Three-Phase Circuits

C. Balanced Wye-Delta Connection

A **balanced Y- Δ** system consists of a balanced Y-connected source feeding a balanced **Δ**-connected load.

The balanced Y-delta system is shown in Fig.1, where the source is Y-connected and the load is **Δ** -connected. There is, of course, no neutral connection from source to load for this case. Assuming the positive sequence, the phase voltages are again

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ \qquad (1)$$

The line voltages are

$$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_p\angle -90^\circ = \mathbf{V}_{BC}$$
$$\mathbf{V}_{ca} = \sqrt{3}V_p\angle -150^\circ = \mathbf{V}_{CA} \qquad (2)$$

Phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} \qquad (3)$$

These currents have the same magnitude but are out of phase with each other by 120^o.

Another way to get these phase currents is to apply KVL. For example, applying KVL around loop ***aABbna*** gives

$$-\mathbf{V}_{an} + \mathbf{Z}_\Delta\mathbf{I}_{AB} + \mathbf{V}_{bn} = 0$$

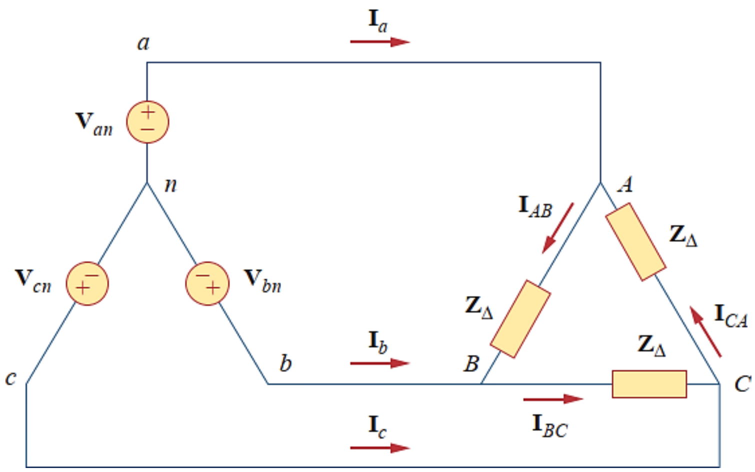

Fig.1 Balanced Y-Δ connection.

or

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{an} - \mathbf{V}_{bn}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_\Delta} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} \qquad (22)$$

This is the more <u>general way of finding the phase currents.</u>

The line currents are obtained from the phase currents by applying KCL at nodes A, B, and C. Thus,

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \qquad (4)$$

Since $I_{CA} = I_{AB}\angle{-240}$

$$\begin{aligned}\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} &= \mathbf{I}_{AB}(1 - 1\angle{-240^\circ}) \\ &= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ}\end{aligned} \qquad (5)$$

showing that the magnitude of the line current I_L is $\sqrt{3}$ times the magnitude I_P of the phase current , or

$$\boxed{I_L = \sqrt{3} I_p} \qquad (6)$$

where

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c| \qquad (7)$$

and

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}| \qquad (8)$$

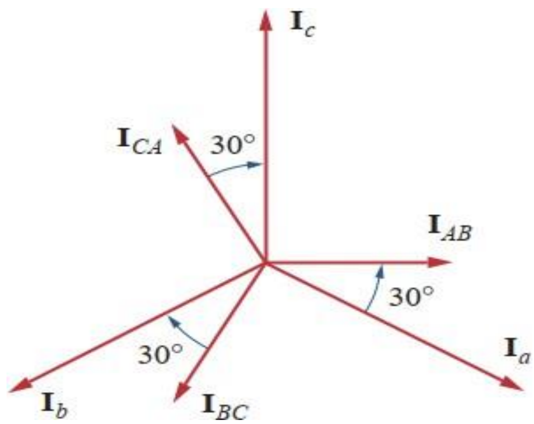

Fig.2 Phasor diagram illustrating the relationship between phase and line currents.

An alternative way of analyzing the circuit is to transform the Δ-connected load to an equivalent Y-connected load.

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} \qquad (9)$$

The three-phase system in Fig. 1 can be replaced by the single-phase equivalent circuit in Fig.3. This allows us to calculate only the line currents. The phase currents are obtained using Eq. (6) and utilizing the fact that each of the phase currents leads the corresponding line current by

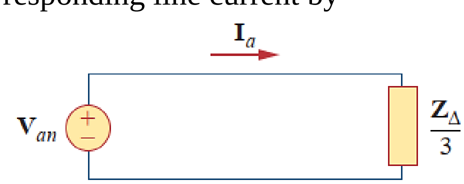

Fig. 3 A single-phase equivalent circuit of a balanced Y-Δ circuit.

