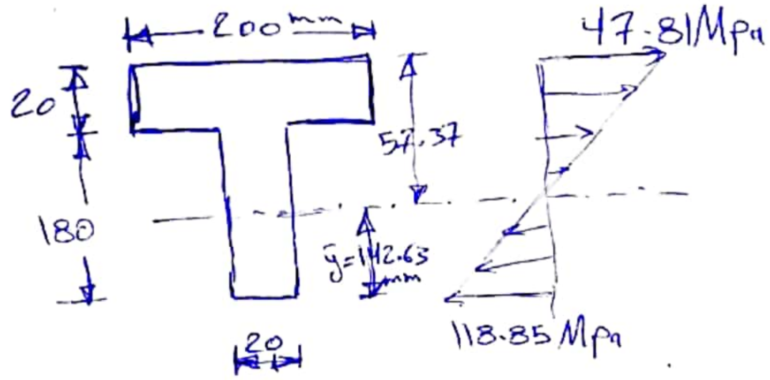
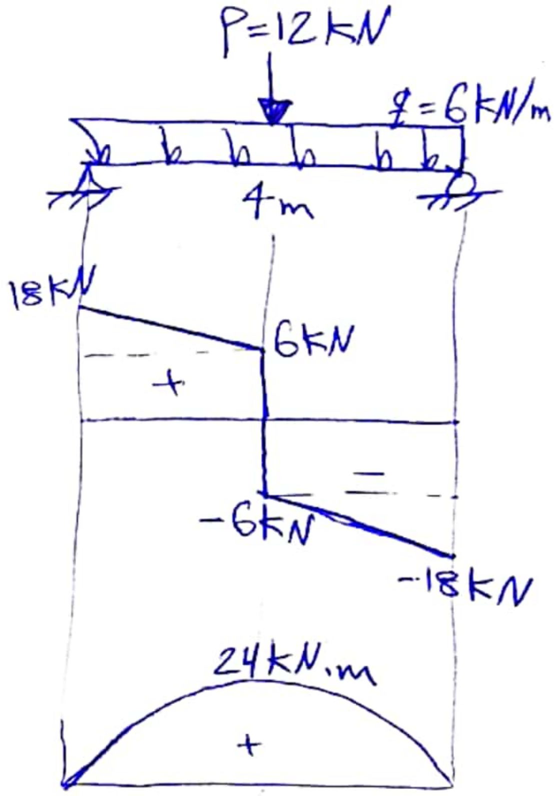
Ex1 :- For the simply supported beam shown in Fig., determine the distribution of bending stresses of critical section.





Sol

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i}$$

$$= \frac{20*200*(180+10)+180*20*90}{20*200+180*20}$$

$$\bar{y} = 142.63\,mm$$

$$I_{N.A} = \Sigma(I_o + A.d^2)$$

$$= \frac{20*(180)^3}{12} + 20*180*(142.63-90)^2$$

$$+ \frac{200*(20)^3}{12} + 20*200*(190-142.63)^2$$

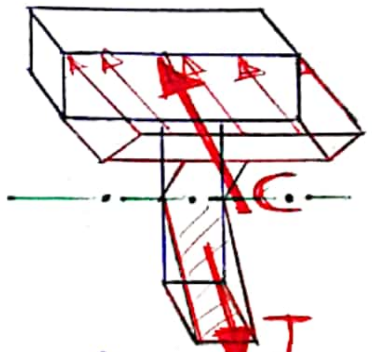
$$I_{N.A} = 2.88*10^7\ mm^4$$

(M+)

$$\sigma_{max}(\text{Top})_{\text{com.}} = \frac{M \cdot c_1}{I} = \frac{24*10^6*(200-142.63)}{2.88*10^7} = 47.81 \text{ Mpa (com.)}$$

$$\sigma_{max}(\text{bottom})_{\text{ten.}} = \frac{M \cdot c_2}{I} = \frac{24*10^6*(142.63)}{2.88*10^7} = 118.85 \text{ Mpa (ten.)}$$

$$\frac{\sigma^*}{(57.37-20)} = \frac{47.81}{57.37} \Rightarrow \sigma^* = 31.143 \text{ Mpa}$$



or

$$\sigma^* = \frac{24*10^6*(57.37-20)}{2.88*10^7} = 31.143 \text{ Mpa}$$

Resultant of compression or tension force = Volume of stress block

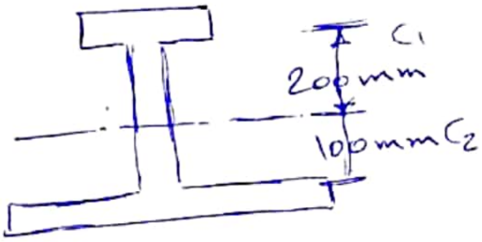
$$T = \frac{1}{2} * 118.85 * (142.63) * 20 * 10^{-3}$$
$$= 169.5 \text{ kN}$$

