Finding Partial Derivative the Easy Way

Since a partial derivative with respect to x is a derivative with the rest of the variables held constant, we can find the partial derivative by taking the regular derivative considering the rest of the variables as constants.

Example:

Let $f(x, y) = 3xy^2 - 2x^2y$ Find:

(1) f_x **(2)** f_y.

Solution:

(1) $f_x = 3y^2 - 4xy.$

(2) $f_y = 6xy - 2x^2.$

Partial Derivative:

Let $z = f(x, y)$ be a function of two variables then we define

The partial derivative as:

The partial derivative of $f(x, y)$ with respect to x is

$$M = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

تسمى العلاقة اعلاة بالمشتقة الاولى ويرمز لها بالرمز $f'(x)$, $\frac{dy}{dx}$, $\frac{df(x)}{dx}$, y'

- نقول ان f(x) دالة قابلة للاشتقاق على الفترة (a , b) اذا كانت f قابلة للاشتقاق عند كل نقطة من نقط (a , b) .
- عندما تكون الغاية في العلاقة السابقة موجودة فأن الدالة f(x) تسمى قابلة للاشتقاق وان $f'(x)$ تسمى مشتقة الدالة الاولى .

Example 1: Using the definition of the derivative to find the derivative of the function $f(x) = 4x - 2$.

Solution: $y' = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{4(x+\Delta x)-2-(4x-2)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{4x+4\Delta x-2-4x+2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(4\Delta x)}{\Delta x} = 4$$

Example 2: Using the definition of the derivative to find the derivative of the function $f(x) = x^2 + 1$.

Solution: $y' = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 + 1 - (x^2+1)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 + 1 - x^2 - 1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta x(2x+\Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

Example 3: Using the definition of the derivative to find the derivative of the function $f(x) = \sqrt{x}$.

Solution: $y' = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sqrt{x+\Delta x} - \sqrt{x}}{\Delta x} \ \frac{\sqrt{x+\Delta x} + \sqrt{x}}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x) - x}{\Delta x(\sqrt{x+\Delta x} + \sqrt{x})} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}$$

Exercises 9 : Using the definition of the derivative to find the derivative of the following functions

1) $f(x) = Ax + B$
2) $f(x) = x^3$
3) $f(x) = \frac{1}{x}$
4) $f(x) = \sqrt[3]{x}$