Chapter 6

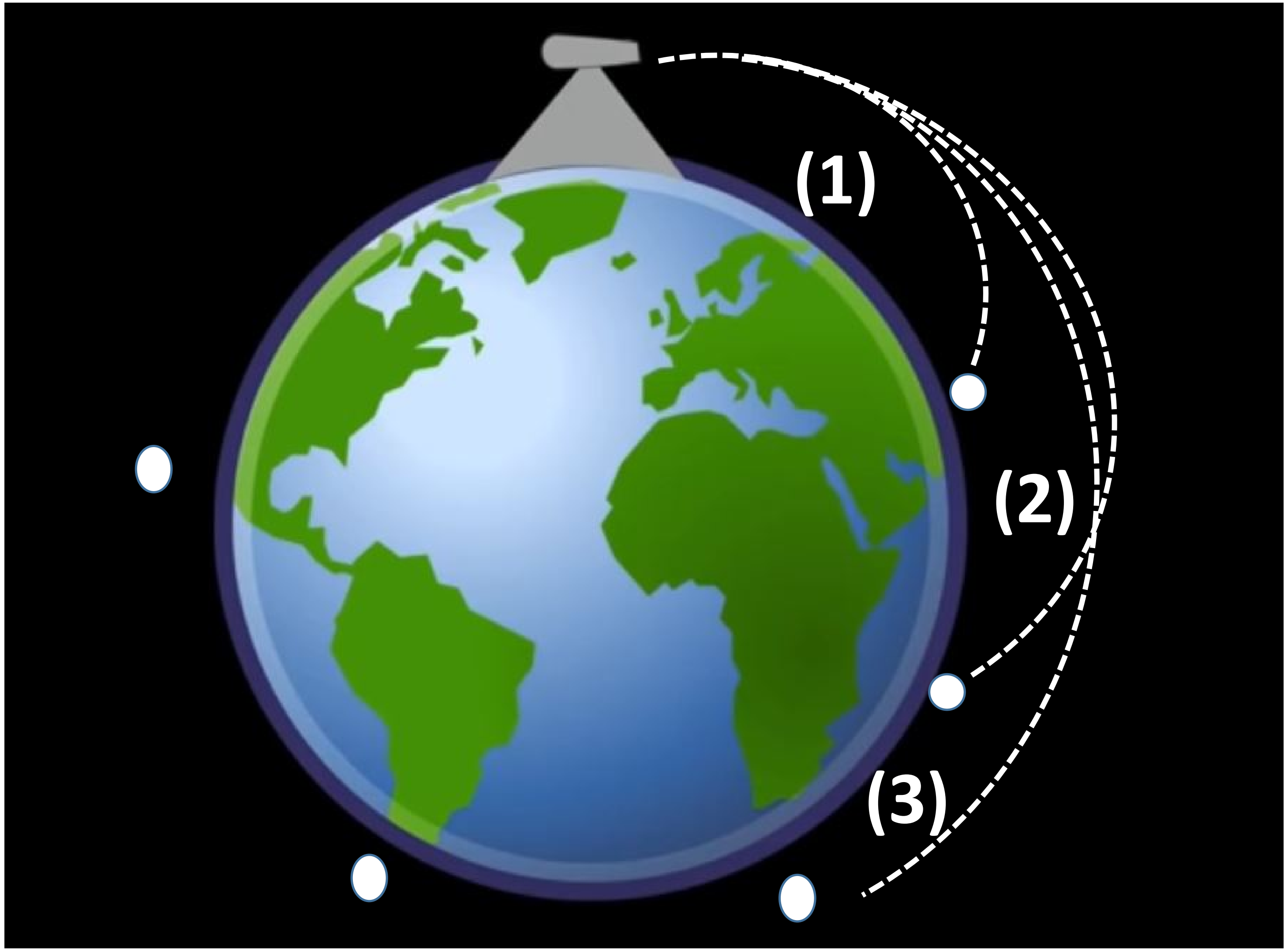
Gravitational and Central Force

6.1 Newton's Law of Universal Gravitation:

((*Every particle in the universe attracts every other particle with a force whose magnitude is proportional to the product of the masses of the two particles and inversely proportional to the square of the distance between them. The direction of the force lies along the straight line connecting the two particles.*))

