جامـــــعة المـــــستـقبـل
AL MUSTAQBAL UNIVERSITY

قســـم الامـــــن الـــــــــسيبرانـــــــي

Department of Cyber Security

Subject:

Recursion and Recurrence Relations

Class:

First

Lecturer:

Asst. lect. Mustafa Ameer Awadh

Lecture: (9)

Lecture Objectives:
By the end of this lecture, students will be able to:

- Understand recursion and how it relates to algorithm design.
- Define recurrence relations and use them to describe recursive algorithms.
- Solve simple recurrence relations using iterative and mathematical methods.
- Apply recurrence relations to time complexity analysis.

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1. What is Recursion?

Definition:
Recursion is a technique where a function calls itself to solve smaller instances of a problem.
Example: Factorial function
csharp
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factorial(n) = n * factorial(n - 1), with base case factorial(0) = 1

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2. Recursive Algorithms

Common examples:

- Factorial
- Fibonacci numbers
- Binary Search
- Merge Sort

Why Use Recursion?

- Simplifies code
- Natural fit for divide-and-conquer problems
- Can mirror mathematical definitions (e.g., tree traversals)

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3. What are Recurrence Relations?

A recurrence relation is an equation that defines a sequence based on previous terms.
Example:
For recursive Fibonacci:
r
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T(n) = T(n - 1) + T(n - 2), with T(0) = 0, T(1) = 1

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4. Solving Recurrence Relations

Iterative Method:

Keep expanding until a pattern is found.

Example:

r

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T(n) = T(n - 1) + 1, with T(1) = 1

T(n) = T(n-1) + 1
= T(n-2) + 2
= T(n-3) + 3
...
= T(1) + (n - 1)
= 1 + (n - 1) = n

Therefore: T(n) = O(n)

Master Theorem:

Used for divide-and-conquer algorithms.

For recurrences of the form:

bash

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T(n) = aT(n/b) + f(n)

Use Master Theorem to determine Big-O.

Example:

Merge Sort:

r

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T(n) = 2T(n/2) + O(n) ⟶ O(n log n)

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5. Applications in Algorithms

Algorithm	Recurrence Relation	Time Complexity
Binary Search	$T(n) = T(n/2) + O(1)$	$O(\log n)$
Merge Sort	$T(n) = 2T(n/2) + O(n)$	$O(n \log n)$
Fibonacci	$T(n) = T(n - 1) + T(n - 2)$	$O(2^n)$ (inefficient)

6. Key Concepts Recap

- Recursion is a method of solving problems using repeated self-calls.
- Recurrence relations model the time or value produced by recursive solutions.
- Solving recurrence relations helps in determining algorithm efficiency.
- Master Theorem is a powerful tool for divide-and-conquer algorithms.