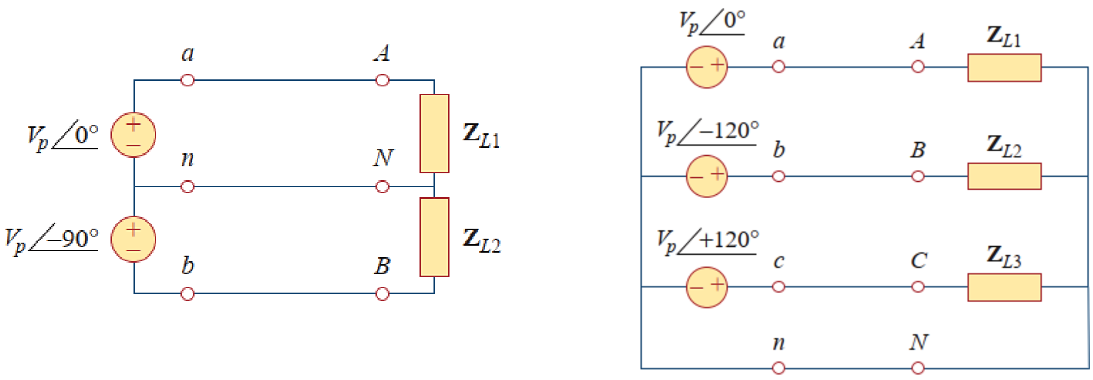
Three-Phase Circuits

Circuits or systems in which the ac sources operate at the same frequency but different phases are known as polyphase. Figure 1(a), shows a two-phase three- wire system, and Fig. 1(b) shows a three-phase four-wire system.



(a) two-phase three-wire system

(b) three-phase four-wire system

Fig.1

As distinct from a single-phase system, a **two-phase system** is produced by a generator consisting of **two coils placed perpendicular to each other** so that the voltage generated by one lags the other by 90°.

By the same token, **a three-phase system** is produced by a generator consisting of three sources having the same amplitude and frequency but out of phase with each other by 120°.

Three-phase systems are important for at least three reasons.

1. First, nearly all electric power is generated and distributed in three-phase, at the operating frequency of 60 Hz (orω = 377 rad/sec) in the United States or 50 Hz (or ω = 314 rad/sec) in some other parts of the world. When one-phase or two-phase inputs are required, they are taken from the three-phase system rather than generated independently. Even when more than three phases are needed—such as in the aluminum industry, where 48 phases are required for melting purposes—they can be provided by manipulating the three phases supplied.
2. Second, the instantaneous power in a three-phase system can be constant (not pulsating). This results in uniform power transmission and less vibration of three-phase machines.
3. Third, for the same amount of power, the three-phase system is more economical than the single-phase. The amount of wire required for a three- phase system is less than that required for an equivalent single-phase system.

We begin with a discussion of balanced three-phase voltages. Then we analyze each of the four possible configurations of balanced three- phase systems. We also discuss the analysis of unbalanced three-phase systems.

1. Balanced Three-Phase Voltages

Three-phase voltages are often produced with a three-phase ac generator (or alternator) whose cross-sectional view is shown in Fig.2.

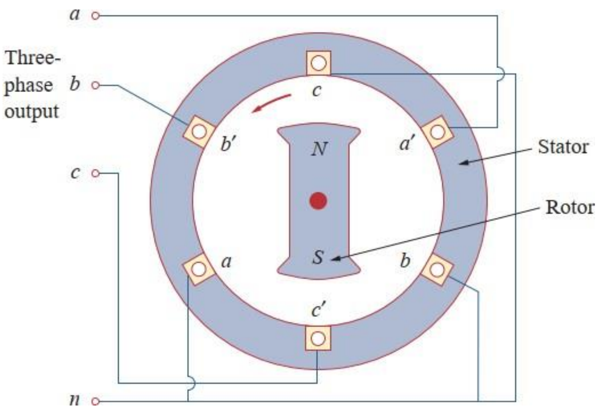


Fig.2 A three-phase generator.