$$F_{ij} = G\frac{m_i m_j}{r_{ij}^2}\left(\frac{r_{ij}}{r_{ij}}\right)$$

$$G = (6.67259 \pm 0.00085 \times 10^{-11}\ \frac{Nm^2}{kg^2}$$



$$F_{ij} = G\frac{m_i m_j}{r_{ij}^2}\left(\frac{r_{ij}}{r_{ij}}\right)$$

- F_{ij} is the force on particle i of mass m_i exerted by particle j of mass m_j.

- The vector r_{ij} is the directed line segment running from particle i to particle j,

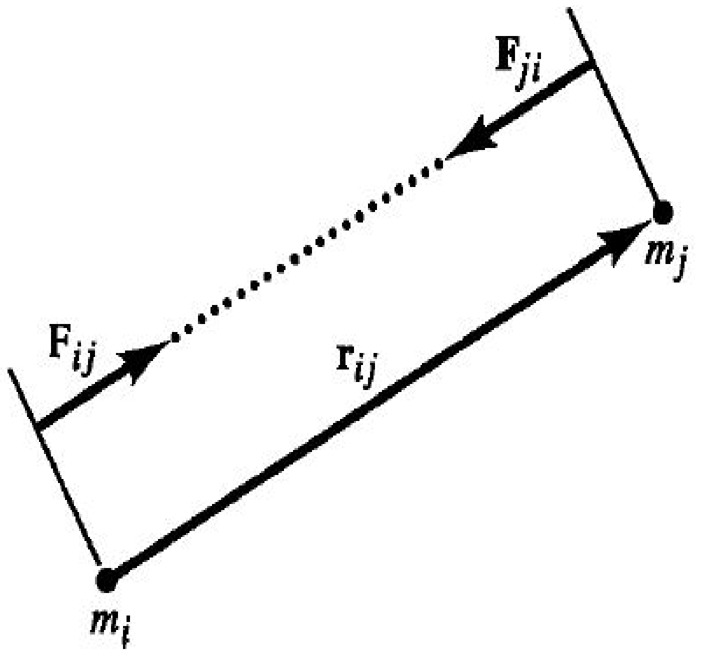
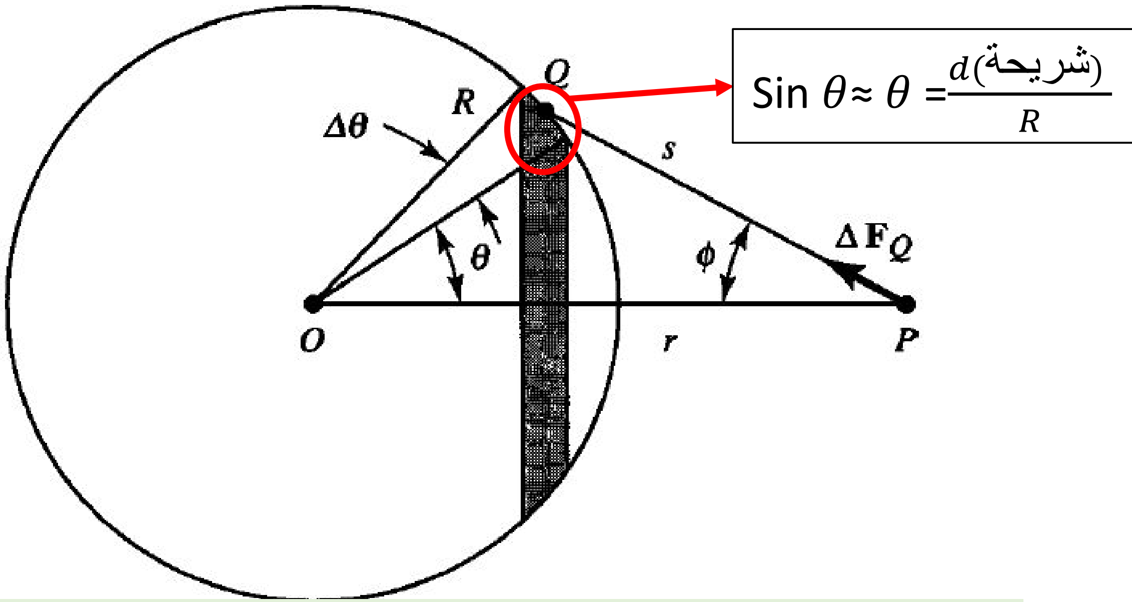