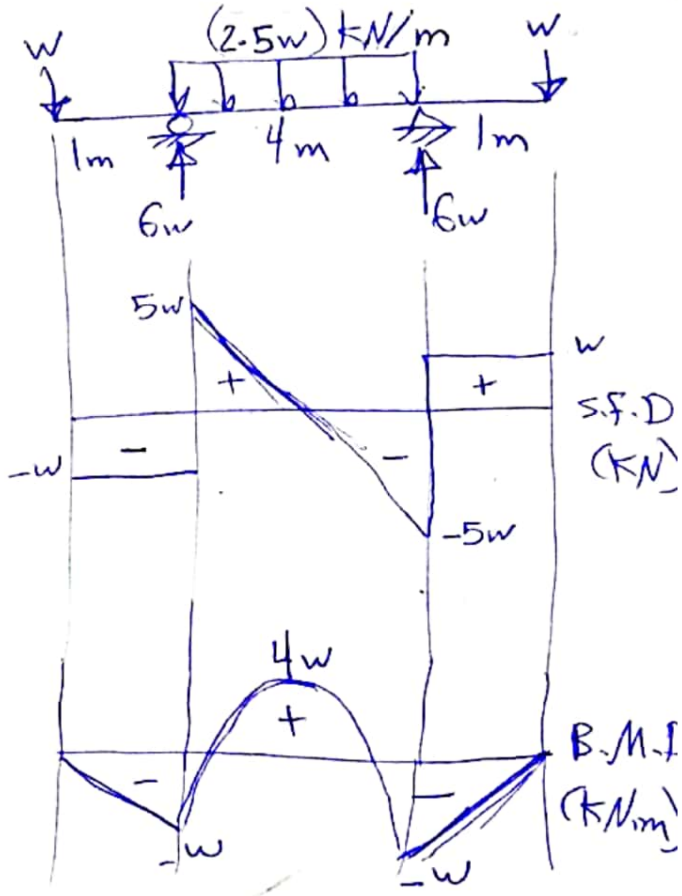
M
- M^-
  - σ_{top} (tension)
  - σ_{bottom} (compression)
- M^+
  - σ_{top} (com.)
  - σ_{bottom} (ten.)

Ex2: Find max. allowable load (w kN/m)? So that stresses will not exceeding (20 Mpa) in tension and (70 Mpa) in compression

$I_{N.A} = 5*10^7 \ mm^4$

Sol





$M^{+}_{max} = 4w \ kN.m$

$M^{-}_{max} = w \ kN.m$

for section of M^{+}_{max} 4w

$$\sigma_{\substack{ten.\\bott.}} = 20 = \frac{4w*10^6*100}{5*10^7}$$

$\therefore w = 2.5 \ kN/m$

$$\sigma_{\substack{com.\\top.}} = 70 = \frac{4w*10^6*200}{5*10^7}$$

$w = 4.375 \ kN/m$

for section of M^{-}_{max}

$$\sigma_{\substack{ten.\\top}} = 20 = \frac{w*10^6*200}{5*10^7} \Rightarrow w = 5 \ kN/m$$

$$\sigma_{\substack{com.\\bott.}} = 70 = \frac{w*10^6*100}{5*10^7} \rightarrow w = 35 \ kN/m$$

$\boxed{\therefore w_{all} = 2.5 \ kN/m \text{ (the least one)}}$

MECHANICS OF MATERIALS (SECOND CLASS)

STEPS FOR DRAWING (S.F.D.) AND (B.M.D.) BY GRAPHICAL METHOD (USING CONSTITUTIVE RELATIONSHIPS)

****

1.REACTIONS OF SUPPORTS :

Draw the **(F.B.D.)** of the beam (or segment) and determine the reactions of supports .Then , **resolve** all the forces and the reactions of the beam into components : **perpendicular** and **parallel** to the longitudinal axis of beam.

2.Select position of coordinate (x) , or (θ) for curved beam ,to extend between two ends of the beam .Usually , it is at the left end of the beam.