The generator basically consists of a rotating magnet (called the rotor) surrounded by a stationary winding (called the stator). Three separate windings or coils with terminals $a - a'$, $b - b'$ and $c - c'$ are physically placed 120^o apart around the stator. Terminals a and a' for example, stand for one of the ends of coils going into and the other end coming out of the page. As the rotor rotates, its magnetic field "cuts" the flux from the three coils and induces voltages in the coils.

Because the coils are placed 120^o apart, the induced voltages in the coils are equal in magnitude but out of phase by 120^o (Fig. 3).

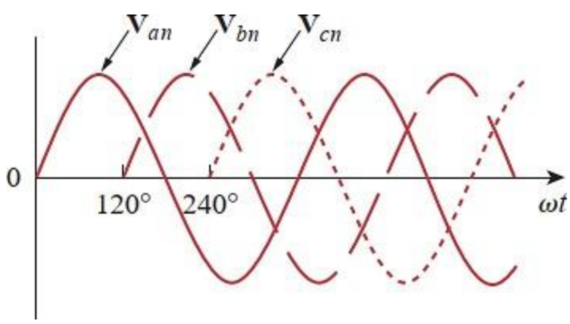


Fig.3 The generated voltages are apart from each other.

Since each coil can be regarded as a single-phase generator by itself, the three- phase generator can supply power to both single-phase and three-phase loads.

A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines). (Three-phase current sources are very scarce.) A three-phase system is equivalent to three single-phase circuits. The voltage sources can be either wye-connected as shown in Fig.4(a) or delta- connected as in Fig.4(b).

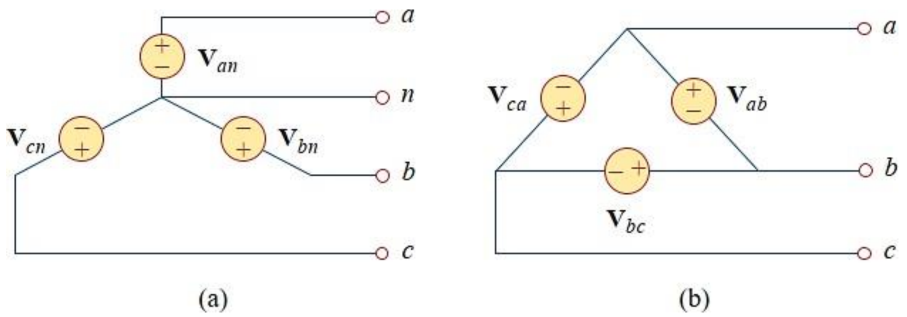


Fig.4 Three-phase voltage sources: (a) Y-connected source, (b) -connected source.

Let us consider the wye-connected voltages in Fig. 4(a) for now. The voltages V_{an} , V_{bn} and V_{cn} are respectively between lines a, b, and c, and the neutral line n. These voltages are called ***phase voltages***.

If the voltage sources have the same amplitude and frequency ω and are out of phase with each other by the voltages are said to be ***balanced.*** This implies that

$$\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} = 0 \quad (1)$$

$$|\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}| \quad (2)$$

Thus,

Balanced phase voltages are equal in magnitude and are out of phase with each other by 120°

Since the three-phase voltages are out of phase with each other, there are two possible combinations. One possibility is shown in Fig.5(a) and expressed mathematically as

$$\begin{aligned} \mathbf{V}_{an} &= V_p\angle 0^\circ \\ \mathbf{V}_{bn} &= V_p\angle -120^\circ \\ \mathbf{V}_{cn} &= V_p\angle -240^\circ = V_p\angle +120^\circ \end{aligned} \quad (3)$$

where V_P is the effective or rms value of the phase voltages. This is known as the ***abc*** sequence or positive sequence. In this phase sequence, V_{an} leads V_{bn} which in turn leads V_{cn}. This sequence is produced when the rotor rotates counterclockwise. The other possibility is shown in Fig. 5(b) and is given by

$$\mathbf{V}_{an} = V_p\angle 0^\circ$$
$$\mathbf{V}_{cn} = V_p\angle -120^\circ$$
$$\mathbf{V}_{bn} = V_p\angle -240^\circ = V_p\angle +120^\circ \qquad (4)$$

This is called the acb sequence or negative sequence. For this phase sequence, V_{an} leads V_{cn} which in turn leads V_{bn}. The ***acb*** sequence is produced when the rotor rotates in the clockwise direction.

