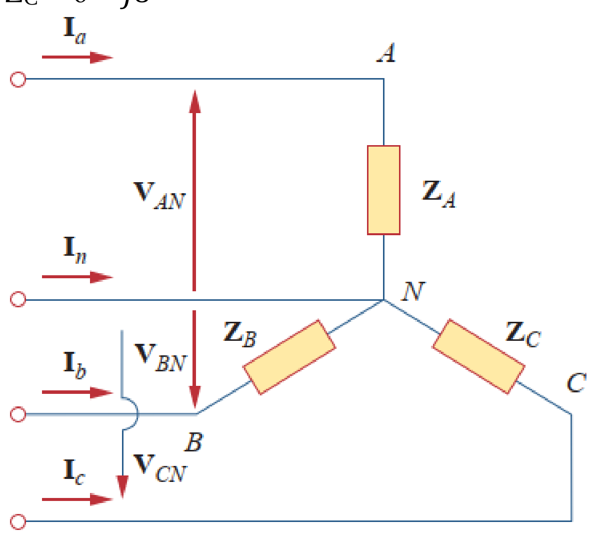

Fig.2

Solution:

The line currents are

$$\mathbf{I}_a = \frac{100\underline{/0^\circ}}{15} = 6.67\underline{/0^\circ}\text{ A}$$

$$\mathbf{I}_b = \frac{100\underline{/120^\circ}}{10 + j5} = \frac{100\underline{/120^\circ}}{11.18\underline{/26.56^\circ}} = 8.94\underline{/93.44^\circ}\text{ A}$$

$$\mathbf{I}_c = \frac{100\underline{/-120^\circ}}{6 - j8} = \frac{100\underline{/-120^\circ}}{10\underline{/-53.13^\circ}} = 10\underline{/-66.87^\circ}\text{ A}$$

the current in the neutral line is

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2)$$
$$= -10.06 + j0.28 = 10.06\underline{/178.4^\circ}\text{ A}$$