Passive Filters

The concept of filters has been an integral part of the evolution of electrical engineering from the beginning. Several technological achievements would not have been possible without electrical filters. Because of this prominent role of filters, much effort has been expended on the theory, design, and construction of filters and many articles and books have been written on them. Our discussion in this chapter should be considered introductory.

A **filter** is a circuit that is designed to pass signals with desired frequencies and reject or attenuate others.

As a frequency-selective device, a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies. Filters are the circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment. A filter is a passive filter if it consists of only passive elements **R, L, and C**. It is said to be an active filter if it consists of active elements (such as transistors and op amps) in addition to passive elements R, L, and C.

LC filters have been used in practical applications for more than eight decades. LC filter technology feeds related areas such as equalizers, impedance-matching networks, transformers, shaping networks, power dividers, attenuators, and directional couplers, and is continuously providing practicing engineers with opportunities to innovate and experiment. Besides the LC filters we study in these sections, there are other kinds of filters—such as digital filters, electromechanical filters, and microwave filters—which are beyond the level of this text .

Type of filters

1. Lowpass Filter

A typical lowpass filter is formed when the output of an RC circuit is taken off the capacitor as shown in Fig. 1

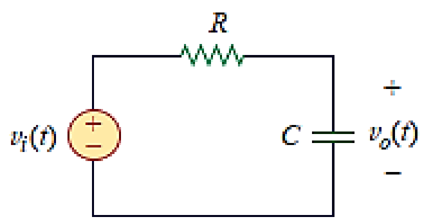

Fig. 1 low pass filter

The transfer function is:

(1)

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{1/j\omega C}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

Note that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 0$. Figure 2 shows the plot of $|H(\omega)|$, along with the ideal characteristic. The half-power frequency, which is equivalent to the corner frequency on the Bode plots but in the context of filters is usually known as the *cutoff frequency* ω_c, is obtained by setting the magnitude of $\mathbf{H}(\omega)$ equal to $1/\sqrt{2}$, thus,

$$H(\omega_c) = \frac{1}{\sqrt{1 + \omega_c^2 R^2 C^2}} = \frac{1}{\sqrt{2}}$$

Or

$$\omega_c = \frac{1}{RC} \tag{2}$$

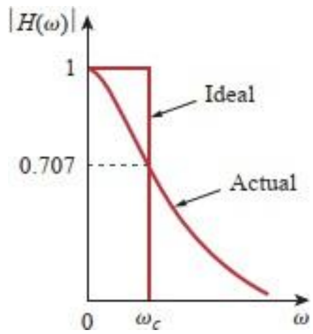

Fig.2 Ideal and actual frequency response of a lowpass filter.

The ***cutoff* frequency** is the frequency at which the transfer function H *drops* in magnitude to *70.71%* of its maximum value. It is also regarded as the frequency at which the power dissipated in a circuit is half of its maximum value.

The cutoff frequency is also called the rolloff frequency.

A lowpass filter is designed to pass only frequencies from dc up to the cutoff frequency ω_c.

A lowpass filter can also be formed when the output of an RL circuit is taken off the resistor. Of course, there are many other circuits for lowpass filters

2. High pass Filter

A highpass filter is formed when the output of an RC circuit is taken off the resistor as shown in Fig. 3

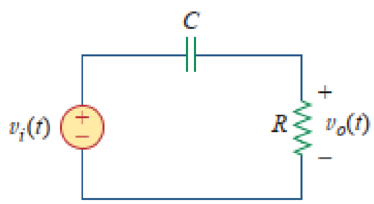

Fig. 3 Highpass filter

The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + 1/j\omega C}$$

$$\mathbf{H}(\omega) = \frac{j\omega RC}{1 + j\omega RC} \qquad (3)$$

Note that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 1$. Figure .3 shows the plot of $|H(\omega)|$. Again, the corner or cutoff frequency is

$$\omega_c = \frac{1}{RC} \qquad (4)$$

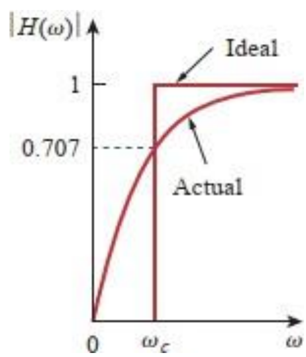

Fig.3 Ideal and actual frequency response of a highpass filter.

A highpass filter is designed to pass all frequencies above its cutoff frequency ω_c.

A highpass filter can also be formed when the output of an RL circuit is taken off the inductor.