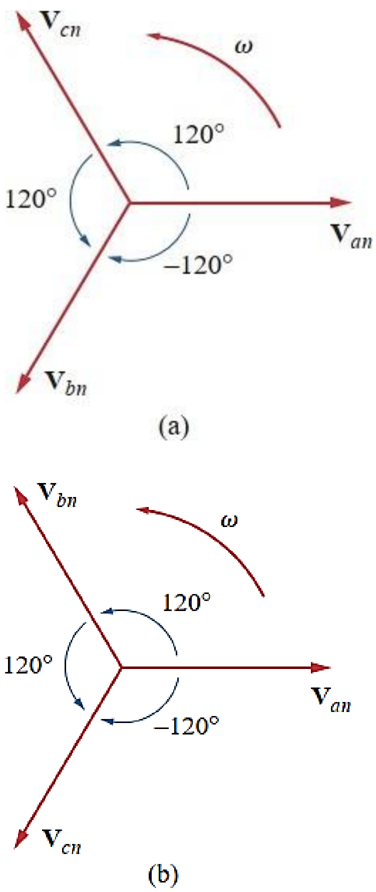


Fig.5 Phase sequences: (a) ***abc*** or positive sequence, (b) ***acb*** or negative sequence.

It is easy to show that the voltages in Eqs. (3) or (4) satisfy Eqs. (1) and (2). For example, from Eq. (3),

$$\begin{aligned}\mathbf{V}_{an} + \mathbf{V}_{bn} + \mathbf{V}_{cn} &= V_p\underline{/0^\circ} + V_p\underline{/-120^\circ} + V_p\underline{/+120^\circ} \\ &= V_p(1.0 - 0.5 - j0.866 - 0.5 + j0.866) \\ &= 0\end{aligned} \tag{5}$$

The **phase sequence** is the time order in which the voltages pass through their respective maximum values.

A three-phase load can be either **wye-connected** or **delta-connected**, depending on the end application.

Fig.6(a) shows a wye-connected load, and Fig. 6(b) shows a delta-connected load.

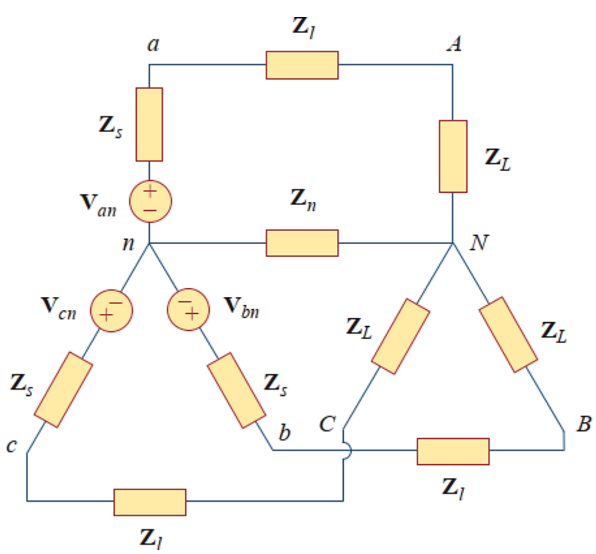


Fig.7 A balanced Y-Y system, showing the source, line, and load impedances.

As illustrated in Fig.7,

z_S denotes the internal impedance of the phase winding of the generator;

z_ℓ is the impedance of the line joining a phase of the source with a phase of the load;

z_L is the impedance of each phase of the load;

z_n is the impedance of the neutral

line. Thus, in general

$$\mathbf{Z}_Y = \mathbf{Z}_s + \mathbf{Z}_\ell + \mathbf{Z}_L \qquad (9)$$

z_S and z_ℓ are often very small compared with z_L , so can assume $z_Y = z_L$ *if no source or line impedance given.*

The Y-Y system in Fig.7 can be simplified to that shown in Fig.8.