Example 1:

A balanced abc-sequence Y-connected source with V_{an}=100∠10°V is connected to Δ -connected balanced load (8+j4)Ω per phase. Calculate the phase and line currents.

Solution:

This can be solved in two ways.

METHOD 1 The load impedance is

$$\mathbf{Z}_\Delta = 8 + j4 = 8.944\underline{/26.57^\circ}\ \Omega$$

If the phase voltage $\mathbf{V}_{an} = 100\underline{/10^\circ}$, then the line voltage is

$$\mathbf{V}_{ab} = \mathbf{V}_{an}\sqrt{3}\underline{/30^\circ} = 100\sqrt{3}\underline{/10^\circ + 30^\circ} = \mathbf{V}_{AB}$$

or

$$\mathbf{V}_{AB} = 173.2\underline{/40^\circ}\ \text{V}$$

The phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{173.2\underline{/40^\circ}}{8.944\underline{/26.57^\circ}} = 19.36\underline{/13.43^\circ}\ \text{A}$$

$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\underline{/-120^\circ} = 19.36\underline{/-106.57^\circ}\ \text{A}$$

$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\underline{/+120^\circ} = 19.36\underline{/133.43^\circ}\ \text{A}$$

The line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\underline{/-30^\circ} = \sqrt{3}(19.36)\underline{/13.43^\circ - 30^\circ} = 33.53\underline{/-16.57^\circ}\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\underline{/-120^\circ} = 33.53\underline{/-136.57^\circ}\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\underline{/+120^\circ} = 33.53\underline{/103.43^\circ}\ \text{A}$$

METHOD 2 Alternatively, using single-phase analysis,

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_\Delta/3} = \frac{100\underline{/10^\circ}}{2.981\underline{/26.57^\circ}} = 33.54\underline{/-16.57^\circ}\ \text{A}$$

as above. Other line currents are obtained using the *abc* phase sequence.

D. Balanced Delta-Wye Connection

A balanced Δ-Y system consists of a balanced Δ -connected source feeding a balanced Y-connected load.

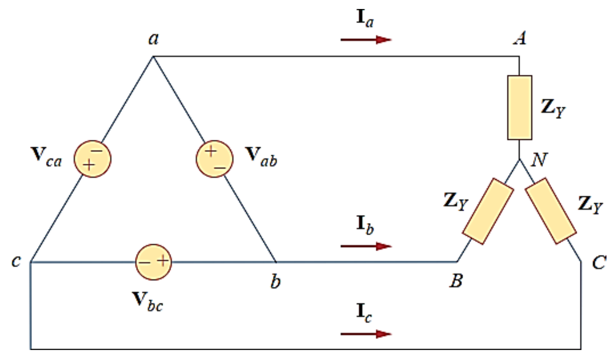

Fig. 4 A balanced Δ-Y connection.

Consider the $\Delta - Y$ circuit in Fig. 4. Again, assuming the ***abc*** sequence, the phase voltages of a delta-connected source are

$$\mathbf{V}_{ab} = V_p\angle 0^\circ, \qquad \mathbf{V}_{bc} = V_p\angle -120^\circ \qquad (10)$$
$$\mathbf{V}_{ca} = V_p\angle +120^\circ$$

These are also the line voltages as well as the phase voltages.

We can obtain the line currents in many ways. One way is to apply KVL to loop *aANBba* in Fig. 4, writing

$$-\mathbf{V}_{ab} + \mathbf{Z}_Y\mathbf{I}_a - \mathbf{Z}_Y\mathbf{I}_b = 0$$

or

$$\mathbf{Z}_Y(\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = V_p\angle 0^\circ$$

Thus,

$$\mathbf{I}_a - \mathbf{I}_b = \frac{V_p\angle 0^\circ}{\mathbf{Z}_Y} \qquad (11)$$

But $\mathbf{I_a}$ lags $\mathbf{I_b}$ by 120^o since we assumed the abc sequence; that is, $I_b = I_a\angle -120^o$ Hence,

$$\mathbf{I}_a - \mathbf{I}_b = \mathbf{I}_a(1 - 1\angle -120^\circ) = \mathbf{I}_a\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \mathbf{I}_a\sqrt{3}\angle 30^\circ \tag{12}$$

Substituting Eq. (12) into Eq. (11) gives

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\angle -30^\circ}{\mathbf{Z}_Y} \tag{13}$$

From this, we obtain the other line currents I_b and I_c using the positive phase sequence, i.e. $I_b = I_a\angle -120^o$, $I_c = I_a\angle +120^o$, The phase currents are equal to the line currents.
Another way to obtain the line currents is to replace the delta-connected source with its equivalent wye-connected source, as shown in Fig. 5.