3.SHEAR FORCE DIAGRAM (S.F.D) :

Establish the (S.F.D.) with X-axis , according to the basic concepts below:

i. Slope of (S.F.) diagram at any point is equal to the intensity of the distributed load (value and sign). + ↑, − ↓

dV/ dX = q (x) ....**(1)**

ii. The change in the value of shear force between any two points on the beam , is equal to the area of the loading diagram between these two points (value and sign)

$\Delta V = \int q(x)\,dx + C_1$ ....**(2)**

iii. Concentrated forces (reactions or point loads) lead to sudden changes (jumps) in (S.F.D.). + ↑, − ↓ (**i.e.** C_1)

iv. If q (x) is a curve of degree (n) , V(x) will be a curve of degree (n+1).

4.BENDING MOMENT DIAGRAM (B.M.D.):

Establish the (B.M.D.) with X-axis according to the basic concepts below:

i. Slope of (B.M.) diagram at any point is equal to the shear force of this point (value and sign).

dM/ dx = V(x) ....**(3)**

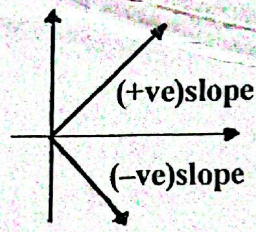
ii. The change in the value of moment between any two points on the beam, is equal to the area of the shear force diagram between these two points (value and sign)

$\Delta M = \int V(x)\,d(x) + C_2$ ....**(4)**

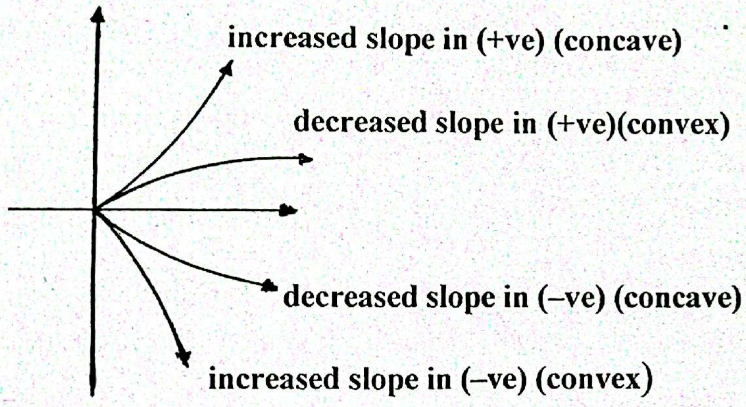
iii. Concentrated moments (couples) lead to sudden changes(jumps) in the (B.M.D.). ↻ +ve , ↺ −ve (**i.e.** C_2).

iv. If V(x) is a curve of degree (n) , M(x) will be a curve of degree (n+1).

5. **The lines** of diagrams may be **straight**



or **curves**

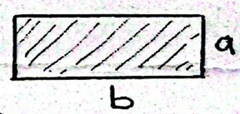


6. **The curvature** of (B.M.) curve equals to the intensity of the distributed load (value and sign). + ↑ (concave) , – ↓ (convex)

$d^2M / dx^2 = q(x)$ ....(5)

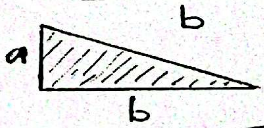
7.Calculation of areas :

i. Rectangular area.



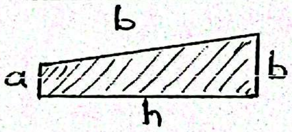
A = a b

ii. Triangular area



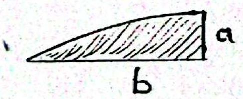
A = (1/2) a b

iii. Trapezoidal area

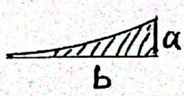


A = (1/2) (a+b) h

iv. Curves of degree (n) with zero slope.



A = [n / (n+1)] a b ,



A = [1/(n+1)] a b

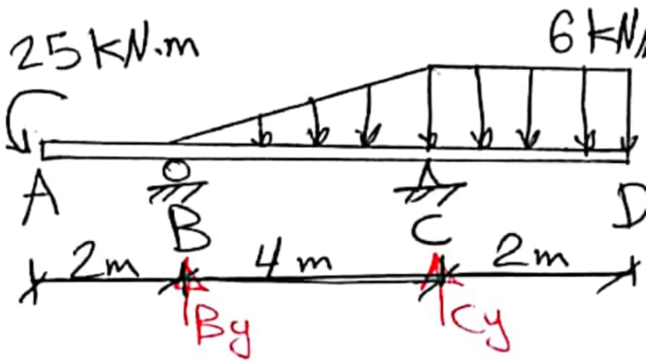
vi. For any curve, you can derive the equation of the curve **V(x)** or **M(x)** by method of sections, and then use integration method to get the required area.