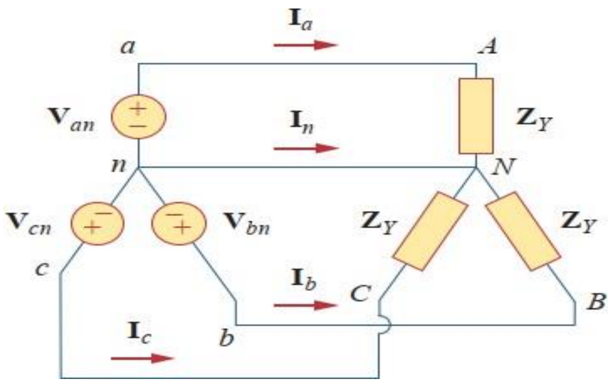


Fig.8 Balanced Y-Y connection.

Assuming the positive sequence, the phase voltages (or line-to-neutral voltages) are

$$\mathbf{V}_{an} = V_p\angle 0^\circ \qquad (10)$$
$$\mathbf{V}_{bn} = V_p\angle -120^\circ, \qquad \mathbf{V}_{cn} = V_p\angle +120^\circ$$

The line-to-line voltages or simply line voltages V_{ab}, V_{bc} and V_{ca} are related to the phase voltages. For example,

$$\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} = \mathbf{V}_{an} - \mathbf{V}_{bn} = V_p\angle 0^\circ - V_p\angle -120^\circ$$
$$= V_p\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}V_p\angle 30^\circ \qquad \text{(11-a)}$$

Similarly we can obtain

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}V_p\angle -90^\circ \qquad \text{(11.b)}$$
$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}V_p\angle -210^\circ \qquad \text{(11.c)}$$

Thus, the magnitude of the line voltages V_L is √3 times the magnitude of the phase voltages V_p, or

$$V_L = \sqrt{3}V_p \tag{12}$$

where

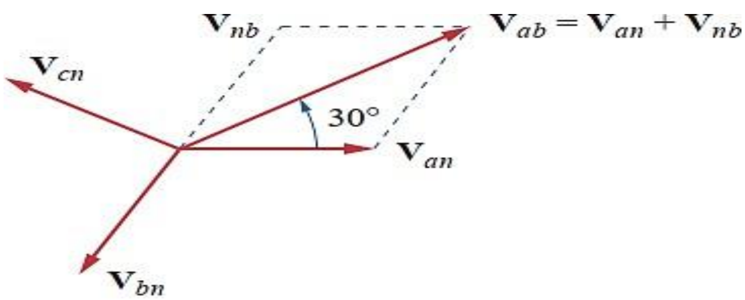
$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}| \tag{13}$$

and

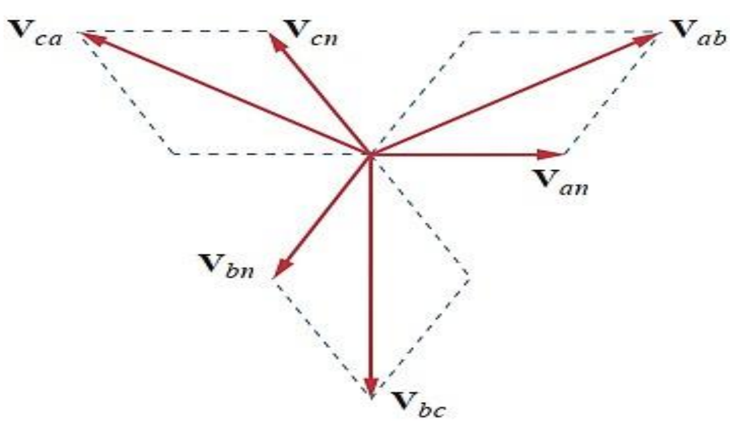
$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}| \tag{14}$$

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$

Also the line voltages lead their corresponding phase voltages by **30^o.**



(a)



(b)

Email (البريد الالكتروني للتدريسي)

Fig.9 Phasor diagrams illustrating the relationship between line voltages and phase voltages.