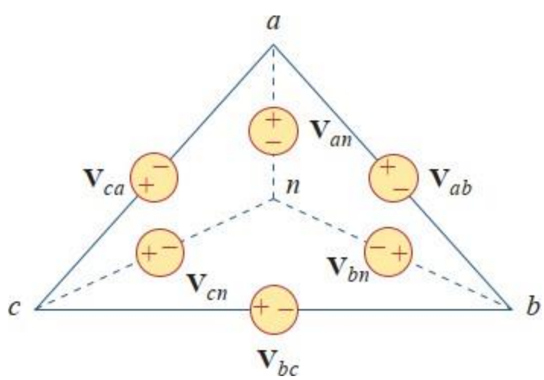

Fig.5 Transforming a *Δ*-connected source to an equivalent Y-connected source.

we obtain each phase voltage of the equivalent wye-connected source by dividing the corresponding line voltage of the delta-connected source by √3 and shifting its phase by 30°. Thus, the equivalent wye-connected source has the phase voltages

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$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30^\circ \tag{14}$$

$$\mathbf{V}_{bn} = \frac{V_p}{\sqrt{3}}\angle -150^\circ, \qquad \mathbf{V}_{cn} = \frac{V_p}{\sqrt{3}}\angle +90^\circ$$

Once the source is transformed to wye, the circuit becomes a wye-wye system. Therefore, we can use the equivalent single-phase circuit shown in Fig. 6,

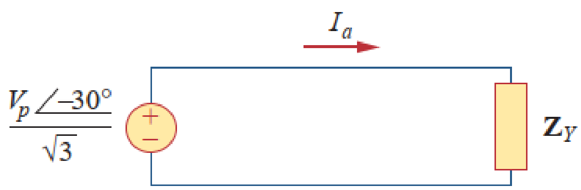

Fig.6 The single-phase equivalent circuit

from which the line current for phase a is

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\angle -30^\circ}{\mathbf{Z}_Y} \tag{15}$$

Note that

$$\mathbf{V}_{AN} = \mathbf{I}_a\mathbf{Z}_Y = \frac{V_p}{\sqrt{3}}\angle -30^\circ \tag{16}$$

$$\mathbf{V}_{BN} = \mathbf{V}_{AN}\angle -120^\circ, \qquad \mathbf{V}_{CN} = \mathbf{V}_{AN}\angle +120^\circ$$

As stated earlier, the delta-connected load is more **desirable than** the wye-connected load. It is easier to alter the loads in any one phase of **the delta-connected loads**, as the individual loads are connected directly across the lines. However, the delta-connected source is hardly used in practice, because any slight imbalance in the phase voltages will result in unwanted circulating currents.

Example 2:

A balanced Y-connected load with a phase impedance of $40 + j25\ \Omega$ is supplied by a balanced, positive sequence Δ-connected source with a line voltage of 210 V. Calculate the phase currents. Use $\mathbf{V}_{ab}$ as reference.

Solution:

The load impedance is

$$\mathbf{Z}_Y = 40 + j25 = 47.17\angle 32^\circ\ \Omega$$

and the source voltage is

$$\mathbf{V}_{ab} = 210\angle 0^\circ\ \text{V}$$

When the Δ-connected source is transformed to a Y-connected source,

$$\mathbf{V}_{an} = \frac{\mathbf{V}_{ab}}{\sqrt{3}}\angle -30^\circ = 121.2\angle -30^\circ\ \text{V}$$

The line currents are

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} = \frac{121.2\angle -30^\circ}{47.12\angle 32^\circ} = 2.57\angle -62^\circ\ \text{A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 2.57\angle -178^\circ\ \text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle 120^\circ = 2.57\angle 58^\circ\ \text{A}$$

which are the same as the phase currents.

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ Same as line currents	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$ $\mathbf{V}_{bn} = V_p\angle -120^\circ$ $\mathbf{V}_{cn} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$ $\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$ $\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$ $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ $\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$ $\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$ $\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	Same as phase voltages $\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$ $\mathbf{V}_{bc} = V_p\angle -120^\circ$ $\mathbf{V}_{ca} = V_p\angle +120^\circ$ Same as line currents	Same as phase voltages $\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$ $\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$ $\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$