Al-Mustaqbal University

College of Engineering & Technology

Building and Construction Techniques Engineering Dep.

Class (2nd)

MECHANICS OF MATERIALS

Lecture 2

Pure Bending

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PURE BENDING

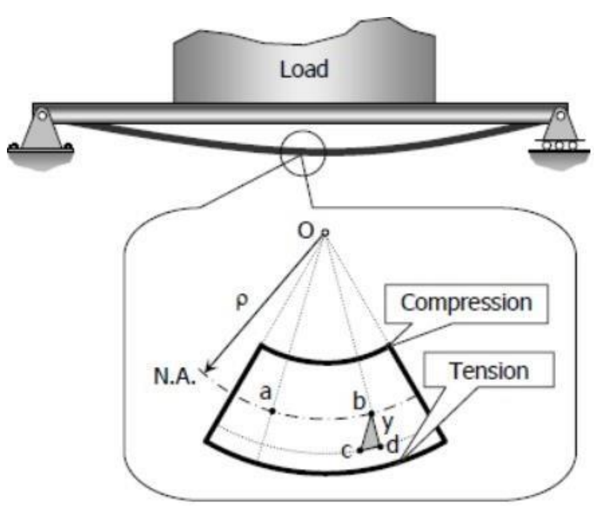
Introduction

Forces and couples acting on the beam cause bending (flexural stresses) and shearing stresses on any cross-section of the beam and deflection perpendicular to the longitudinal axis of the beam. If couples are applied to the ends of the beam and no forces act on it, the bending is called **pure bending**. If forces produce the bending, the bending is called **ordinary bending**.

In using the following formulas for flexural and shearing stresses, it is assumed that a plane section of the beam normal to its longitudinal axis prior to loading remains plane after the forces and couples have been applied, and that the beam is initially straight and of uniform cross section and that the moduli of elasticity in tension and compression are equal.

Flexure Formula

Stresses caused by the bending moment are known as flexural or bending stresses. Consider a beam to be loaded as shown in figure.



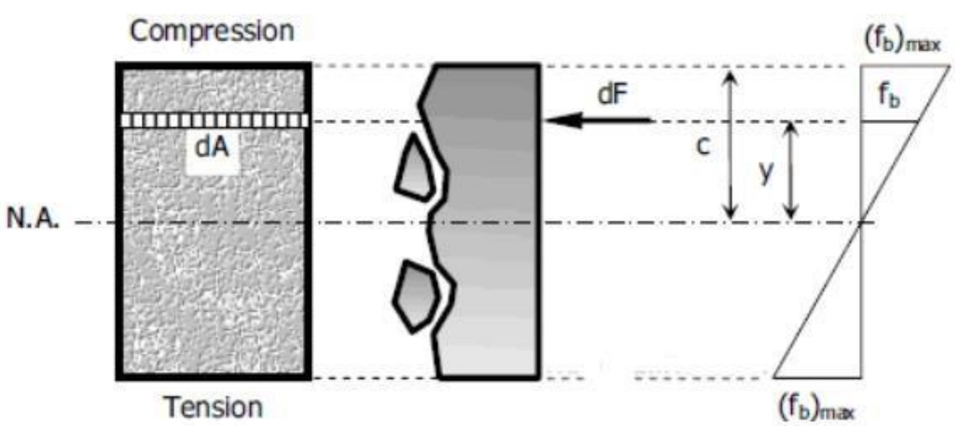
Consider a fiber at a distance y from the neutral axis, because of the beam's curvature, as the effect of bending moment, the fiber is stretched by an amount of cd. Since the curvature of the beam is very small, bcd and oba are considered as similar triangles. The strain on this fiber is:

$$\varepsilon = \frac{cd}{ab} = \frac{y}{\rho} \qquad \text{.....................(1)}$$

By Hooke's law, ($\varepsilon = \dfrac{\sigma}{E}$) then:

$$\frac{\sigma}{E} = \frac{y}{\rho} \;\Rightarrow\; \sigma_m = \frac{y}{\rho} * E \qquad \text{.....................(2)}$$

which means that the stress is proportional to the distance y from the neutral axis.



Considering a differential area dA at a distance y from N.A., the force acting over the area:

$$dF = \sigma_m * dA \qquad \text{.....................(3)}$$

Sub. Eq. (2) in Eq. (3)

$$dF = \frac{y}{\rho} * E * dA$$

$$dF = \frac{E}{\rho} * y * dA \qquad \text{.....................(4)}$$

The resultant of all the elemental moment about N.A. must be equal to the bending moment on the section.

$$M = \int dM = \int y\, dF = \int y \left(\frac{E}{\rho} y\, dA\right)$$

$$M = \frac{E}{\rho} \int y^2\, dA$$

but $\int y^2\, dA = I$, then

$$M = \frac{EI}{\rho} \text{ or } \rho = \frac{EI}{M}$$

From Eq. (2) , $\because \rho = \frac{y}{\sigma_m} * E$

$$\therefore \frac{y\ \not{E}}{\sigma_m} = \frac{\not{E}\ I}{M}$$

$$\sigma_m = \frac{M\ y}{I} \qquad \text{.....................(5)}$$

Where:

ρ is the radius of curvature of the beam in mm (*in*).