Figure 9(a) shows how to determine V_{ab} from the phase voltages, while Fig. 9(b) shows the same for the three line voltages. Notice that V_{ab} leads V_{bc} by **120^o** and V_{bc} leads V_{ca} by **120^o** so that the line voltages **sum up to zero as** do the phase voltages.

Applying KVL to each phase, we obtain the line currents as:

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}, \qquad \mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle{-120^\circ}}{\mathbf{Z}_Y} = \mathbf{I}_a\angle{-120^\circ} \tag{15}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\angle{-240^\circ}}{\mathbf{Z}_Y} = \mathbf{I}_a\angle{-240^\circ}$$

We can readily infer that the line currents add up to zero,

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0 \tag{16}$$

so that

$$\mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0 \tag{17-a}$$

or

$$\mathbf{V}_{nN} = \mathbf{Z}_n\mathbf{I}_n = 0 \tag{17-b}$$

that is, the voltage across the neutral wire is zero. The neutral line can thus be *removed without affecting the system.*

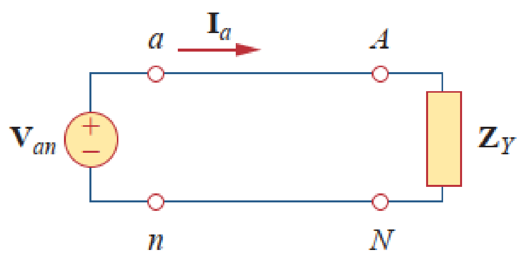


Fig.10 A single-phase equivalent circuit

The single-phase analysis from fig.10 yields the line current $\mathbf{I_a}$ as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y} \tag{18}$$

From $\mathbf{I_a}$ we use the phase sequence to obtain other line currents. Thus, as long as the system is balanced, we need only analyze one phase. We may do this even if the neutral line is absent, as in the three-wire system.

Example 1:

Calculate the line currents in the three-wire Y-Y system of Fig.11

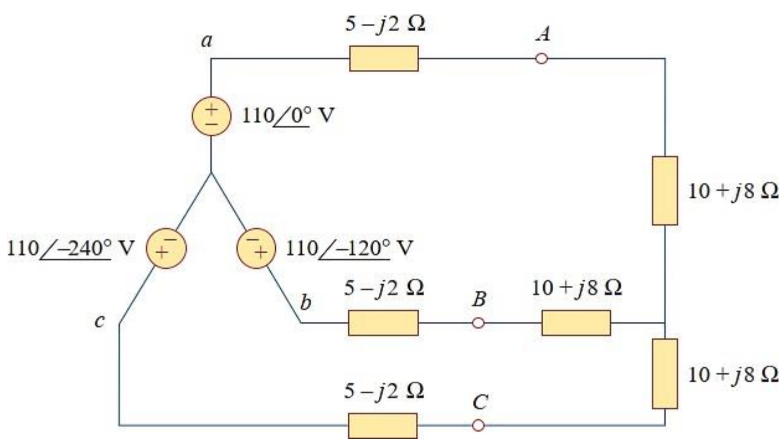


Fig. 11

Solution:

The three-phase circuit in Fig.11 is balanced; we may replace it with its single-phase equivalent circuit such as in Fig.12 . We obtain $\mathbf{I_a}$ from the single-phase analysis as

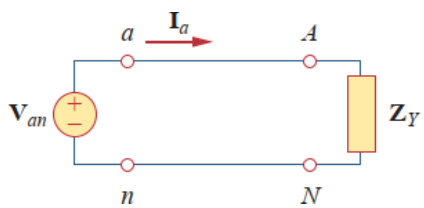


Fig.12

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

where $\mathbf{Z}_Y = (5 - j2) + (10 + j8) = 15 + j6 = 16.155\angle 21.8^\circ$. Hence,

$$\mathbf{I}_a = \frac{110\angle 0^\circ}{16.155\angle 21.8^\circ} = 6.81\angle -21.8^\circ \text{ A}$$

$$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ = 6.81\angle -141.8^\circ \text{ A}$$

$$\mathbf{I}_c = \mathbf{I}_a\angle -240^\circ = 6.81\angle -261.8^\circ \text{ A} = 6.81\angle 98.2^\circ \text{ A}$$

B. Balanced Delta-Delta Connection

A **balanced Δ – Δ** system is one in which both the balanced source and balanced load are Δ -connected.