Ex1: For the beam shown in Fig. draw (S.F) and (B.M) diagrams, and illustrate all details.

sol

Find Reactions

The whole beam as F.B.D



$\circlearrowleft + \sum M_C = 0$

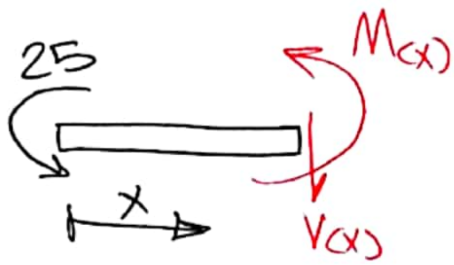
$4B_y + 6*2*1 - 25 - \frac{1}{2}*4*6*\frac{4}{3} = 0$

$\therefore \boxed{B_y = 7.25\ kN}$

$+\uparrow \sum F_y = 0$

$\boxed{C_y = 16.75\ kN}$

Segment AB $0 \leq x \leq 2m$



$+\downarrow \sum F_y = 0$

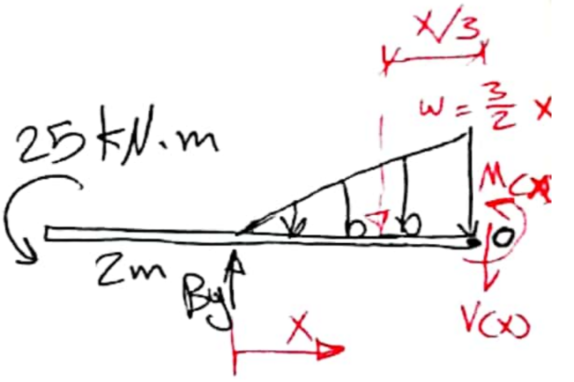
$\boxed{V(x) = 0}$ $0 \leq x \leq 2$

$\circlearrowleft + \sum M_o = 0$

$M(x) + 25 = 0$

$\boxed{M(x) = -25}$ $0 \leq x \leq 2m$

Segment BC $0 \leq x \leq 4m$



$\frac{6}{4} = \frac{w}{x} \Rightarrow w = \frac{3}{2}x$

$\Sigma F_y = 0$

$V(x) - B_y + \frac{1}{2}x \cdot \frac{3}{2}x = 0$

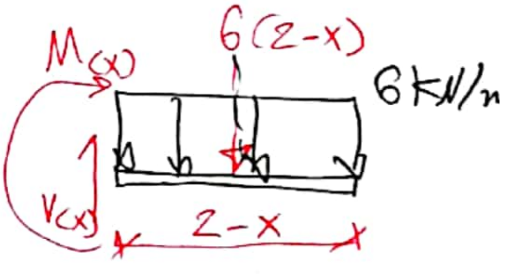
$\boxed{V(x) = 7.25 - \frac{3}{4}x^2}$ $0 \leq x \leq 4m$