M is the bending moment in *N·mm* (*lb·in*).

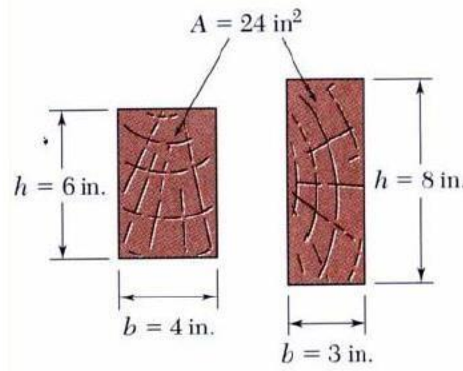
σ_m is the flexural stress in *MPa* (psi).

I is the centroidal moment of inertia in mm^4 (in^4).

y is the distance from the neutral axis to the outermost fiber in *mm* (*in*).

Beam Section Properties

- The maximum normal stress due to bending.



$$\sigma_{\max} = \frac{M\,c}{I} = \frac{M}{S} \qquad \text{…………(6)}$$

$$\textbf{Section modulus } (S) = \frac{I}{c} \qquad \text{…………(7)}$$

Where:

$\sigma_{\max}$ is the maximum bending stress in *MPa* (psi).

S is the section modulus in mm^3 (in^3).

c is the maximum distance from the neutral axis to the outermost fiber in *mm* (*in*).

Deformations in a Transverse Cross Section

Deformation due to bending moment M is quantified by the curvature of the neutral surface

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{\sigma_m}{Ec} = \frac{1}{Ec}\frac{Mc}{I} = \frac{M}{EI}$$

The beam curvature is:

$$k = \frac{1}{\rho}$$

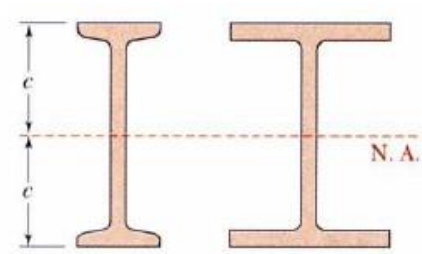
A beam section with a larger section modulus will have a lower maximum stress

- Consider a rectangular beam cross section,

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^2 = \frac{1}{6}Ah$$

Between two beams with the same cross sectional area, the beam with the greater depth will be more effective in resisting bending.

- Structural steel beams are designed to have a large section modulus.

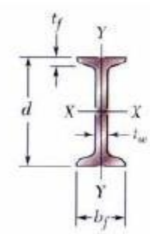


(*a*) S-beam (*b*) W-beam

Properties of American Standard Shapes

Appendix C. **Properties of Rolled-Steel Shapes**
(SI Units)

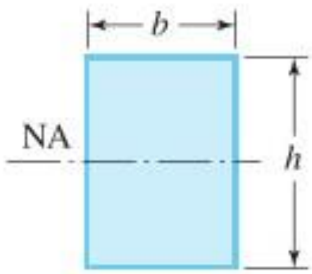
S Shapes
(American Standard Shapes)



Designation†	Area A, mm²	Depth d, mm	Flange: Width b_f, mm	Flange: Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X: I_x 10^6 mm⁴	Axis X-X: S_x 10^3 mm³	Axis X-X: r_x mm	Axis Y-Y: I_y 10^6 mm⁴	Axis Y-Y: S_y 10^3 mm³	Axis Y-Y: r_y mm
S610 × 180	22900	622	204	27.7	20.3	1320	4240	240	34.9	341	39.0
158	20100	622	200	27.7	15.7	1230	3950	247	32.5	321	39.9
149	19000	610	184	22.1	18.9	995	3260	229	20.2	215	32.3
134	17100	610	181	22.1	15.9	938	3080	234	19.0	206	33.0
119	15200	610	178	22.1	12.7	878	2880	240	17.9	198	34.0
S510 × 143	18200	516	183	23.4	20.3	700	2710	196	21.3	228	33.9
128	16400	516	179	23.4	16.8	658	2550	200	19.7	216	34.4
112	14200	508	162	20.2	16.1	530	2090	193	12.6	152	29.5
98.3	12500	508	159	20.2	12.8	495	1950	199	11.8	145	30.4
S460 × 104	13300	457	159	17.6	18.1	385	1685	170	10.4	127	27.5
81.4	10400	457	152	17.6	11.7	333	1460	179	8.83	113	28.8
S380 × 74	9500	381	143	15.6	14.0	201	1060	145	6.65	90.8	26.1
64	8150	381	140	15.8	10.4	185	971	151	6.15	85.7	27.1