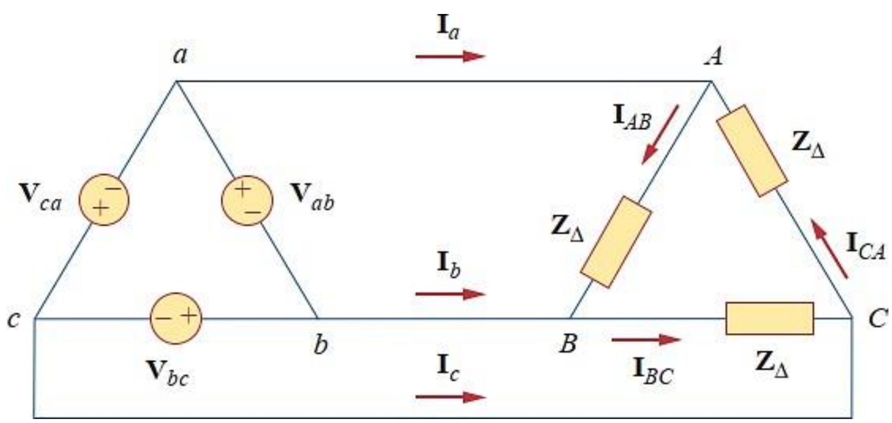


Fig.13 a balanced $\Delta - \Delta$ connection

Assuming a positive sequence, the phase voltages for a delta-connected source are

$$\mathbf{V}_{ab} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bc} = V_p\angle -120^\circ, \qquad \mathbf{V}_{ca} = V_p\angle +120^\circ$$

The line voltages are the same as the phase voltages. From Fig.1, assuming there is no line impedances, the phase voltages of the delta-connected source are equal to the voltages across the impedances; that is,

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA}$$

Hence, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta} = \frac{\mathbf{V}_{ab}}{Z_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{Z_\Delta} = \frac{\mathbf{V}_{bc}}{Z_\Delta}$$
$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{Z_\Delta} = \frac{\mathbf{V}_{ca}}{Z_\Delta}$$

Since the load is delta-connected just as in the previous section, some of the formulas derived there apply here. The line currents are obtained from the phase

currents by
at nodes A, B, and C, as we did in the previous section:

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC}$$

Also, as shown in the last section, each line current lags the corresponding phase current by 30^o; the magnitude I_L of the line current is $\sqrt{3}$ times the magnitude I_P of the phase current,

$$I_L = \sqrt{3} I_p$$

Example 2:

A balanced Δ-connected load having an impedance (20-j15) is connected to a Δ-connected, positive-sequence generator having $V_{ab}=330\angle 0^o$V. Calculate the phase currents of the load and the line currents.

The load impedance per phase is

$$\mathbf{Z}_\Delta = 20 - j15 = 25\angle{-36.87^\circ}\ \Omega$$

Since $\mathbf{V}_{AB} = \mathbf{V}_{ab}$, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta} = \frac{330\angle 0^\circ}{25\angle{-36.87}} = 13.2\angle 36.87^\circ \text{ A}$$
$$\mathbf{I}_{BC} = \mathbf{I}_{AB}\angle{-120^\circ} = 13.2\angle{-83.13^\circ} \text{ A}$$
$$\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle{+120^\circ} = 13.2\angle 156.87^\circ \text{ A}$$

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude $\sqrt{3}$ times that of the phase current. Hence, the line currents are

$$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle{-30^\circ} = (13.2\angle 36.87^\circ)(\sqrt{3}\angle{-30^\circ})$$
$$= 22.86\angle 6.87^\circ \text{ A}$$
$$\mathbf{I}_b = \mathbf{I}_a\angle{-120^\circ} = 22.86\angle{-113.13^\circ} \text{ A}$$
$$\mathbf{I}_c = \mathbf{I}_a\angle{+120^\circ} = 22.86\angle 126.87^\circ \text{ A}$$