$\circlearrowleft + \Sigma M_o = 0$

$M(x) + 25 + \frac{1}{2}x \cdot \frac{3}{2}x \cdot \frac{x}{3} - B_y(x) = 0$

$\boxed{M(x) = \frac{-x^3}{4} + 7.25x - 25}$ $0 \leq x \leq 4m$

Segment CD $0 \leq x \leq 2m$



$+\uparrow \Sigma F_y = 0$

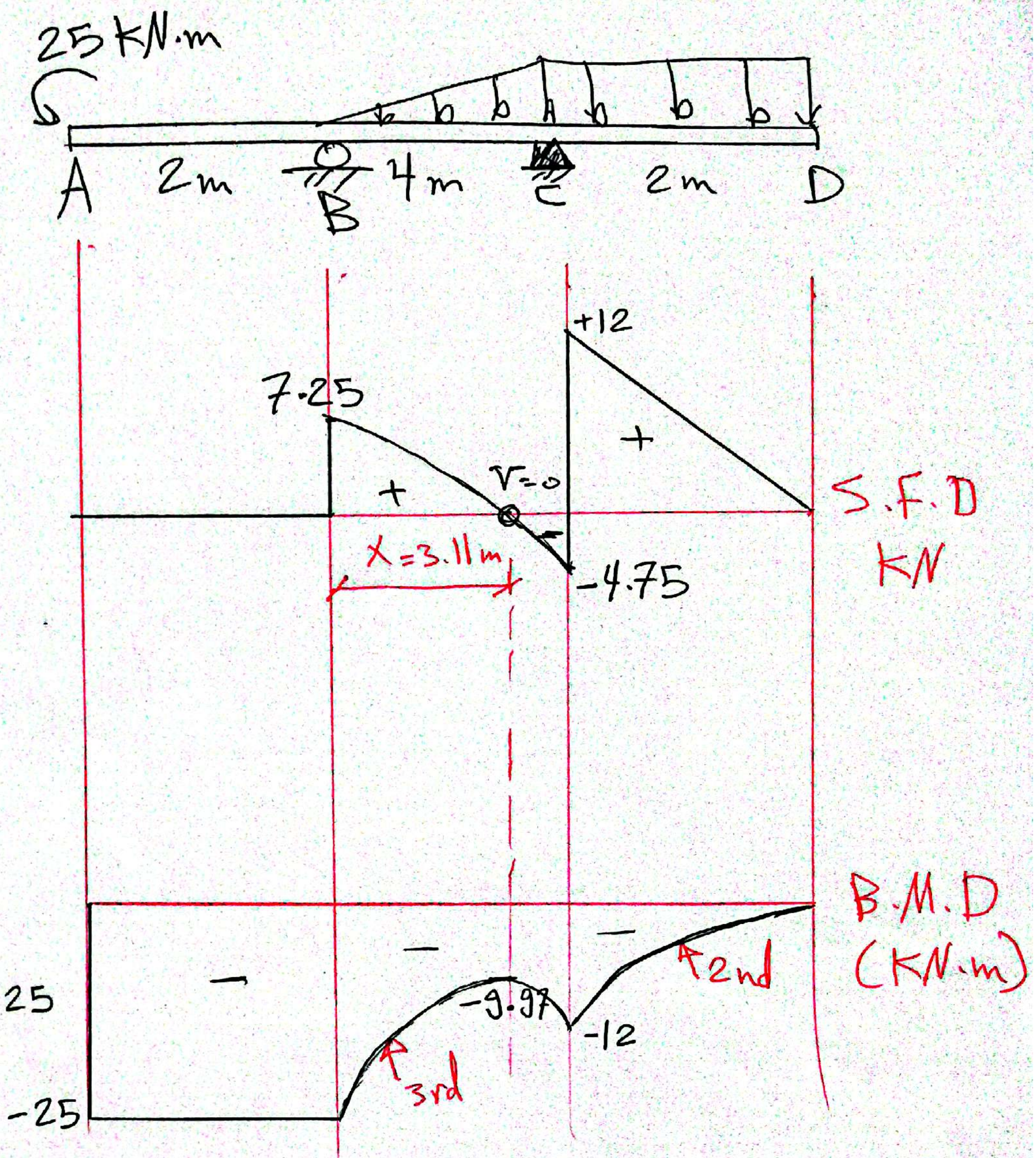
$V(x) - 6(2-x) = 0$

$\boxed{V(x) = 6(2-x)}$ $0 \leq x \leq 2$

$\circlearrowleft + \Sigma M_o = 0$

$M(x) + 6(2-x) \cdot \frac{(2-x)}{2} = 0$

$\boxed{M(x) = -3(x-2)^2}$ $0 \leq x \leq 2m$



segment AB

$V(0) = 0$

$V(2) = 0$

$M(0) = -25$

$M(2) = -25$

segment BC

$V(0) = 7.25 - \frac{3}{4}(0)^2$

$= 7.25$

$V(4) = 7.25 - \frac{3}{4}(4)^2$

$= -4.75$

$M(0) = 0 + 7.25(0) - 25$

$= -25$

$M(4) = \frac{-(4)^3}{4} + 7.25(4) - 25$

$= -12$

$M(3.11) = \frac{-(3.11)^3}{4} + 7.25(3.11) - 25$

$= -7.52 + 22.55 - 25$

$= -9.97$

segment CD

$V(0) = 6(2-0)$

$= 12$

$V(2) = 6(2-2)$

$= 0$

$M(0) = -3(0-2)^2$

$= -12$

$M(2) = -3(2-2)^2$

$= 0$

to find the value of (X)
sub. (V = 0) @ eq. of shear
in seg. (BC)

$V(x) = 7.25 - \frac{3}{4}x^2$

$0 = 7.25 - \frac{3}{4}x^2$

$\frac{3}{4}x^2 = 7.25$

$x^2 = 9.67$

$x = \pm 3.11$