3.Bandpass Filter

The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Fig. 4.

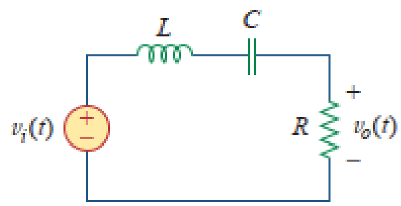

Fig. 4 bandpass filter

The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)} \qquad (5)$$

We observe that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 0$. Figure 6 shows the plot of $|H(\omega)|$. The bandpass filter passes a band of frequencies ($\omega_1 < \omega < \omega_2$) centered on ω_0, the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (6)$$

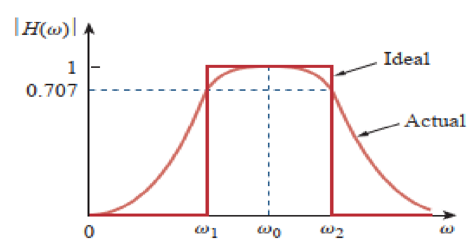

Fig.6 Ideal and actual frequency response of a bandpass filter.

A bandpass filter is designed to pass all frequencies within a band of frequencies, $\omega_1 < \omega_0 < \omega_2$.

4.Bandstop Filter

A filter that prevents a band of frequencies between two designated values($\omega_1 and\ \omega_2$) from passing is variably known as a bandstop, bandreject, or notch filter. A bandstop filter is formed when the output RLC series resonant circuit is taken off the LC series combination as shown in Fig. 7. The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)} \quad (7)$$

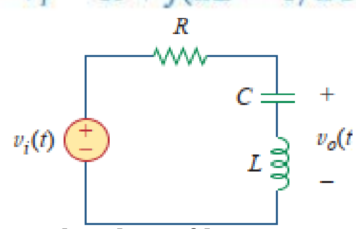

Fig.7 bandstop filter

Notice that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 1$. Figure 8 shows the plot of $|H(\omega)|$. Again, the center frequency is given by

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad (8)$$

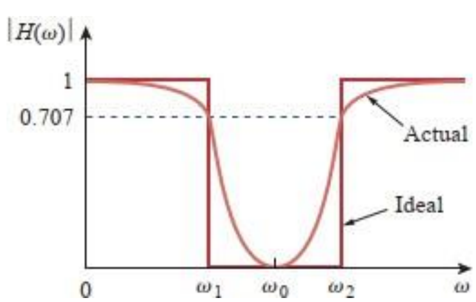

Fig.8 Ideal and actual frequency response of a bandstop filter.

Here, $\omega_0$is called the frequency of rejection, while the corresponding bandwidth (B= $\omega_2 - \omega_1$)is known as the bandwidth of rejection. Thus,

A bandstop filter is designed to stop or eliminate all frequencies within a band of frequencies, $\omega_1 < \omega_0 < \omega_2$.

Example 1:

Determine what type of filter is shown in Fig. 9. Calculate the corner or cutoff frequency. Take $R = 2k\Omega$,$L = 2\ H\ and\ C = 2\mu F$

Solution:

Since $\mathbf{H}(0) = 1$ and $\mathbf{H}(\infty) = 0$, the circuit is second order lowpass filter, the magnitude pf H is:

$$H = \frac{R}{\sqrt{(R - \omega^2 RLC)^2 + \omega^2 L^2}} \qquad \text{(b)}$$

The corner frequency is the same as the half-power frequency, i.e., where $\mathbf{H}$ is reduced by a factor of $1/\sqrt{2}$. Since the dc value of $H(\omega)$ is 1, at the corner frequency, Eq. (b) becomes after squaring

$$H^2 = \frac{1}{2} = \frac{R^2}{(R - \omega_c^2 RLC)^2 + \omega_c^2 L^2}$$

or

$$2 = (1 - \omega_c^2 LC)^2 + \left(\frac{\omega_c L}{R}\right)^2$$

Substituting the values of R, L, and C, we obtain

$$2 = (1 - \omega_c^2\, 4 \times 10^{-6})^2 + (\omega_c 10^{-3})^2$$

Assuming that ω_c is in krad/s,

$$2 = (1 - 4\omega_c^2)^2 + \omega_c^2 \qquad \text{or} \qquad 16\omega_c^4 - 7\omega_c^2 - 1 = 0$$

Solving the quadratic equation in ω_c^2, we get $\omega_c^2 = 0.5509$ and -0.1134. Since ω_c is real,

$$\omega_c = 0.742 \text{ krad/s} = 742 \text{ rad/s}$$