Figure 6.1.1 Action and reaction in Newton's law of gravity.

6.2 Gravitational Force between a Uniform Sphere and a Particle:

Consider first a thin uniform shell of mass M and radius R. Let r be the distance from the centre O to a test particle P of mass m (Fig. 6.2.1). We assume that r > R. We **shall divide the shell into circular rings** of width $R\,\Delta\,\theta$. Where, as shown in the figure, the angle

- The angle *POQ* is denoted by θ, *Q* being a point on the ring.
- Where S is the distance PQ (the distance from the particle P to the ring) as shown in above Figure.
- we can write the force between a shell and the particle as :

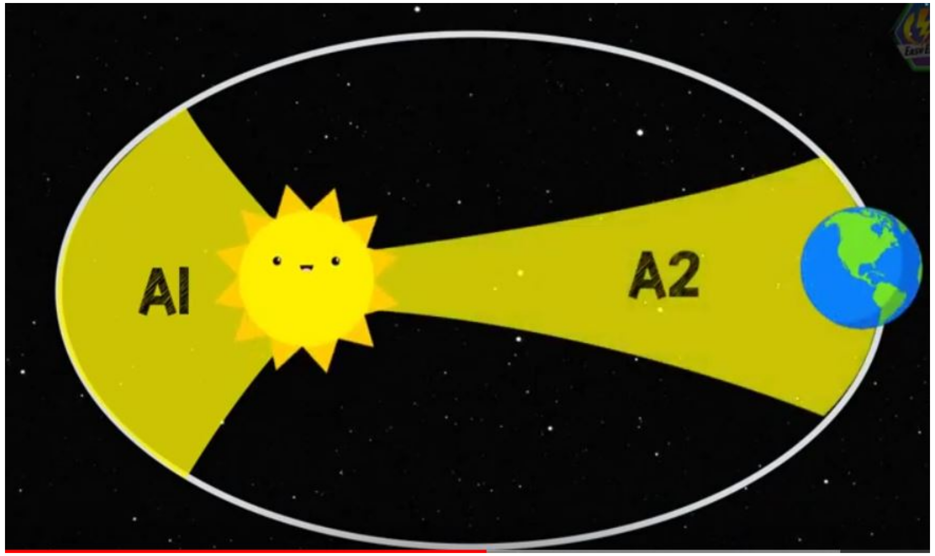


$$F = -G\,\frac{Mm}{r^2}\,e_r$$

e_r : is the radial vector from origin O.

- The gravitational force on a particle located inside a uniform spherical shell is zero.

6.3 Kepler's Laws of Planetary Motion:



I. Law of Ellipses (1609)

The orbit of each planet is an ellipse, with the Sun located at one of its foci (البؤرة)

II. Law of Equal Areas (1609)

A line drawn between the Sun and the planet sweeps out equal areas in equal times as the planet orbits the Sun.