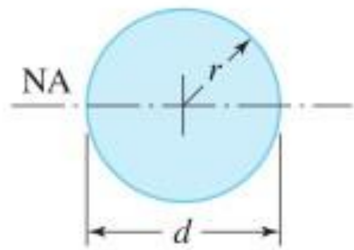
Common cross sections

Rectangle



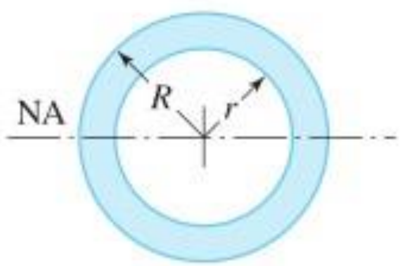
$$I = \frac{bh^3}{12}, \quad c = \frac{h}{2}$$

Solid circle



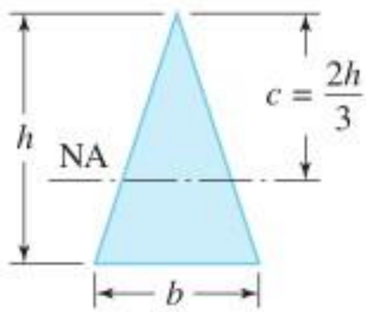
$$I = \frac{\pi r^4}{4}, \quad c = r$$

Tube



$$I = \frac{\pi}{4}(R^4 - r^4), \quad c = R - r$$

Triangle



$$I = \frac{bh^3}{36}$$

Solved Problems in Bending

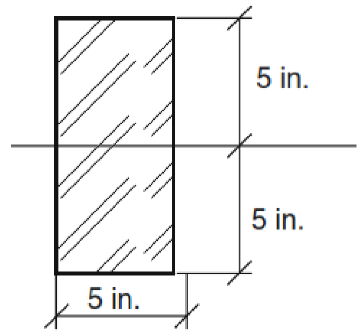
Example 1 : Calculate the maximum bending stress of the beam if the cross-sectional dimension is 5 in. × 10 in. and the bending moment is 50,000 lb-in.

$$\sigma = Mc/I$$

$$I = (1/12)\,bh^3 = (1/12)(5 \times 10^3) = 416.67\,\text{in.}^4$$

$$\sigma = (50,000 \times 5)/416.67$$

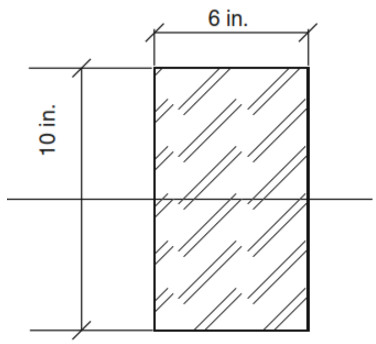
$$\sigma = 600\,\text{psi}$$



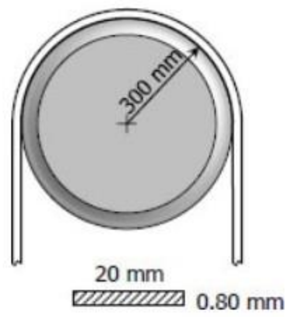
Example 2:: A nominal size piece of structural timber 6 in. × 10 in. is subjected to a vertical loading . Determine the section modulus of the beam if the maximum bending stress is established.

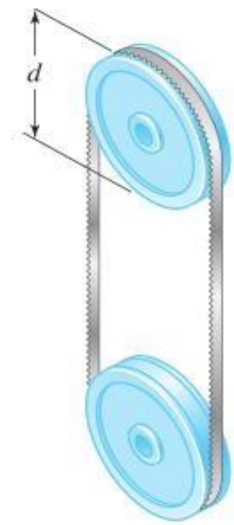
$$S = I/c$$
$$S = (bh^3/12)/h/2 = bh^3/6$$

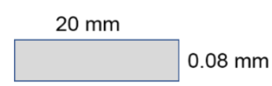
$$S = (5\tfrac{1}{2})(9\tfrac{1}{2})^2/6 = 82.7\,\text{in.}^3$$



Ex 3: A steel band saw, 20 mm wide and 0.8 mm thick, runs over pulleys of diameter d. (a) Find the maximum bending stress in the saw if $d = 600$ mm. (b) What is the smallest value of d for which the bending stress in the saw does not exceed 400 MPa? Use $E = 200$ GPa for steel.







Flexural stress developed:

$$M = \frac{EI}{\rho}$$

$$f_b = \frac{Mc}{I} = \frac{(EI/\rho)c}{I}$$

$$f_b = \frac{Ec}{\rho} = \frac{200000(0.80/2)}{300}$$

$$f_b = 266.67\,\text{MPa}$$

Minimum diameter of pulley:

$$f_b = \frac{Ec}{\rho}$$

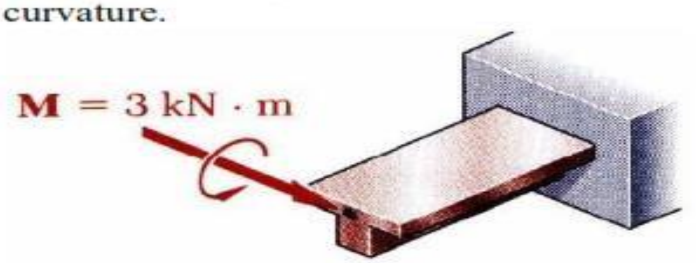
$$400 = \frac{200000(0.80/2)}{\rho}$$

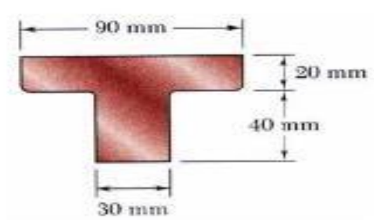
$$\rho = 200\,\text{mm}$$

Diameter, d = 400 mm *answer*

Example 4:

A cast-iron machine part is acted upon by a 3 kN-m couple. Knowing E = 165GPa and neglecting the effects of fillets, determine (a) the maximum tensile and compressive stresses, (b) the radius of curvature.

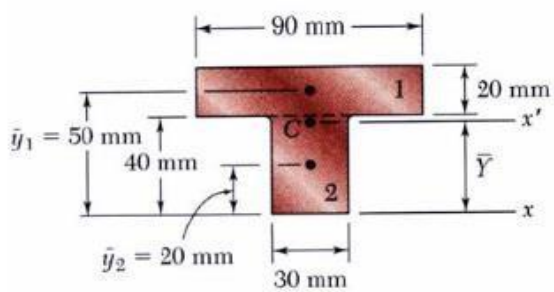




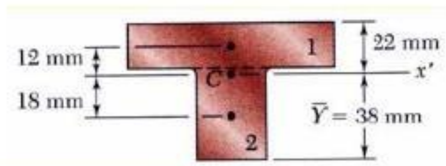
SOLUTION:

- Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

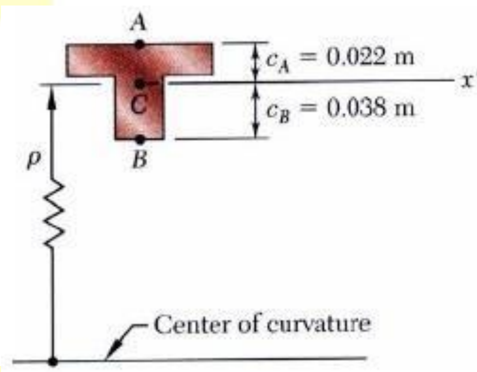
	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$20\times90=1800$	50	90×10^3
2	$40\times30=1200$	20	24×10^3
	$\sum A=3000$		$\sum\bar{y}A=114\times10^3$



$$\bar{Y}=\frac{\sum\bar{y}A}{\sum A}=\frac{114\times10^3}{3000}=38\,\text{mm}$$



$$I_{x'}=\sum\left(\bar{I}+Ad^2\right)=\sum\left(\tfrac{1}{12}bh^3+Ad^2\right)$$
$$=\left(\tfrac{1}{12}90\times20^3+1800\times12^2\right)+\left(\tfrac{1}{12}30\times40^3+1200\times18^2\right)$$
$$I=868\times10^3\,\text{mm}=868\times10^{-9}\,\text{m}^4$$



- Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m=\frac{Mc}{I}$$

$$\sigma_A=\frac{Mc_A}{I}=\frac{3\,\text{kN}\cdot\text{m}\times0.022\,\text{m}}{868\times10^{-9}\,\text{mm}^4}\qquad \boxed{\sigma_A=+76.0\,\text{MPa}}$$

$$\sigma_B=-\frac{Mc_B}{I}=-\frac{3\,\text{kN}\cdot\text{m}\times0.038\,\text{m}}{868\times10^{-9}\,\text{mm}^4}\qquad \boxed{\sigma_B=-131.3\,\text{MPa}}$$

- Calculate the curvature

$$\frac{1}{\rho}=\frac{M}{EI}$$
$$=\frac{3\,\text{kN}\cdot\text{m}}{(165\,\text{GPa})\left(868\times10^{-9}\,\text{m}^4\right)}$$

$$\frac{1}{\rho}=20.95\times10^{-3}\,\text{m}^{-1}$$
$$\rho=47.7\,\text{m}$$