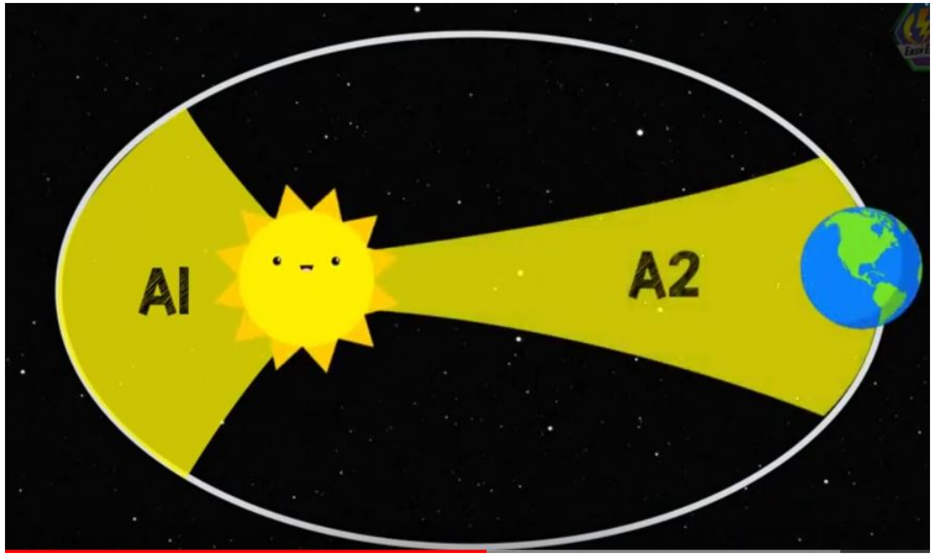
III. Harmonic Law (1618)

The square of the **sidereal period** الفترة الفلكية of a planet (the time it takes a planet to complete one revolution about the Sun relative to the stars) is directly proportional to the cube of the semi-major axis of the planet's orbit.

6.4 Kepler's Second Law: Equal Areas:

L = r x p

$$\frac{dL}{dt} = \frac{d(r \times p)}{dt} = \frac{dr}{dt} \times p + r \times \frac{dp}{dt}$$



- But $\frac{dr}{dt} = v$, so the first term in right became $v \times p = v \times mv = m(v \times v) = m(vv\, sin\theta) = 0 \quad as\ \theta = 0$

- And $\frac{dp}{dt} = F$ from 2nd law of **Newton,**

$$\frac{dL}{dt} = r \times F \qquad (6.4)$$

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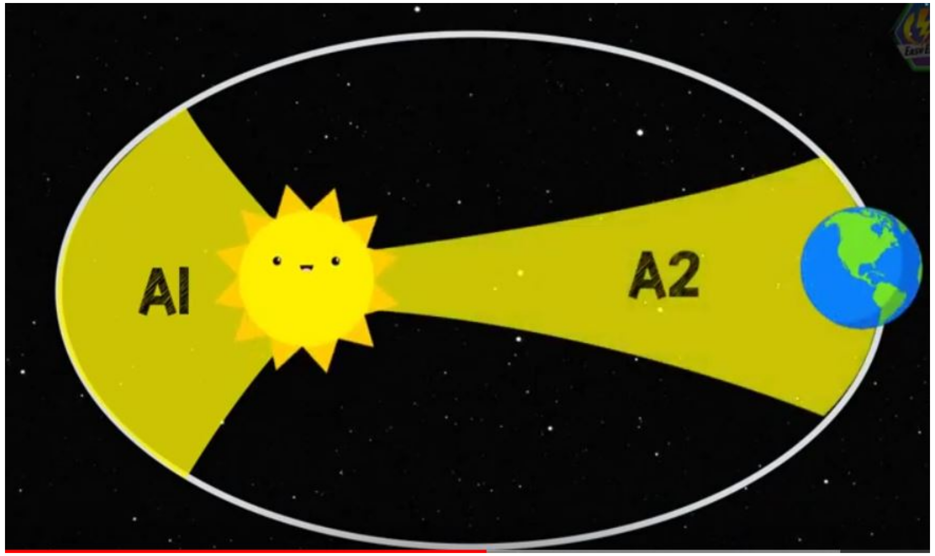

- $N = r\ x\ F$: **moment of force**, or **torque**, on the particle about the origin of the coordinate system.

- If r and F are collinear, this cross product vanishes and so does L(i.e. dL/dt=0), so , **the angular momentum L, in such cases, is a constant of the motion.**